



Probing low energy neutrino backgrounds with neutrino capture on beta decaying nuclei

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Abstract: We study the interaction of low energy neutrinos on nuclei that spontaneously undergo beta decay showing that the product of the cross section times neutrino velocity takes values as high as $10^{-42} \text{ cm}^2 c$ for some specific nuclei that decay via allowed transitions. The absence of energy threshold and the value of the cross section single out these processes towards the direct detection of very low energy neutrino backgrounds such as the cosmological relic neutrinos.

Introduction

The interaction of an electron (anti)neutrino with a nucleus N naturally undergoing beta (positron) decay to the daughter nucleus N'

$$\bar{\nu}_e^{(-)} + N \rightarrow N' + e^{\pm} \quad (1)$$

shows the remarkable property of having no energy threshold on the value of the incoming neutrino energy. Indeed, in these cases the energy balance of the corresponding beta decay reactions is such that $M(N) - M(N') = Q_\beta > 0$ with $M(N)$, $M(N')$ the mass of neutral atoms. Neutrino interaction of this type is thus always energetically allowed, no matter the value of the incoming neutrino energy E_ν . This reaction will be denoted in the following as Neutrino Capture on Beta decaying nuclei (NCB). In the limit of vanishing values of neutrino mass m_ν and E_ν the neutrino contributes to NCB uniquely via its lepton flavor quantum number and in this case the electron in the final state has exactly the beta decay endpoint energy Q_β . However, for finite m_ν the electron kinetic energy is $Q_\beta + E_\nu \geq Q_\beta + m_\nu$, while electrons emerging from the analogous beta decay has at most an energy $Q_\beta - m_\nu$, neglecting nucleus recoil energy. A minimum gap of $2m_\nu$ is thus present and this at least in principle allows to distinguish between beta decay and NCB interaction.

In neutrino physics, NCB represents to date the only known reaction able to unambiguously detect electron (anti)neutrino having arbitrary low energies. The idea of using NCB to measure the cosmological relic neutrino background predicted in the framework of the Hot Big Bang model was already advocated many years ago in [9] even though the original motivations have been ruled out. However, as we already mentioned, in case of massive neutrinos a gap around Q_β is expected of the order of twice the neutrino mass. Presently, neutrino mass in the eV range is still allowed by data. Indeed, oscillation experiments only provide a lower limit to the mass of (at least) one neutrino mass eigenstate of the order of 0.05 eV [6], while direct measurements of electron energy spectrum in ^3H decay gives $m_\nu < 2$ eV [3, 5]. As we will argue in the following, if m_ν is in the eV range, future NCB experiments could represent an almost unique way to detect cosmological neutrinos.

Neutrino cross section on β^\pm decaying nuclei

NCB and its corresponding beta decay are essentially the same phenomenon. In the case of unobserved polarization the two processes have the same invariant squared amplitude. In order to de-

rive an expression for the NCB cross section we make use of the beta decay formalism, evaluating invariant amplitudes using the description of the fermions in terms of spherical waves and the computation of the corresponding nuclear state transition matrix elements.

Following the formalism of [1], for spin averaged initial state and unobserved polarization we have that for an incoming neutrino having momentum p_ν (v_ν being the modulus of neutrino velocity)

$$\sigma_{\text{NCB}} v_\nu = \frac{G_\beta^2}{\pi} p_e E_e F(Z, E_e) C(E_e, p_\nu)_\nu, \quad (2)$$

where $F(Z, E_e)$ is the Fermi function and $E_e = E_\nu + Q_\beta + m_e = E_\nu + m_\nu + W_o$ with W_o the corresponding beta decay endpoint; $C(E_e, p_\nu)_\nu$ is the nuclear shape factor, an angular momentum weighted average of nuclear state transition amplitudes, which depends upon the nuclear properties of the parent and daughter nuclei, thus involving the calculation of nuclear matrix elements. Its general expression can be found in [1]. NCB rate is strongly related to the corresponding beta decay process, whose rate is given by the well known expression in terms of the beta decay shape factor $C(E_e, p_\nu)_\beta$. It is worth notice here that a simple relation holds between the beta decay and the NCB shape factors $C(E_e, p_\nu)_\nu = C(E_e, -p_\nu)_\beta$ though both variables have different kinematical domains in the two processes.

The beta decay rate can be expressed using the relation

$$\lambda_\beta^{-1} = 2\pi^3 / (G_\beta^2 f \bar{C}_\beta) \quad (3)$$

where the integrated Fermi function (f) and the mean shape factor (\bar{C}_β) are evaluated using the known value of W_o . We therefore obtain

$$\sigma_{\text{NCB}} v_\nu = \frac{2\pi^2 \ln 2}{\mathcal{A} \cdot t_{1/2}}, \quad (4)$$

where the quantity \mathcal{A} is given by

$$\mathcal{A} = \frac{f \bar{C}_\beta}{p_e E_e F(Z, E_e) C(E_e, p_\nu)_\nu}, \quad (5)$$

where the two factors $C(E_e, p_\nu)_\nu$ and \bar{C}_β depend upon the same nuclear transition matrix elements. As we will see, in some relevant cases the evaluation of \mathcal{A} is particularly simple so that Eq. (4) can

be computed in an exact way. In all cases where this is not possible, systematic uncertainties affecting the nuclear matrix element evaluation are expected to largely cancel in the shape factor ratio appearing in \mathcal{A} .

In case of superallowed transitions the NCB cross section evaluation is particularly simple. Since the decay occurs inside the same isospin multiplet ($J^\pi \rightarrow J^\pi$) both the vector and axial form factors can contribute to the decay; the shape factor is thus a function of the vector and axial form factors and can be evaluated using the initial and final state spin and isospin wavefunctions. However, a precise calculation requires the evaluation of higher order correction to nuclear matrix elements. On the other hand, we notice that since the form factors do not depend on E_e we can write $\mathcal{A} = f/p_e E_e F(Z, E_e)$ and we get for NCB processes

$$\sigma_{\text{NCB}} v_\nu = 2\pi^2 \ln 2 \frac{p_e E_e F(Z, E_e)}{f t_{1/2}}, \quad (6)$$

which only depends on the half-life and the Q_β of the corresponding beta decay.

NCB cross section for nuclei that undergo allowed beta decay can be evaluated only using a first order approximation since the beta decay shape factor depends also on higher order terms. If only the leading terms are taken into account we have that $C(E_e, p_\nu)_\beta = C(E_e, p_\nu)_\nu$ and from (2) one derives an order of magnitude estimate for the NCB interaction cross section. The effect of the higher order terms is however, not negligible and more precise estimates require nuclear matrix element evaluation for each specific transition. Nevertheless, we expect that at a higher level of accuracy with respect to the leading approximation as above can be obtained using (4) and the fact that for the shape factor *ratio* is reasonable to assume $C(E_e, p_\nu)_\beta / C(E_e, p_\nu)_\nu \simeq 1$. Within this approximation \mathcal{A} is obtained using expression used for superallowed decays.

In case of K-unique forbidden NCB takes a simple form since there is only one nuclear form factor involved in the shape factor. Defining the following functions $u_i(p_e, p_\nu)$ for the i -forbidden unique decays

$$u_1 = p_\nu^2 + \lambda_2 p_e^2$$

$$u_2 = p_\nu^4 + \frac{10}{3}\lambda_2 p_\nu^2 p_e^2 + \lambda_3 p_e^4 \quad (7)$$

$$u_3 = p_\nu^6 + 7\lambda_2 p_\nu^4 p_e^2 + 7\lambda_3 p_\nu^2 p_e^4 + \lambda_4 p_e^6$$

the corresponding shape factors are then given by

$$C(E_e, p_\nu)_\beta^i = \frac{R^{2i} |{}^A F_{(i+1)i}^{(0)}|^2}{[(2i+1)!!]^2} u_i(p_e, p_\nu). \quad (8)$$

Since there are only even powers of the neutrino momentum p_ν these expressions are valid both for beta decay and NCB interaction. As can be easily seen the quantity \mathcal{A} does not depend on the form factor.

Estimating the cross section

We have evaluated NCB cross section for several nuclei using parametrization of the Fermi function and of the radial wave function coefficients λ_n in the shape factor as in [1] and [10] to account for finite nuclear size effect. We have performed an extensive calculation of the NCB cross section for all beta decaying transitions listed in the ENSDF database [4]. A total of 14543 decays have been analyzed, 6288 β^- and 8255 electron capture and β^+ . We restricted our attention to both allowed and unique forbidden decays having branching ratios greater than 5%, namely 1272 β^- decays and 799 β^+ decays. A detailed report of NCB cross section evaluation can be found in [2]. As we will discuss in details in the next Section, any use of NCB processes to study low energy neutrino fluxes is crucially related to the issue of rejection of background events represented by the corresponding beta decay process. For a given incident neutrino flux, the ratio of the NCB to decay events is proportional to $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$, so that nuclei with the highest value for this combination might give the best chances if used in a future low energy neutrino detection experiment. We report these nuclei in Table 1.

NCB versus β decay: the case of cosmological relic neutrinos

We now consider a possible application of NCB to detection of the background of cosmological

relic neutrinos. Actually, this represents one of the most ambitious challenges in modern cosmology. Presently, we know that these neutrinos have a number density of order $n_\nu \sim 50 \text{ cm}^{-3}$ neutrino (or antineutrino) per flavor and are characterized by a very small mean kinetic energy, of order 10^{-4} eV. There are two intertwined issues which should be discussed, i.e. the event to background rate and the energy resolution. First of all, though as we stressed several times the NCB process is with no energy threshold, nevertheless, the ratio of NCB event number to corresponding beta decay events is typically very small. Using expression (4) we find that $\lambda_\nu/\lambda_\beta = 2\pi^2 n_\nu/\mathcal{A}$ where we have used the fact that relic neutrinos have a very small mean momentum of order T_ν with a spread of the same order of magnitude, and the fact that the product of NCB cross section times neutrino velocity gets an asymptotic constant value for small neutrino energies. In the case of ${}^3\text{H}$ we get $\lambda_\nu({}^3\text{H}) = 0.66 \cdot 10^{-23} \lambda_\beta({}^3\text{H})$. Despite of that the experimental signature of NCB events is in principle unambiguous since the electron (positron) in the final state has a kinetic energy at least $2m_\nu$ above the beta decay endpoint energy. However, the finite energy resolution of any experimental apparatus and the extremely low cross section make relic neutrino detection via NCB a real challenge due to the large potential amount of background. In an optimistic scenario with comparable values of neutrino masses and experimental energy resolution, the situation could be promising. As an example, we consider a future experiment reaching an energy resolution Δ , and neutrino masses in the eV range. From Eq.3 the ratio of the event rate $\lambda_\beta(\Delta)$ for the last beta decay electron energy bin $W_o - \Delta < E_e < W_o$, compared with the total NCB event rate can be easily calculated, giving

$$\frac{\lambda_\nu}{\lambda_\beta(\Delta)} = \frac{9}{2} \zeta(3) \left(\frac{T_\nu}{\Delta} \right)^3 \frac{1}{(1 + 2m_\nu/\Delta)^{3/2}}, \quad (9)$$

where we have used the fact that $n_\nu = 3\zeta(3)T_\nu^3/(4\pi^2)$ and in the case of $Q_\beta \gg \Delta$. We have checked that this expression is accurate at percent level for nuclei that undergo both allowed and unique-forbidden transitions with endpoint energy in the range $10^{-3} < Q_\beta < 10$ MeV. This gives for example, the value $\lambda_\nu/\lambda_\beta(\Delta) \sim 2.2 \cdot 10^{-10}$ for $\Delta = 0.2$ eV and $m_\nu = 0.5$ eV. An estimate of the

Isotope	Decay	Half-life (seconds)	$\sigma_{\text{NCB}}(v_\nu/c)$ (10^{-41} cm^2)
^3H	β^-	3.8878×10^8	7.84×10^{-4}
^{63}Ni	β^-	3.1588×10^9	1.38×10^{-6}
^{93}Zr	β^-	4.952×10^{13}	2.39×10^{-10}
^{106}Ru	β^-	3.2278×10^7	5.88×10^{-4}
^{107}Pd	β^-	2.0512×10^{14}	2.58×10^{-10}
^{187}Re	β^-	1.3727×10^{18}	4.32×10^{-11}
^{11}C	β^+	1.226×10^3	4.66×10^{-3}
^{13}N	β^+	5.99×10^2	5.3×10^{-3}
^{15}O	β^+	1.224×10^2	9.75×10^{-3}
^{18}F	β^+	6.809×10^3	2.63×10^{-3}
^{22}Na	β^+	9.07×10^7	3.04×10^{-7}
^{45}Ti	β^+	1.307×10^4	3.87×10^{-4}

Table 1: Beta decaying nuclei that present the largest product of $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$ for low neutrino momentum and have a β^\pm decay branching fraction larger than 80%.

signal to background ratio can be easily obtained in case of Gaussian errors on the measured electron energy. A signal to noise ratio of order 3 is for example obtained if $\Delta = 0.2$ eV for $m_\nu = 0.7$ eV, while a smaller neutrino mass of 0.3 eV requires $\Delta = 0.1$ eV. In these cases a total event number of order 10 is needed to get a $5\text{-}\sigma$ *discovery* claim. Presently, this energy resolution seems very hard to get. Nevertheless, if a large neutrino mass will be found by ongoing beta decay experiments such as KATRIN [7], it is not inconceivable that a future generation of experiments might reach energy resolution as low as 0.1 eV. Finally, we estimate the order of magnitude of the mass of detector required to see neutrino events from the cosmological background using NCB. It can be shown that the expected total event rate is given by

$$2.85 \cdot 10^{-2} \frac{\sigma_{\text{NCB}} v_\nu / c}{10^{-45} \text{ cm}^2} \text{ yr}^{-1} \text{ mol}^{-1}. \quad (10)$$

As an interesting example, we consider the case of ^3H . From (10) and using the results of Table 1, we estimate 7.5 events per year of data taking for a mass of 100 g.

Conclusions

In this paper we have reported a careful analysis of neutrino capture on beta decaying nuclei. These

processes have the remarkable property of having no energy threshold on the incoming neutrino energy and thus they might represent a good class of interactions suitable for low energy neutrino detection. Our study has been inspired by the fact that we presently know that neutrinos are massive particles and might have masses m_ν as large as a fraction of eV. Indeed, there is a gap of $2m_\nu$ in the electron or positron energy spectrum separating the (few) events induced by e.g. relic neutrinos, from the large background of standard beta decays. At least in principle, this allows to disentangle the two processes. Of course, any possibility to translate this idea into a real experimental technique depends upon two crucial issues, namely the expected order of magnitude of NCB event rate as well as the required energy resolution on the outgoing electron (positron). We have investigated in detail these issues showing that NCB cross section could give rise to a remarkably large event rate and that a reasonable background rejection is achievable in case detector energy resolution is better than the value of the neutrino mass. We remark here that a further enhancement could be obtained in case of massive neutrino gravitational clustering as described in [8].

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