Muonic Hydrogen Lamb Shift

The Proton Radius Puzzle

A. Antognini

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Collaboration

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Y.-W. Liu	National Tsing Hua University, Hsinchu, Taiwan
A. Dax, P.E. Knowles, L. Ludhova, F. Mulhasuer	former members

Non-motivating motivation

Spectroscopy	Fraunhofer, Amstrong, Balmer, Rydberg (1800-1900)	
	Bohr, Heisenberg, Schrödinger, Born, Pauli (1925)	QM
H_α doublet, $\vec{S}_{e^-}$	Michelson (1891), Goudsmit, Dirac (1930)	α, spin, antimatter
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p/d/He charge radii	Lamb shift in μp/μd/μHe	QED test, R_∞, lattice QCD/few-nucleon th. Discrepancy, New physics?

The role of hydrogen in history of physics

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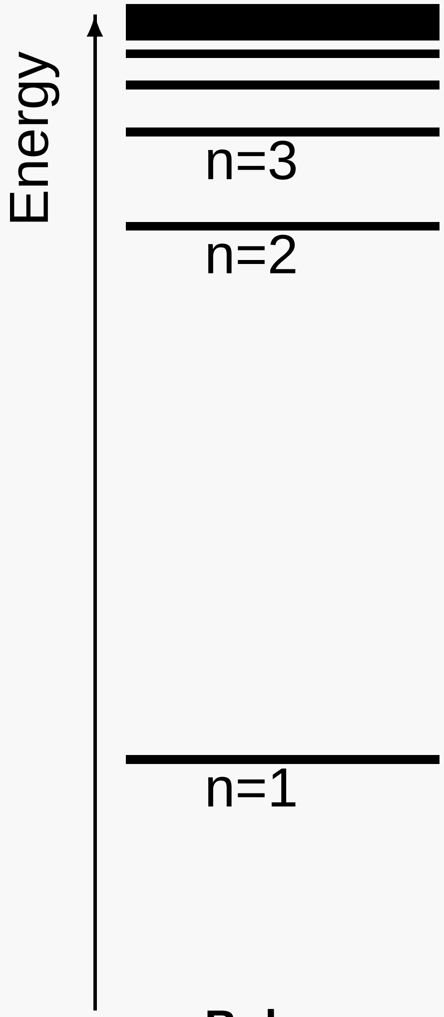
p, d, ^{3,4}He radii	Lamb shift in μp, μd,	QED test, R_∞, lattice QCD/few-nucleon th. 5σ deviation, New physics?

Hydrogen energy levels

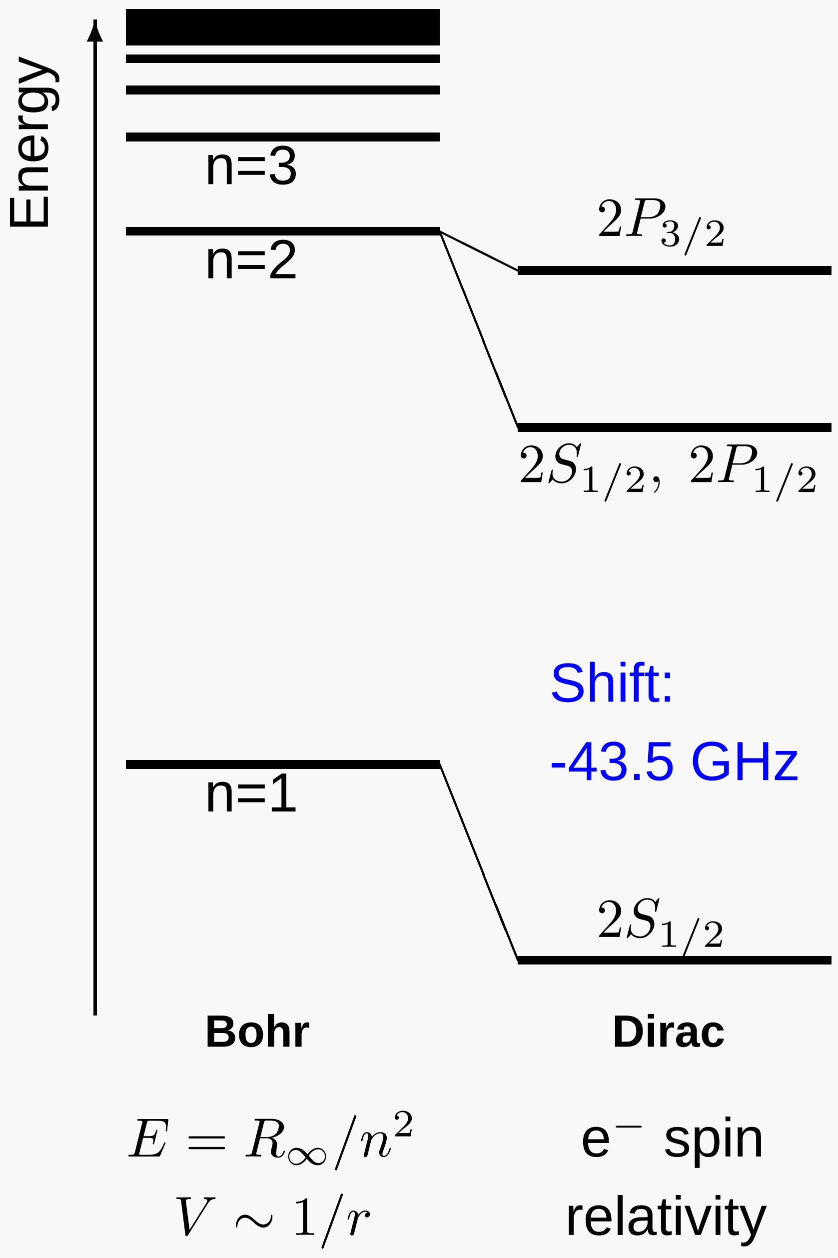


Bohr

$$E = R_\infty / n^2$$

$$V \sim 1/r$$

Hydrogen energy levels



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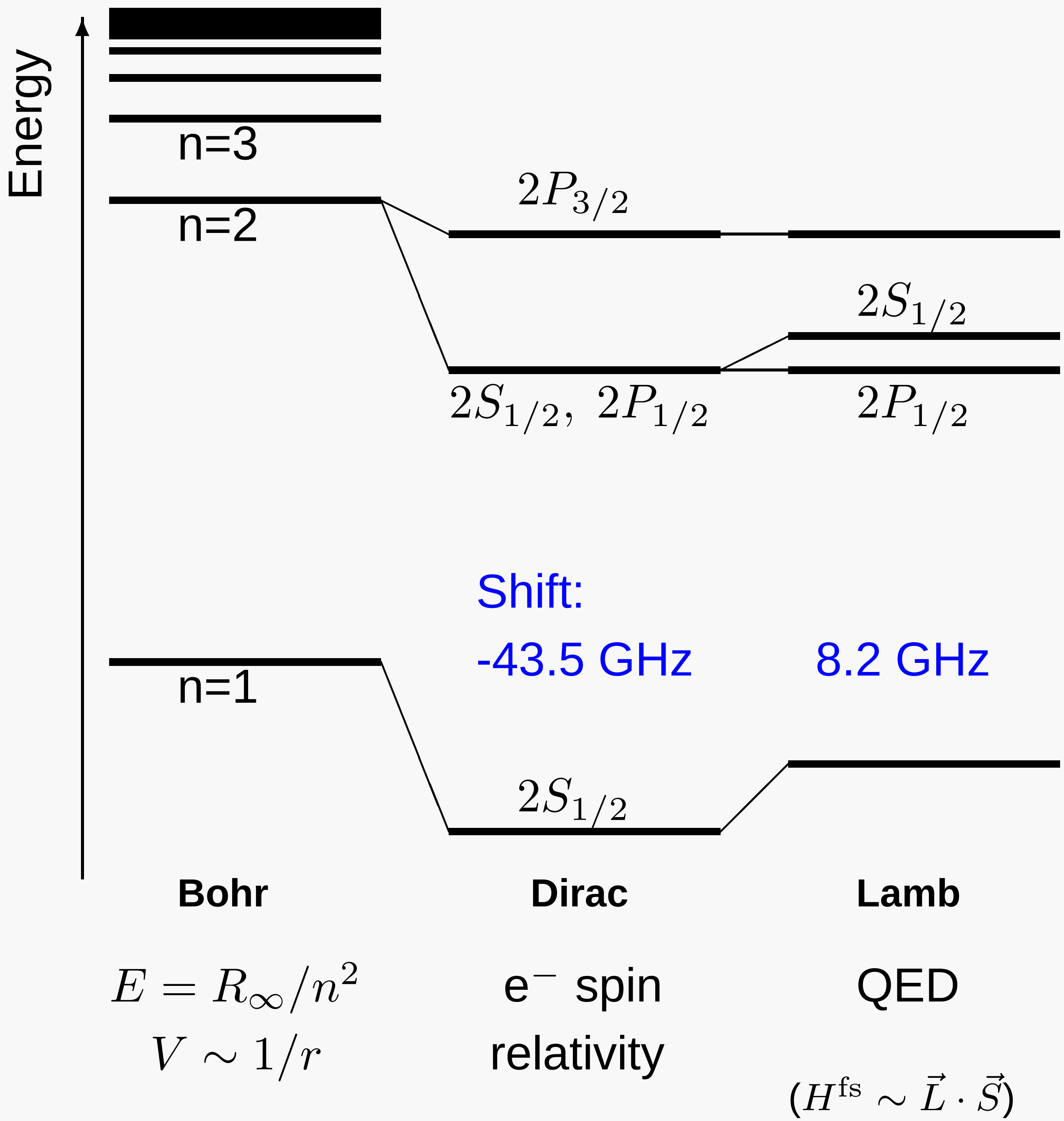

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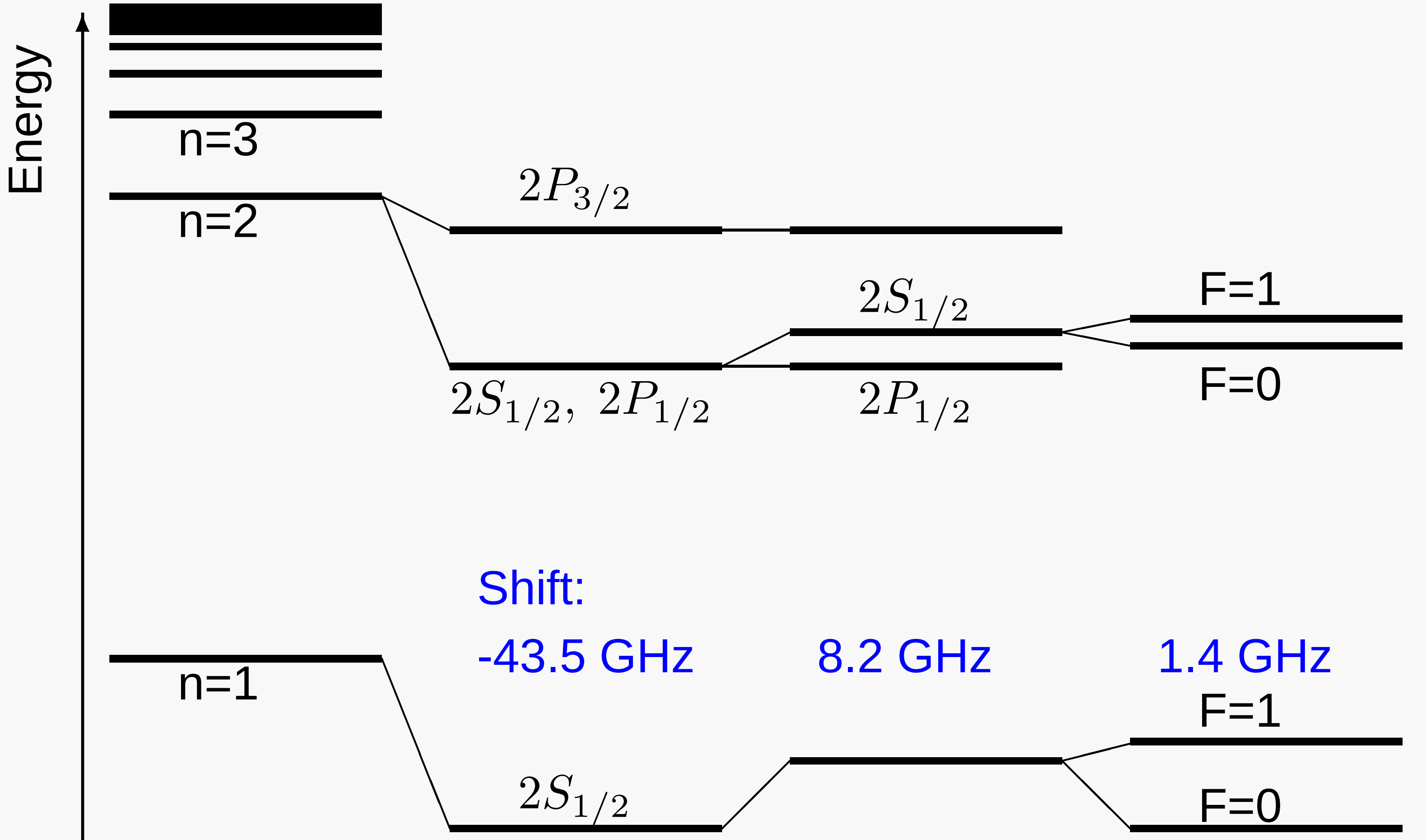

Bohr	Dirac	Lamb	hfs-splitting
$E = R_\infty/n^2$	e^- spin	QED	proton-spin
$V \sim 1/r$	relativity	($H^{\mathrm{fs}} \sim \vec{L} \cdot \vec{S}$)	$H^{\mathrm{hfs}} \sim \vec{\mu}_p \cdot \vec{\mu}_e$

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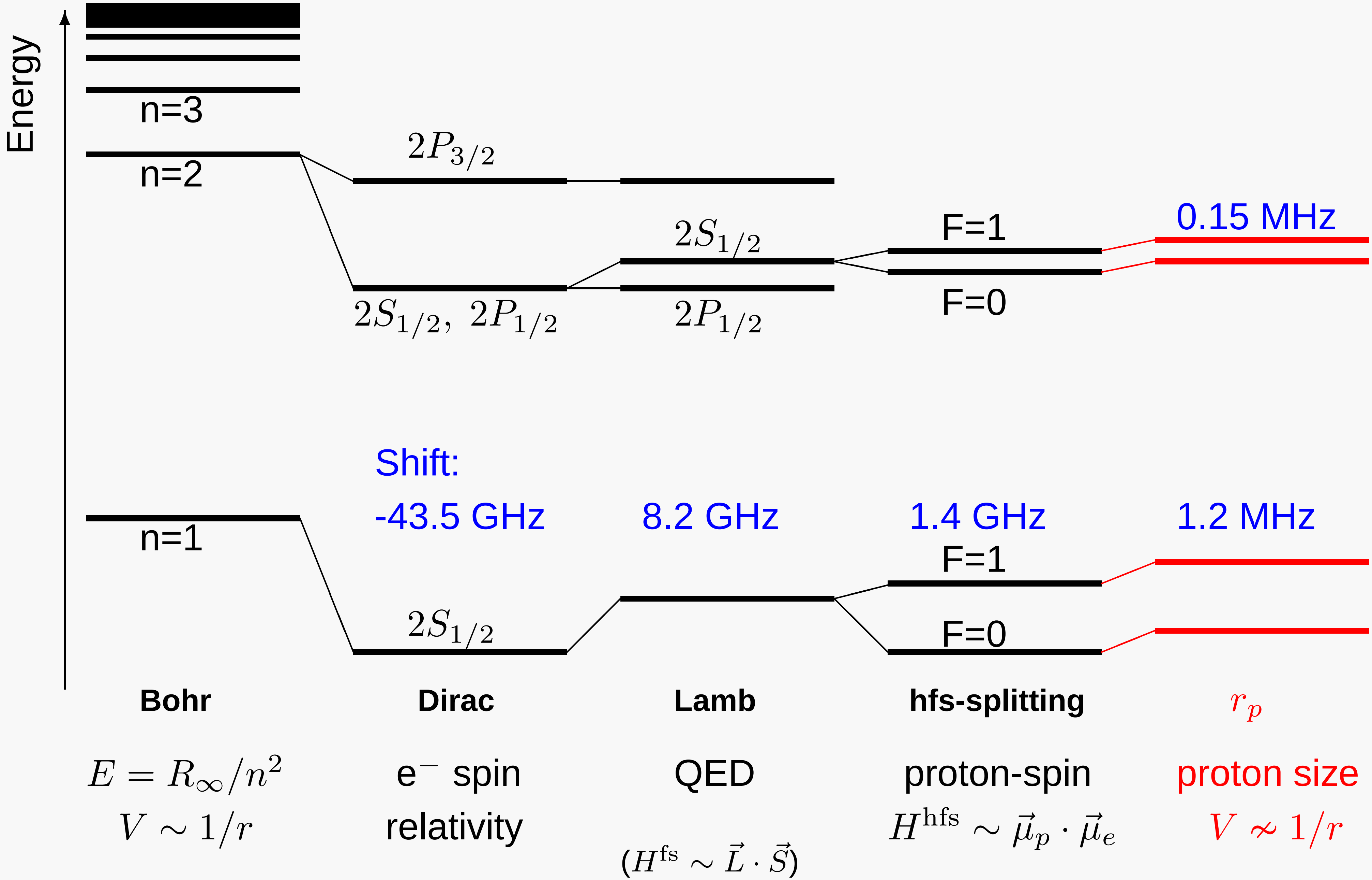

Proton charge radius determinations

- Electron-proton scattering (1963, ... 2003, ...)

  $\rightarrow r_{\mathrm{p}}$ with $u_r = 2\%$

Proton charge radius determinations

- Hydrogen spectroscopy (1989, . . .)
  - very precise measurements: $d\nu/\nu = 1 \times 10^{-14}$ (1S-2S)
  - interpretation of the measurements need $r_{\rm p}$
  - conversely assuming correctness of bound-state QED $\rightarrow r_{\rm p}$
  - however finite size effect small ($u_r \sim 10^{-10}$) $\rightarrow r_{\rm p}$ with $u_r = 1\%$

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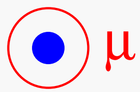
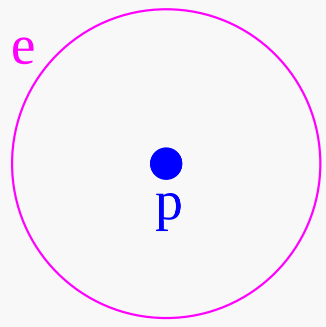
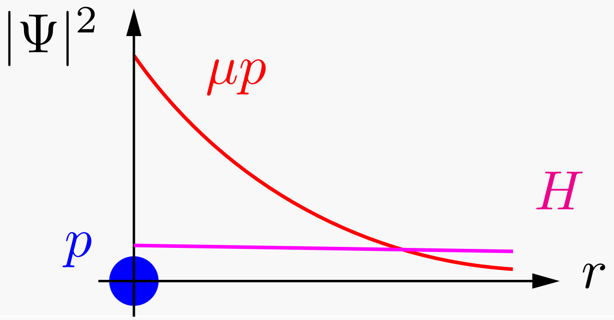
- Muonic hydrogen spectroscopy (2009 . . .)

  $m_\mu/m_e \approx 200 \rightarrow \mu^-$ “orbit” is 200 times smaller than e$^-$ “orbit”

  $\rightarrow$ large finite size effect $\rightarrow r_{\mathrm{p}}$ with $u_r = 0.1\%$

Muonic hydrogen 

Hydrogen 



$$\frac{|\Psi_\mu(0)|^2}{|\Psi_e(0)|^2} = \left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$$

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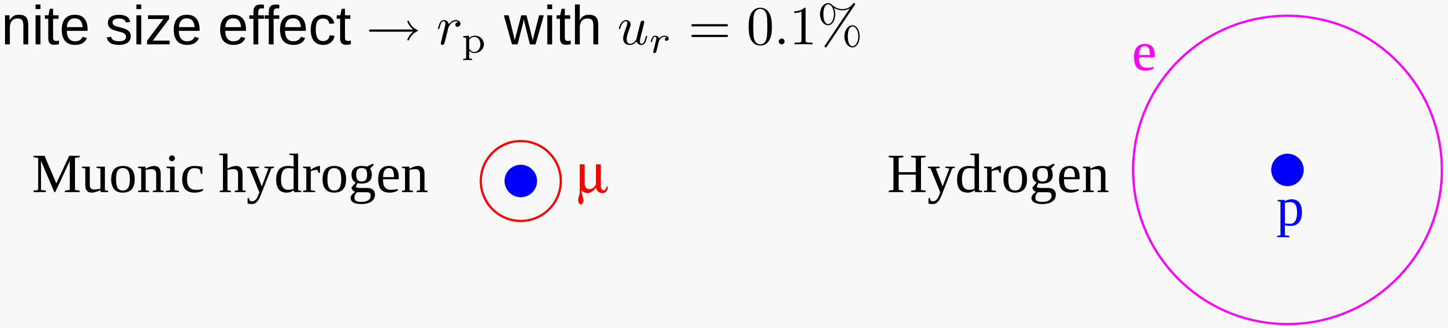


$r_{\rm p}^2 = \int d^3r\, \rho(r) r^2 \quad \longrightarrow \quad r_{\rm p}$ is the rms charge radius

Outline

- Apparatus
- Measured resonances
- Interpretation of results

Aim of the experiment

- Measure the $2S - 2P$ energy difference (Lamb shift) in μp

  $\Delta E(2S - 2P) = 209.978(5) - 5.226\, r_{\mathrm{p}}^2 + 0.0347\, r_{\mathrm{p}}^3$ meV

  with 30 ppm precision.

- Extract r_{p} with $u_r \approx 10^{-3}$ (rel. accuracy)

  $\rightarrow$ bound-state QED test in hydrogen

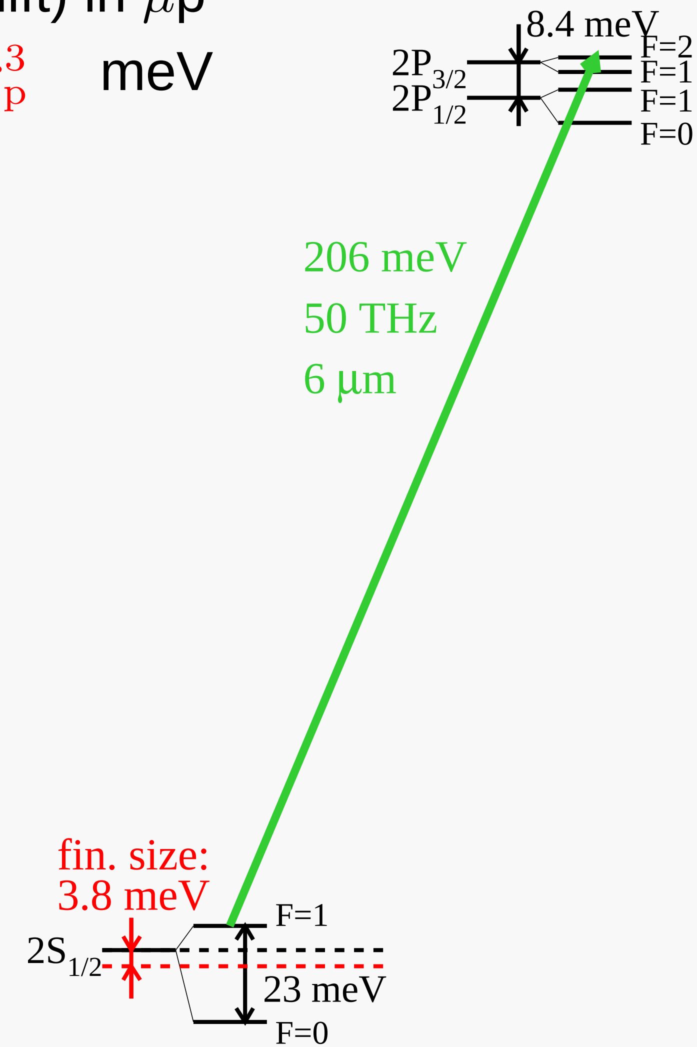
  to a level of $u_r \approx 3 \times 10^{-7}$ $(10\times$ better)

  $\rightarrow$ improve Rydberg constant ($R_\infty = mc\alpha^2/2h$)

  to a level of $u_r \approx 1 \times 10^{-12}$ $(6\times$ better)

  $\rightarrow$ benchmark for lattice QCD calculations

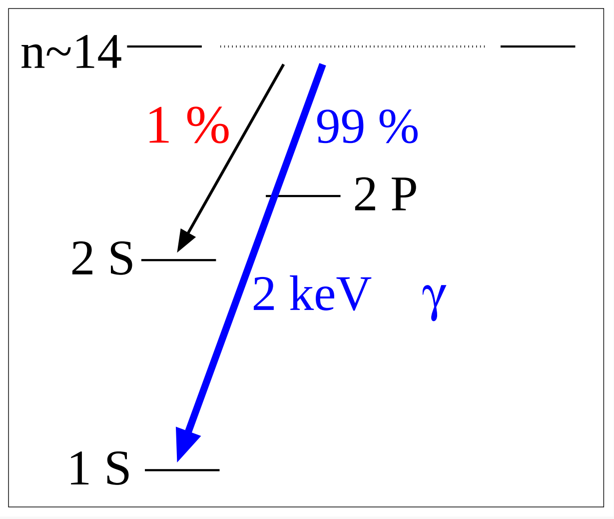
  $\rightarrow$ confront with electron scattering results



Principle of the experiment

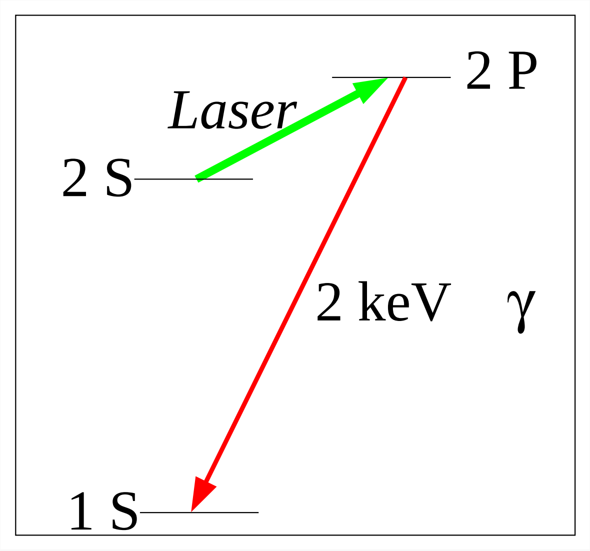
- μ^- are produced with the PSI accelerator (p$\rightarrow \pi^- \rightarrow \mu^-$)
- μ^- stop in a 1 mbar hydrogen target whereby muonic hydrogen is formed
- Before stopping, the μ^- trigger the laser system
- The laser pulse excites the $2S - 2P$ transition
- 2 keV X-ray are detected as a signature of the laser-induced transition

μp formation



1%: long-lived μp($2S$)
with 1 μs lifetime (@ 1 mbar)

μp spectroscopy



Laser at $\lambda \approx 6\ \mu$m
Signature: 2 keV X-ray

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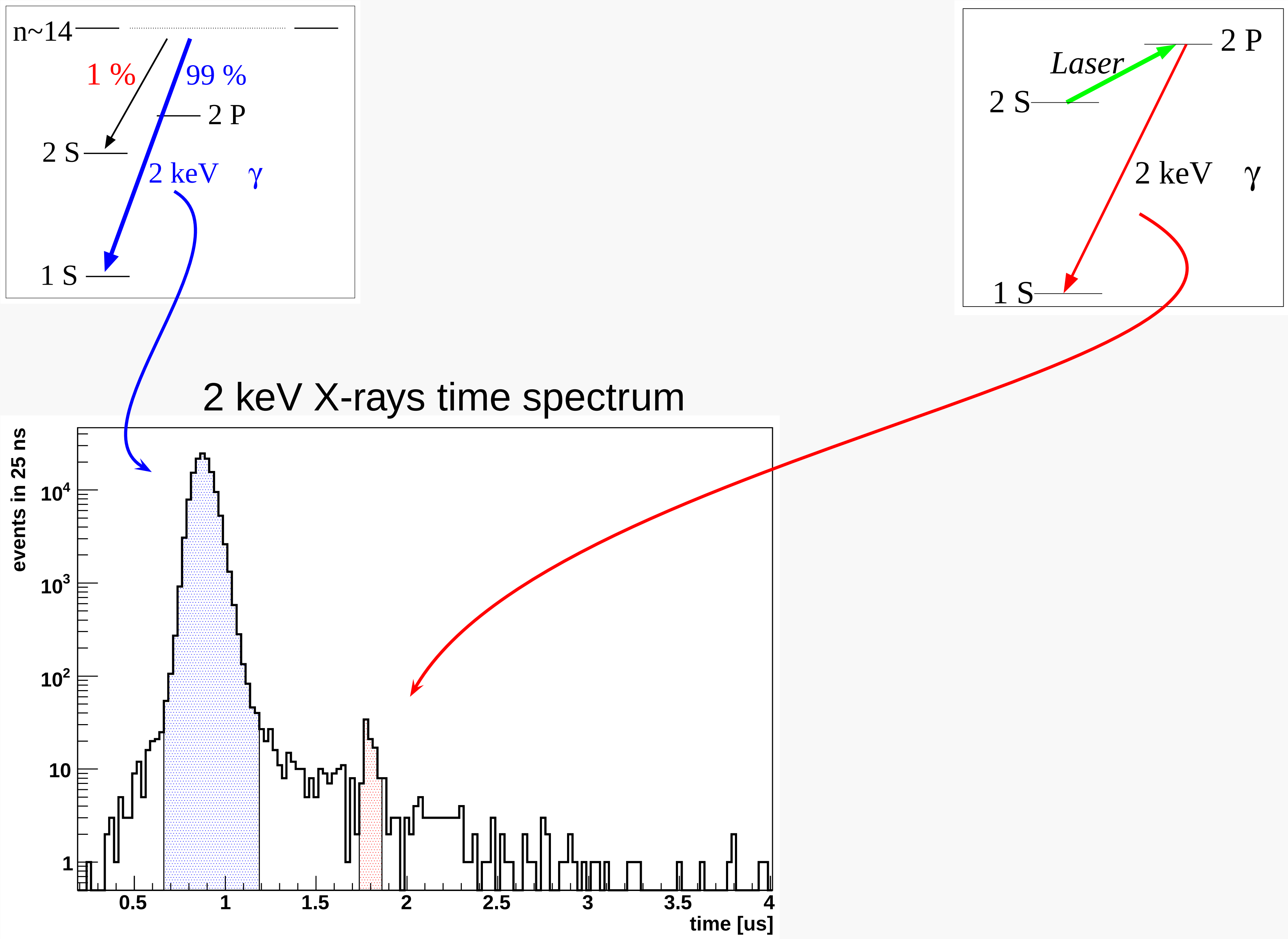

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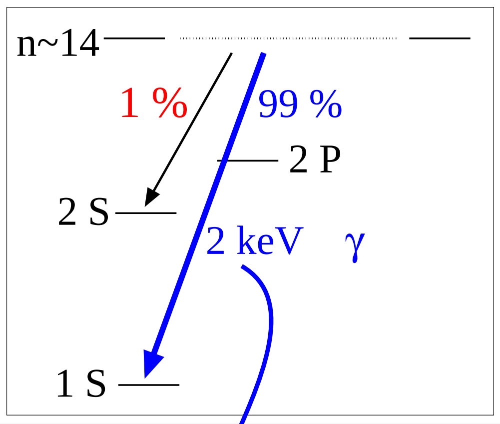
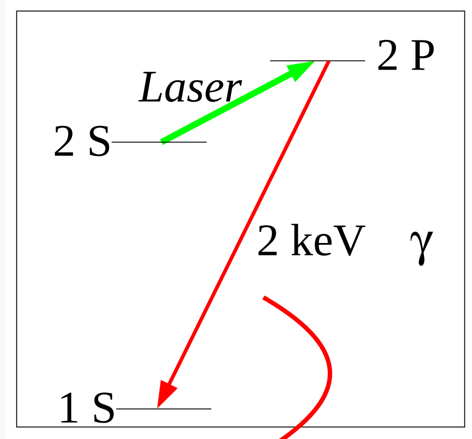
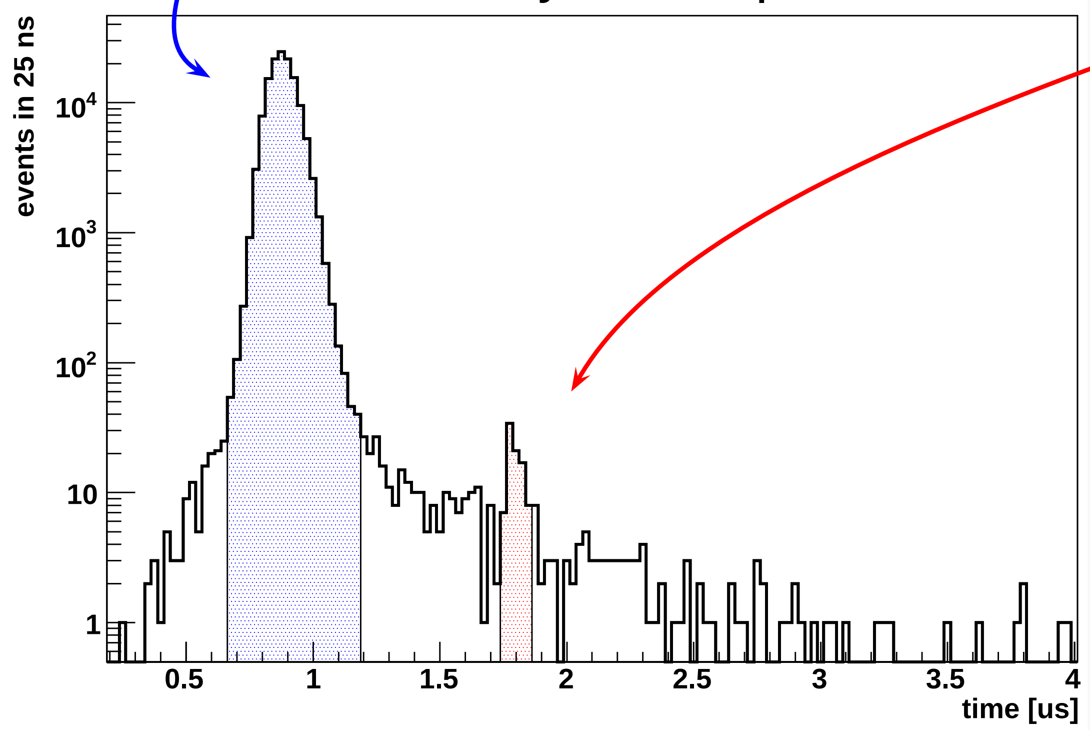
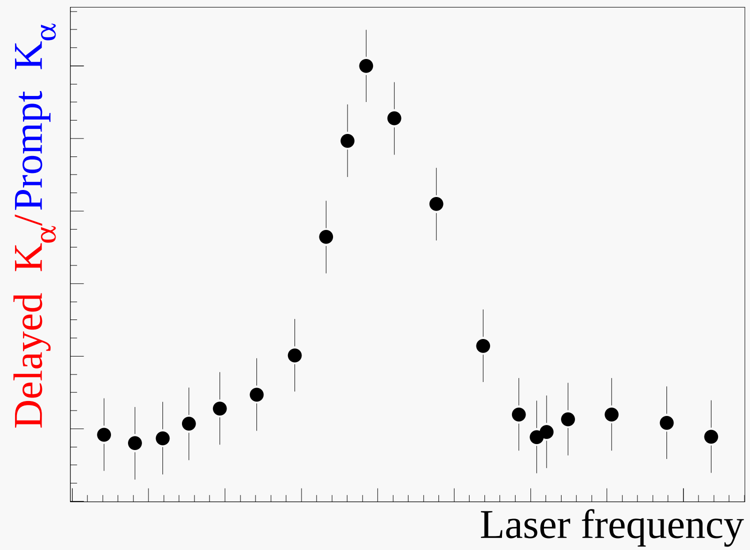




2 keV X-rays time spectrum



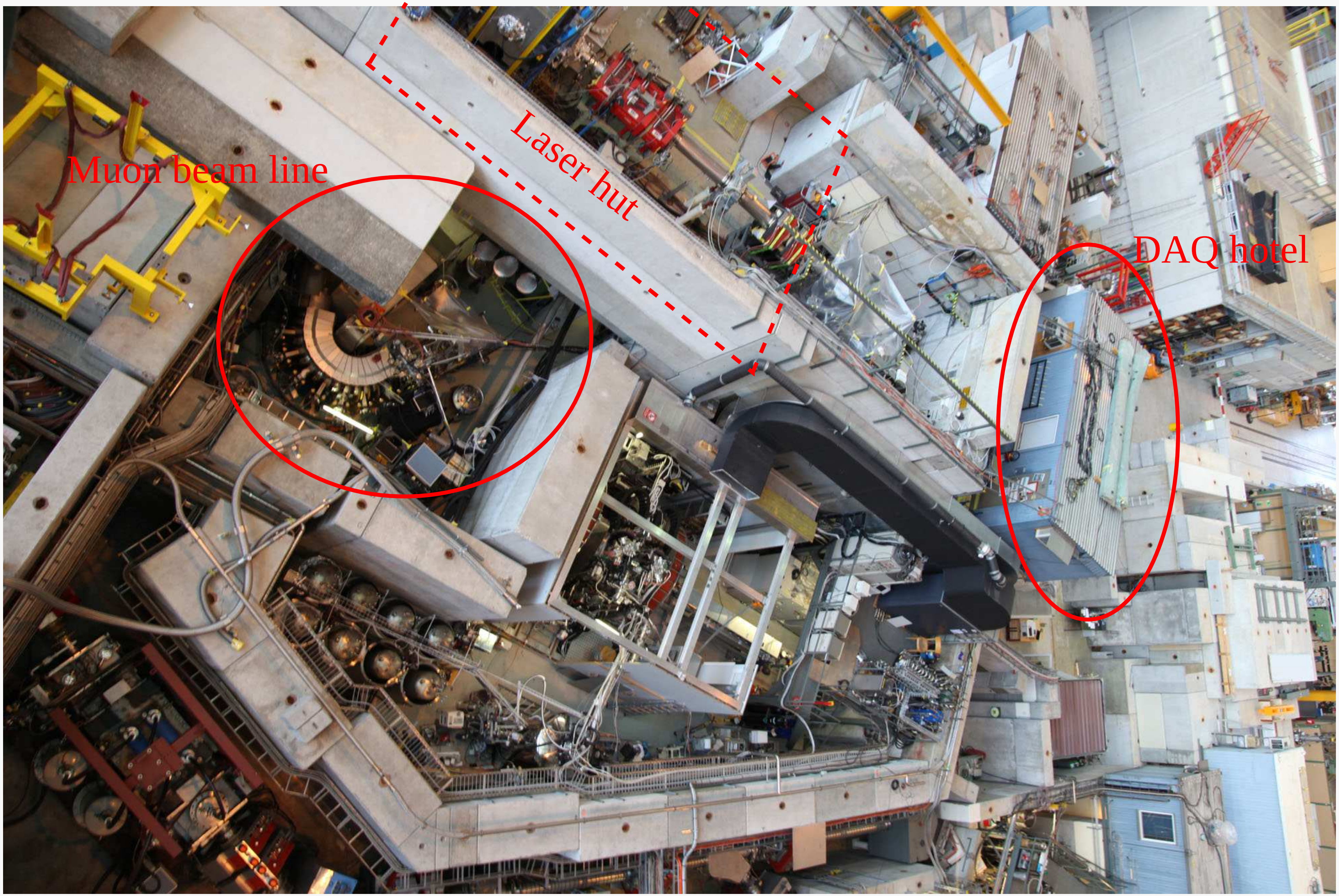
Cartoon of the resonance



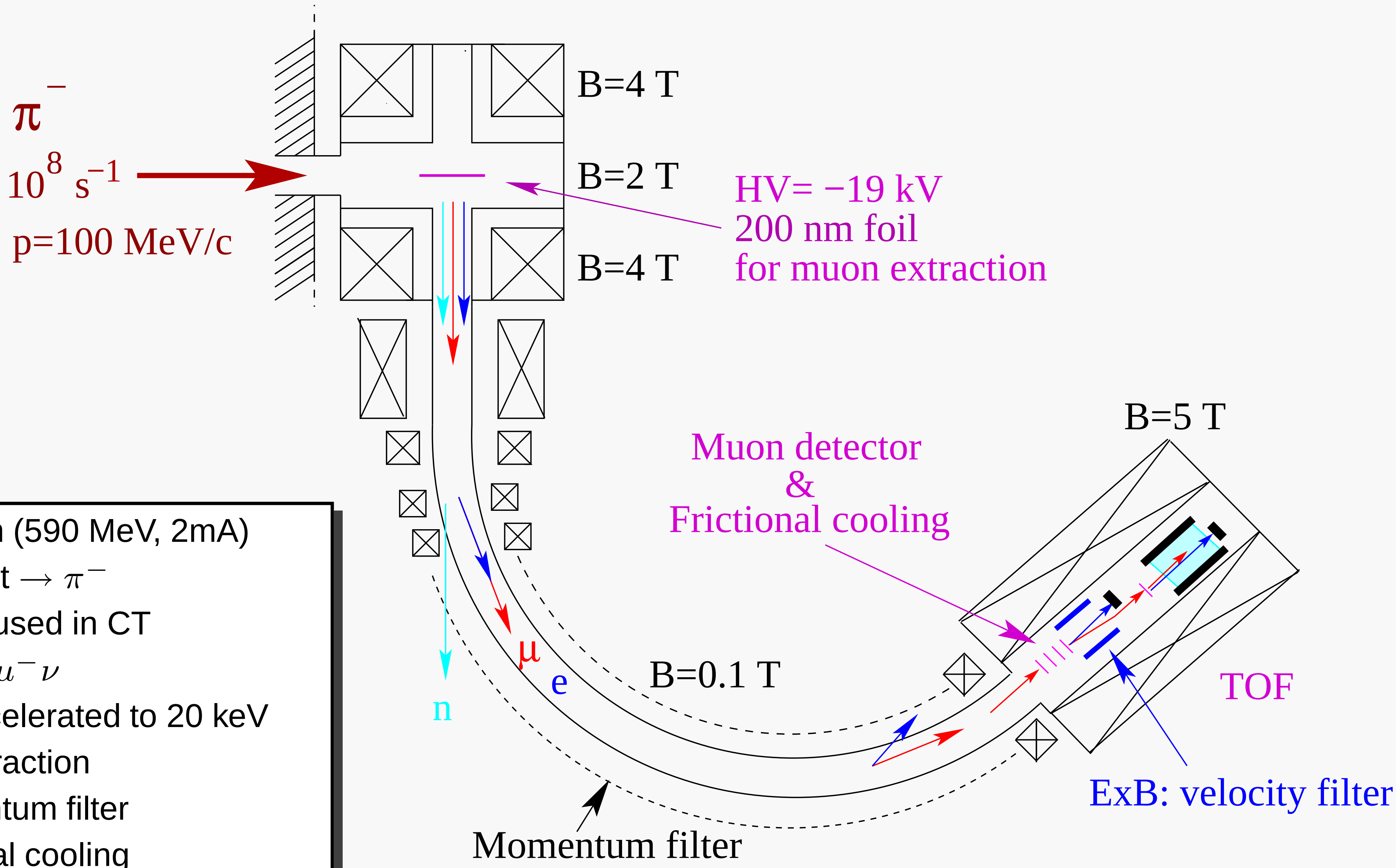
Apparatus

- Muon beam line
- Laser system
- Detectors and DAQ

The experimental hall at PSI: our paradise

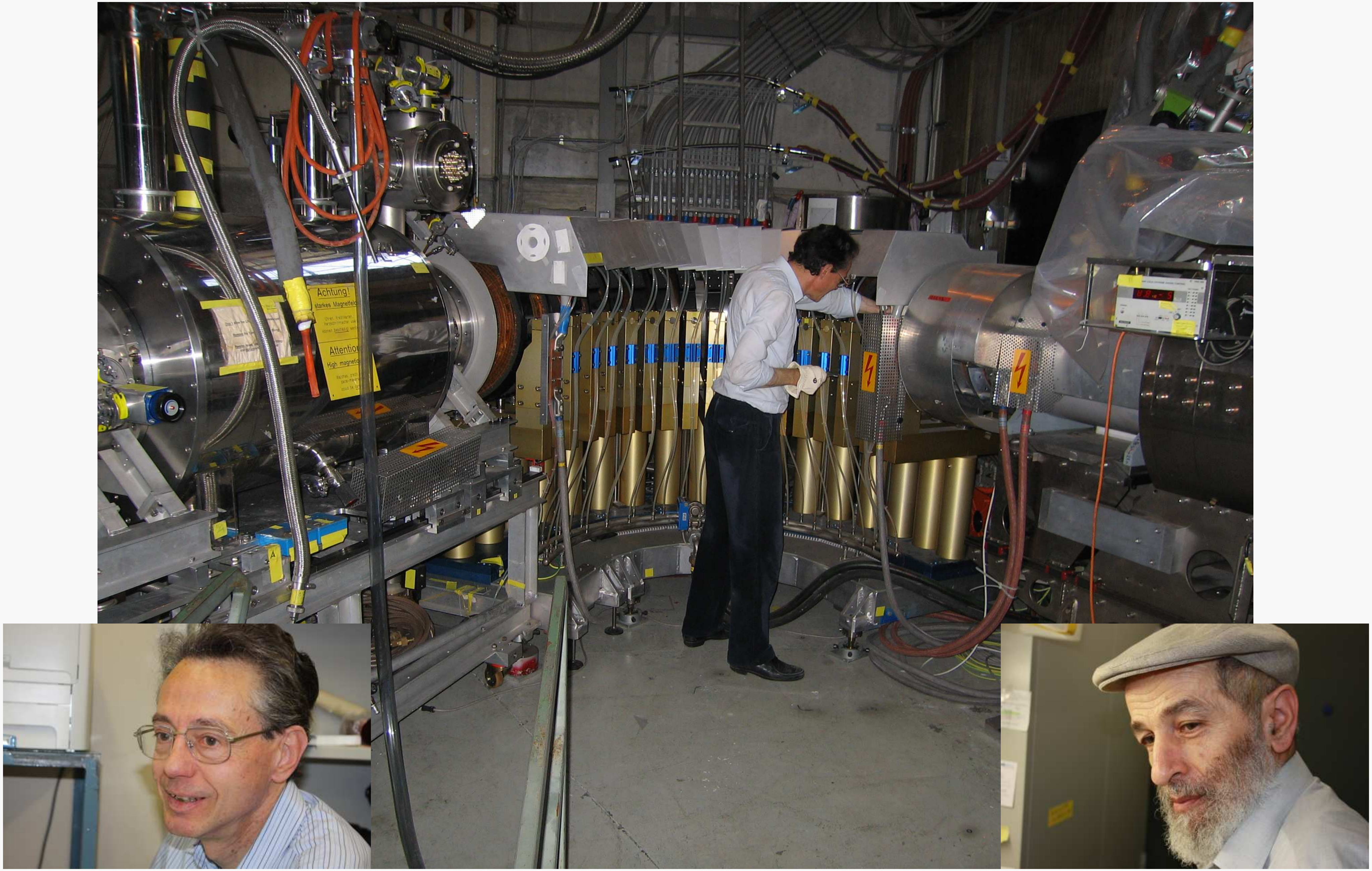


Low energy μ^- beam line



- p-beam (590 MeV, 2mA)
- C target $\rightarrow \pi^-$
- π^- focused in CT
- $\pi^- \rightarrow \mu^- \nu$
- μ^- decelerated to 20 keV
- μ^- extraction
- momentum filter
- frictional cooling
- non-destructive μ^- detection

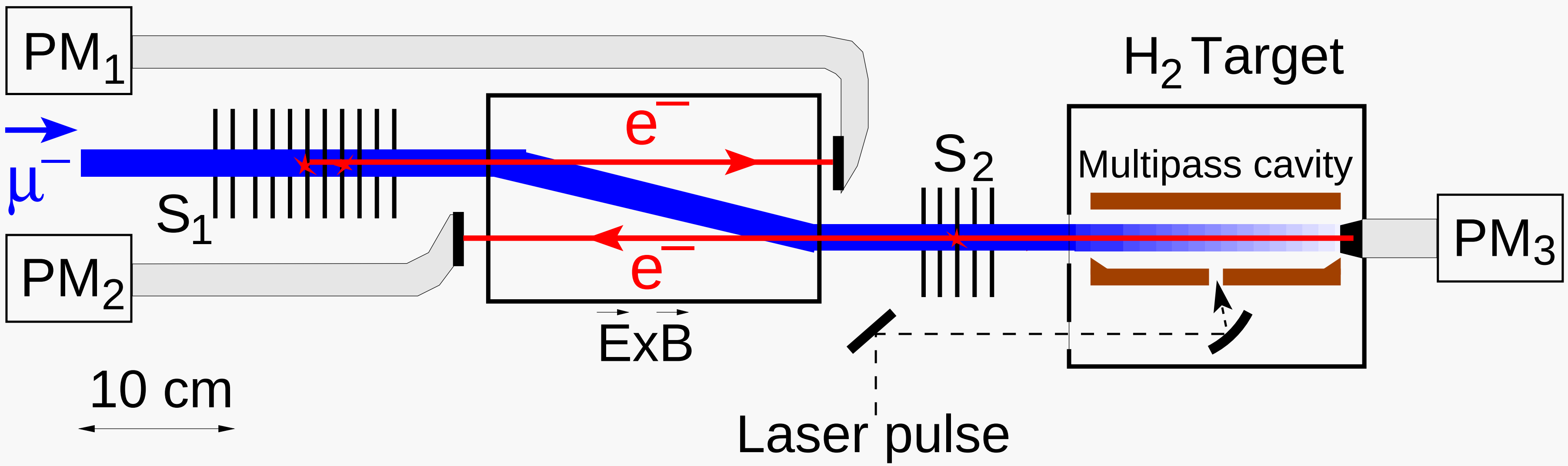
Muon beam line



Muon beam line

Target entrance: 5 keV μ^-, 400 s^{-1}

- From the muon extraction channel: 20-50 keV μ^-
  slowing down + frictional cooling + e^- emission + $E \times B$ + TOF + trigger
- $\epsilon_{S_1} = 85\%$, $\epsilon_{S_2} = 35\%$, $\epsilon_{S_3} = 55\%$
- Stopping volume in 1 hPa H_2: $= 5 \times 15 \times 190$ mm^3



Animation muon beam

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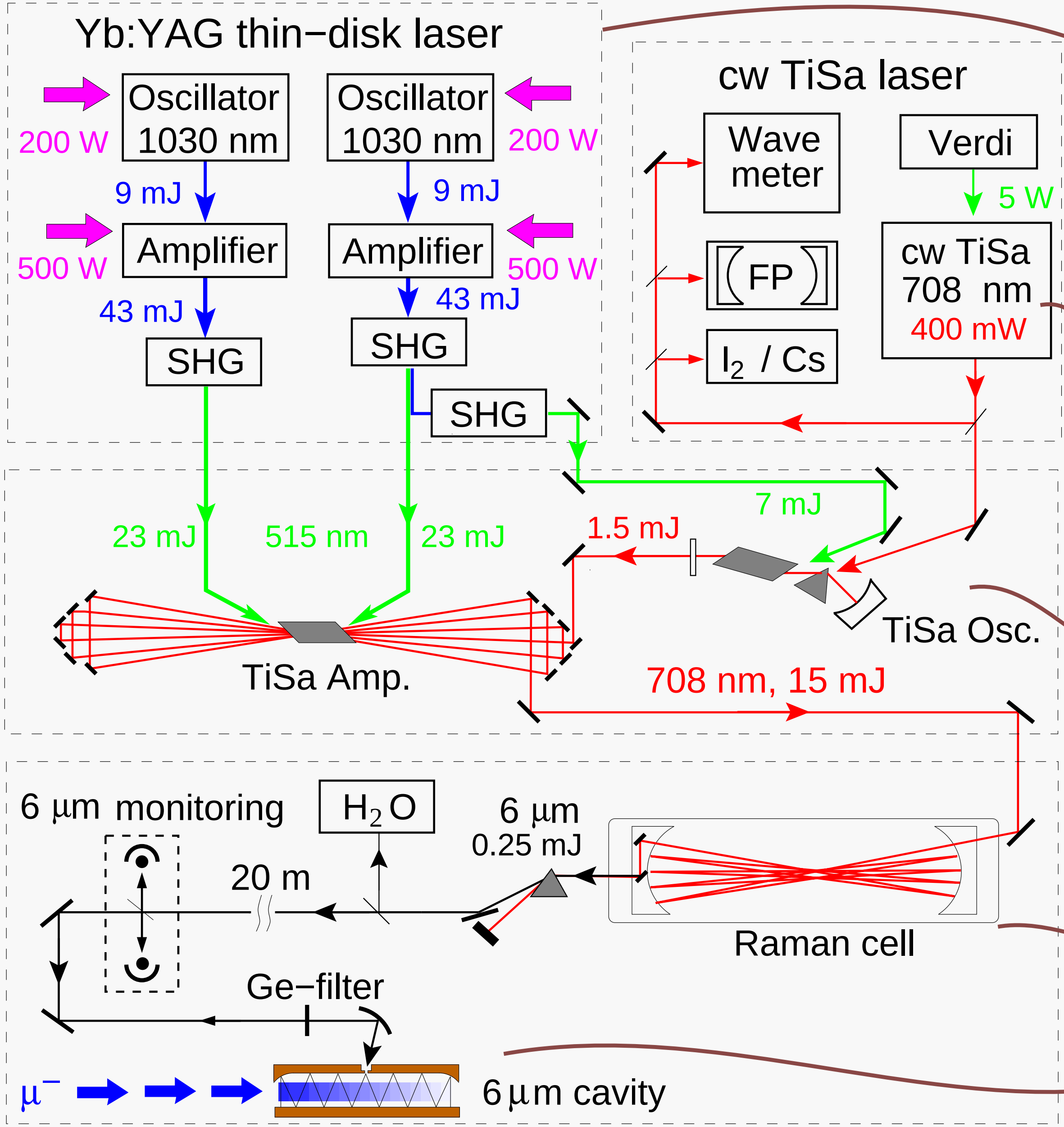
(T.W. Hänsch)

The laser system



Main components:

- Thin-disk laser
- Frequency doubling
- TiSa laser:
  - cw frequency stabilized laser
  - injected seeded oscillator
  - multipass amplifier
- Raman cell
- Target cavity

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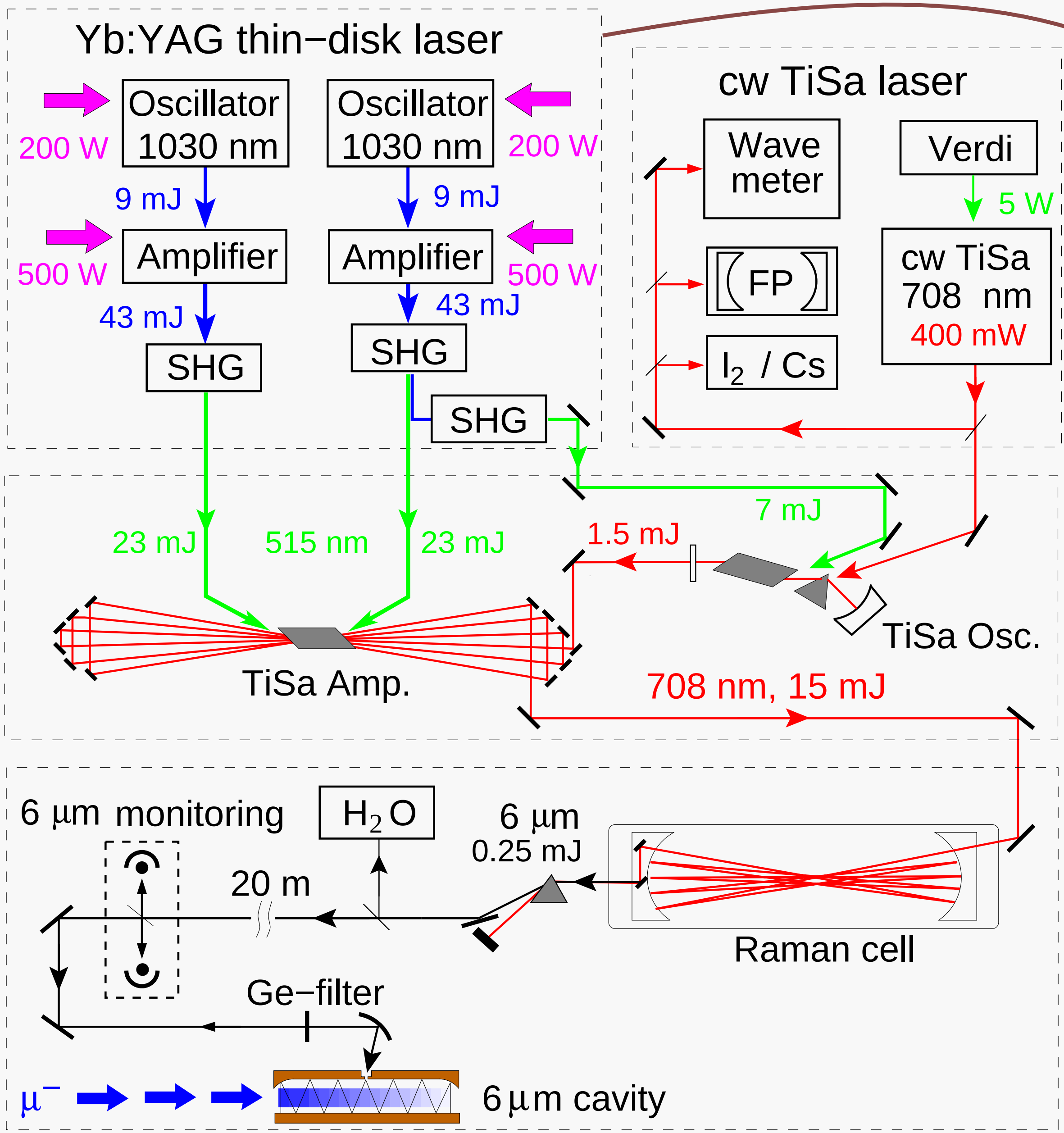

Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single μ^- triggers the laser system
- $2S$ lifetime $\approx 1\ \mu$s $\rightarrow$ short laser delay

A. Antognini *et. al.*, IEEE J. Quant. Electr. Vol. 45, No. 8, 993-1005 (2009).

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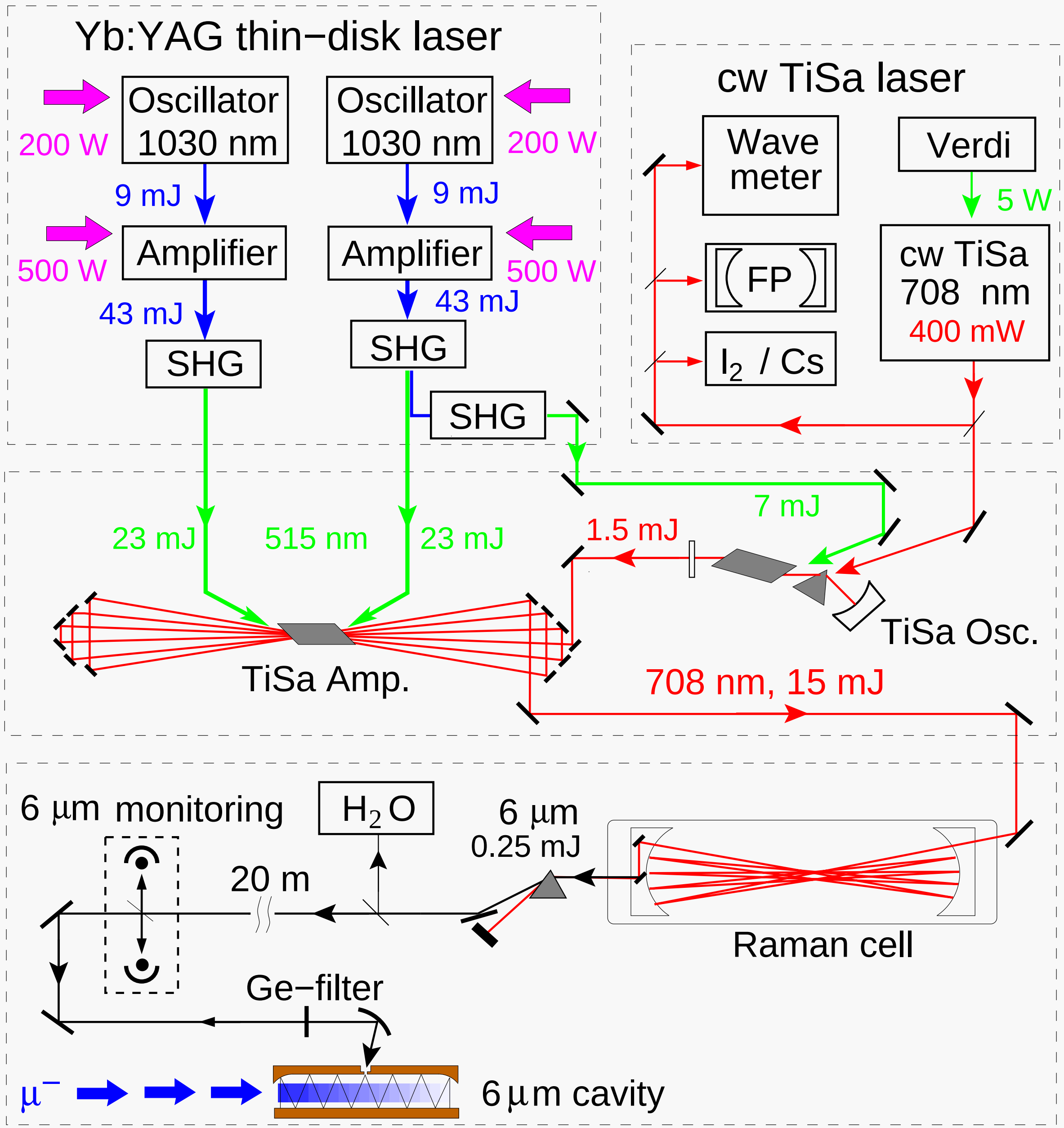


MOPA TiSa laser:

Cw frequency stabilized laser

- referenced to a stable FP cavity
- FP cavity calibrated with I_2, Rb, Cs lines

$\nu_{\mathrm{FP}} = N \cdot FRS$

$FRS = 1497.344(6)$ MHz, $N \approx 2 \times 10^5$.

$\nu^{\mathrm{cw}}_{\mathrm{TiSa}}$ absolutely know with $\sigma = 30$ MHz

$\Gamma_{\mathrm{2P-2S}} = 18.6$ GHz

Seeded oscillator

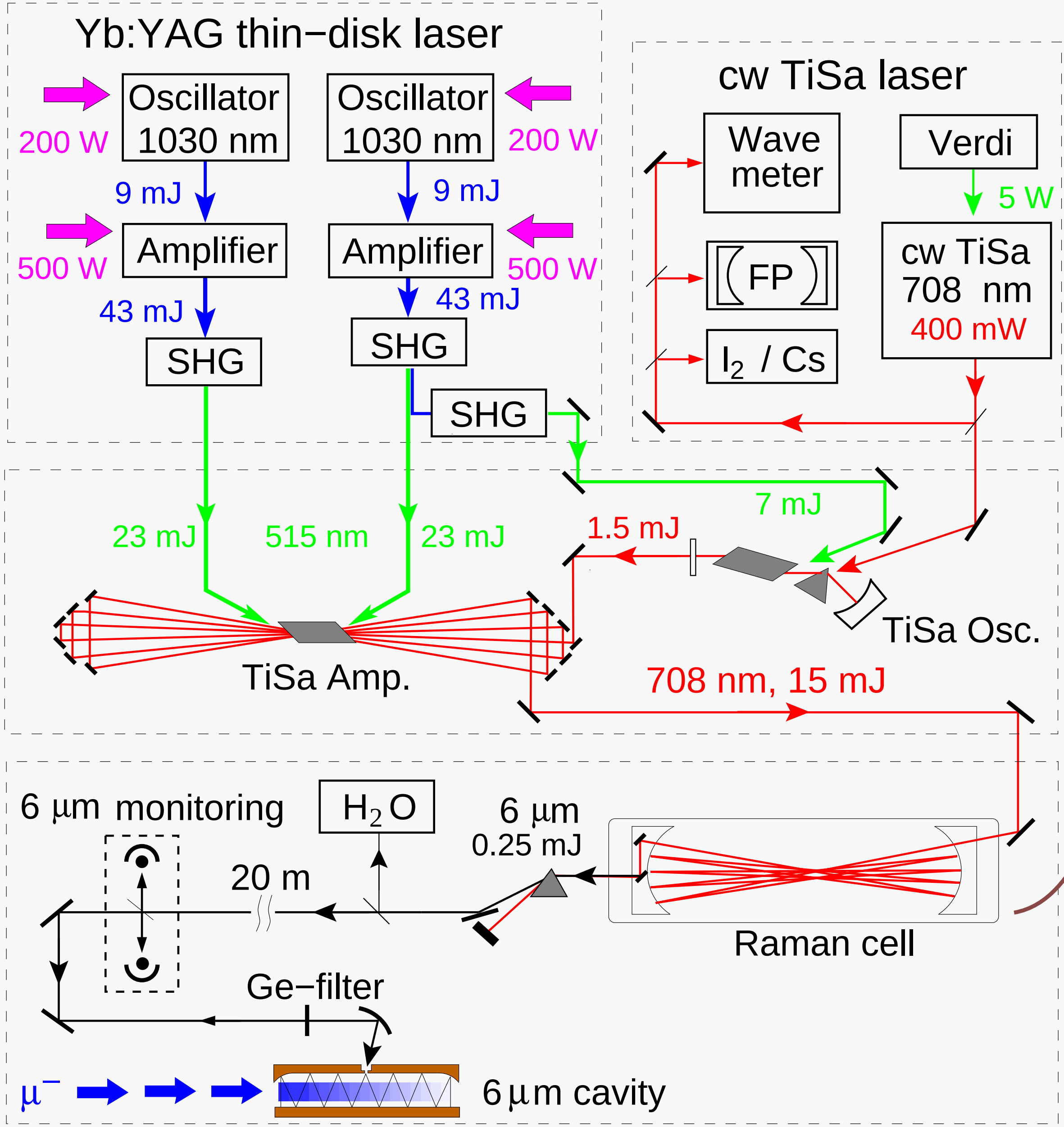
$\rightarrow \nu^{\mathrm{pulsed}}_{\mathrm{TiSa}} = \nu^{\mathrm{cw}}_{\mathrm{TiSa}}$

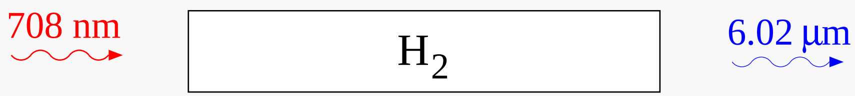
(frequency chirp ≤ 100 MHz)

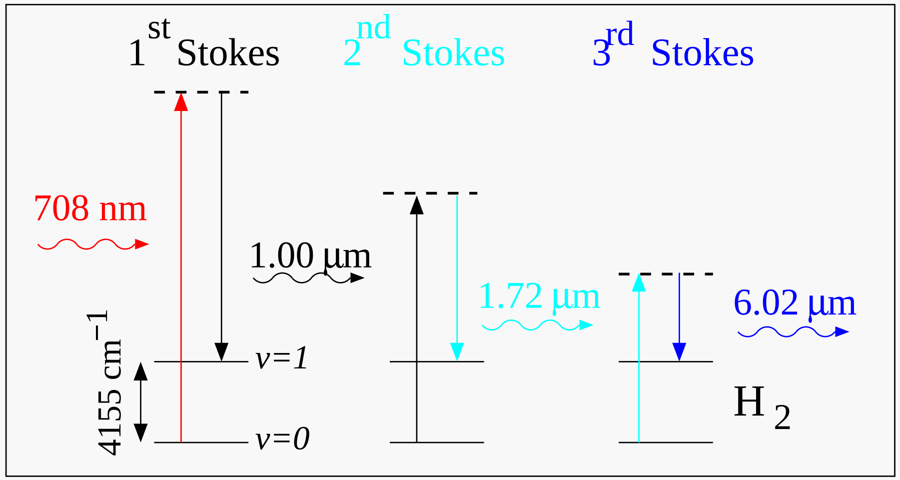
Multipass amplifier (2f- configuration)

gain=10

The laser system





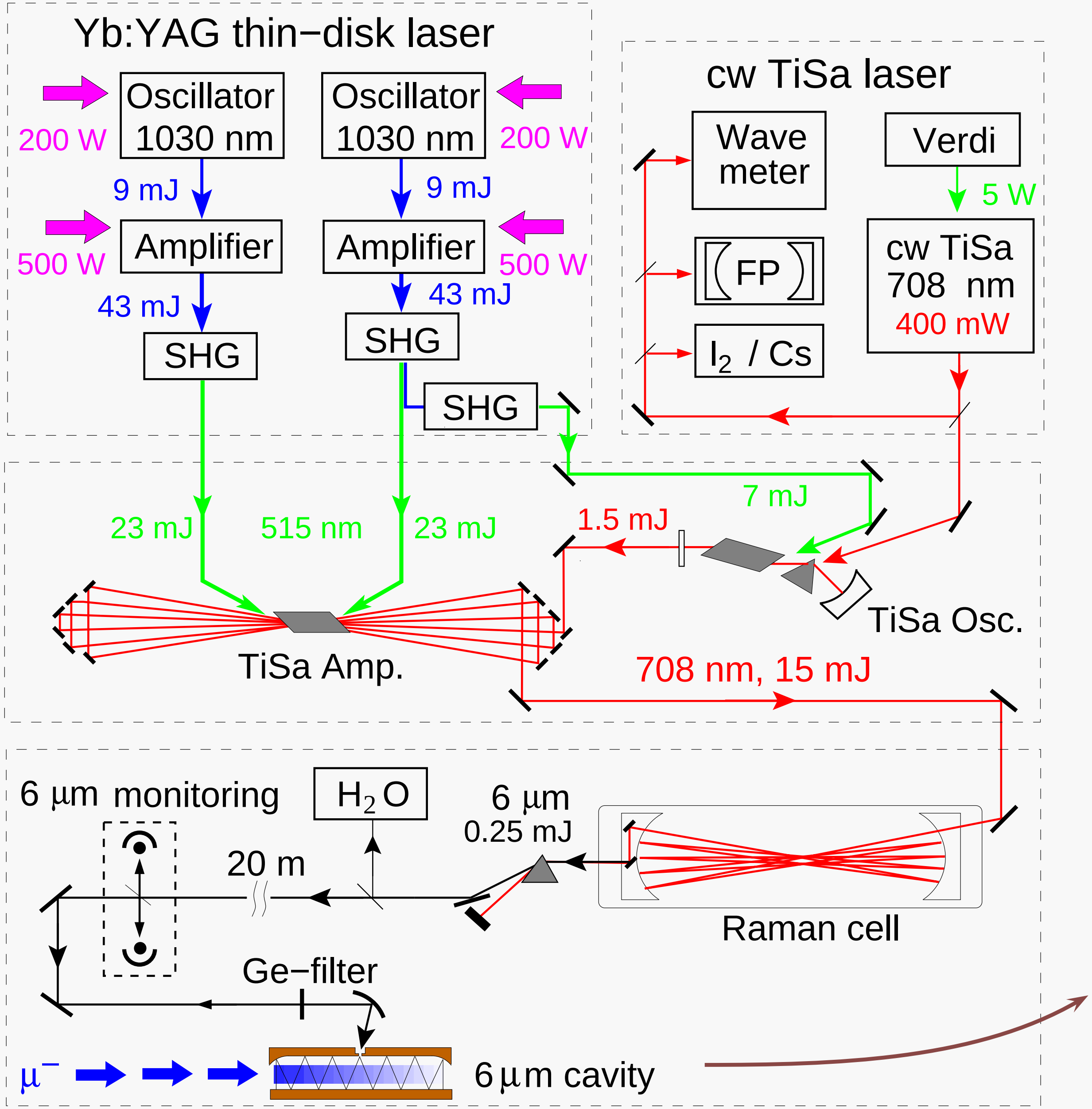


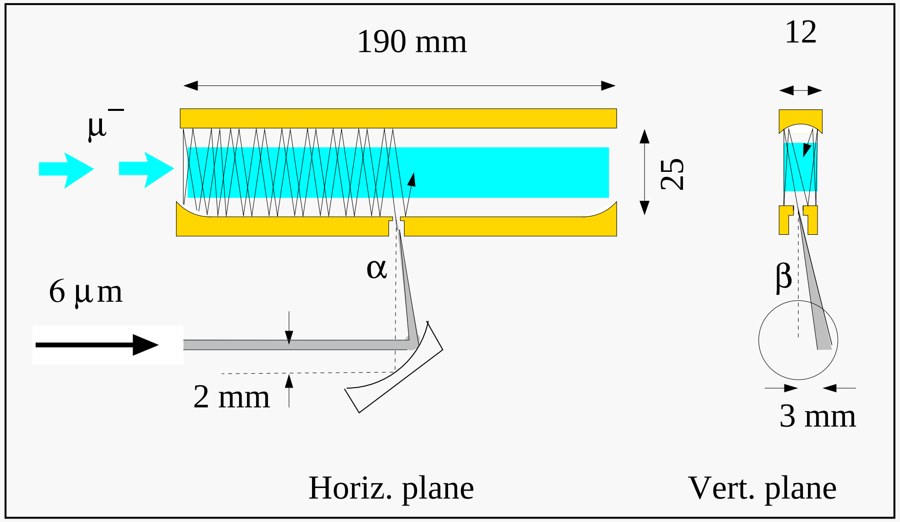
$$\nu^{6\mu m} = \nu^{708\text{nm}} - 3 \cdot \hbar\omega_{\text{vib}}$$

tunable

$\omega_{\text{vib}}(p, T) =$ const

The laser system





Design: insensitive to misalignment

Transverse illumination

Large volume

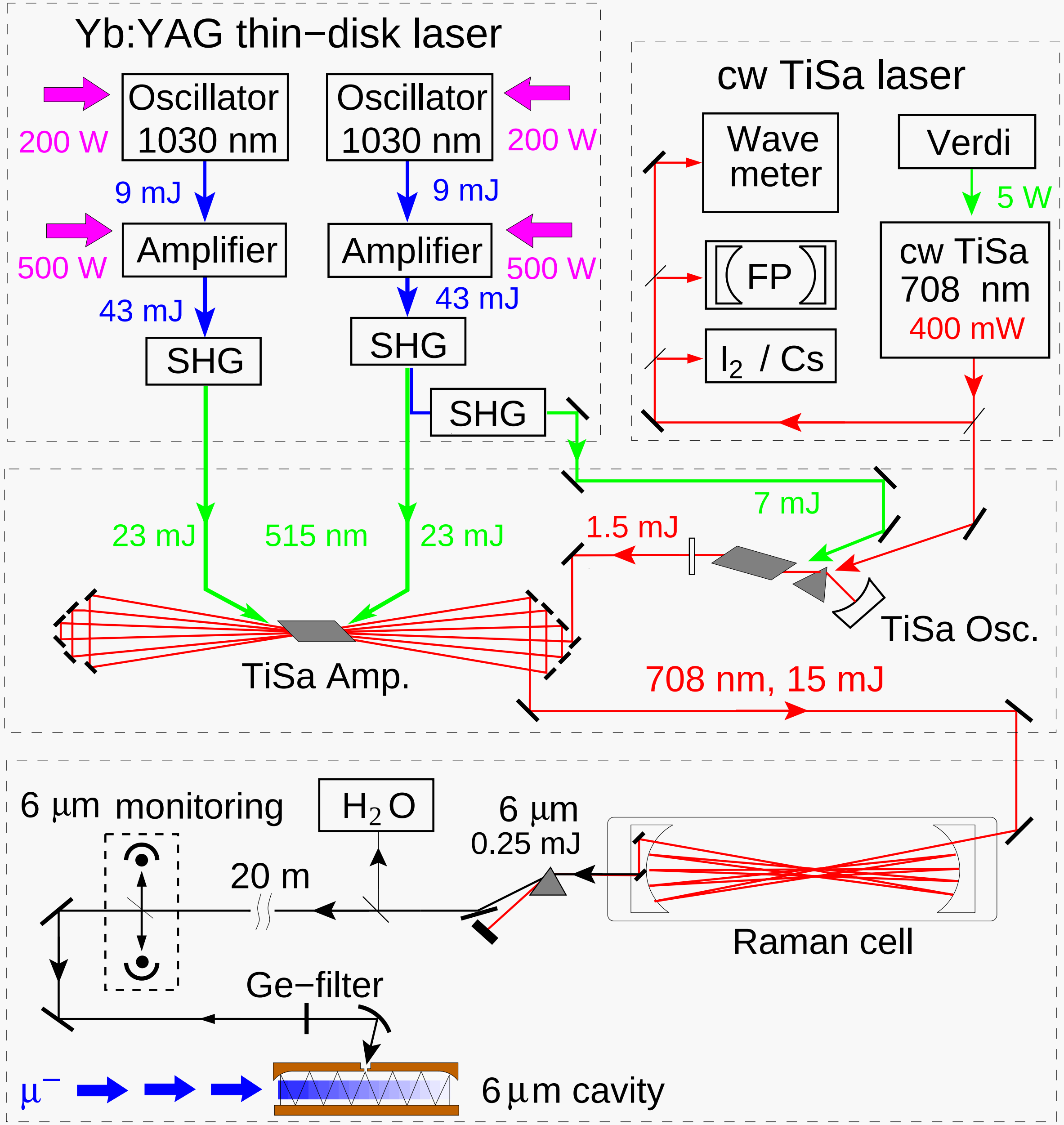
Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

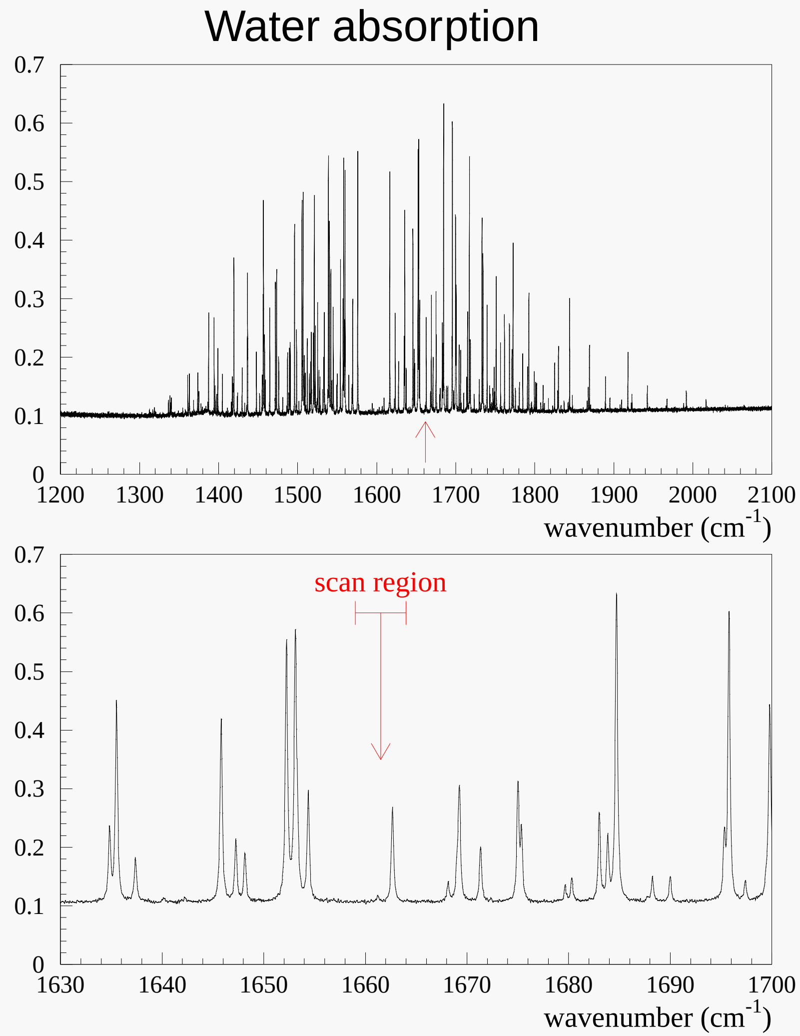
→ Light makes 1000 reflections

→ Light is confined for 50 ns

→ 0.15 mJ saturates the $2S - 2P$ transition

The laser system

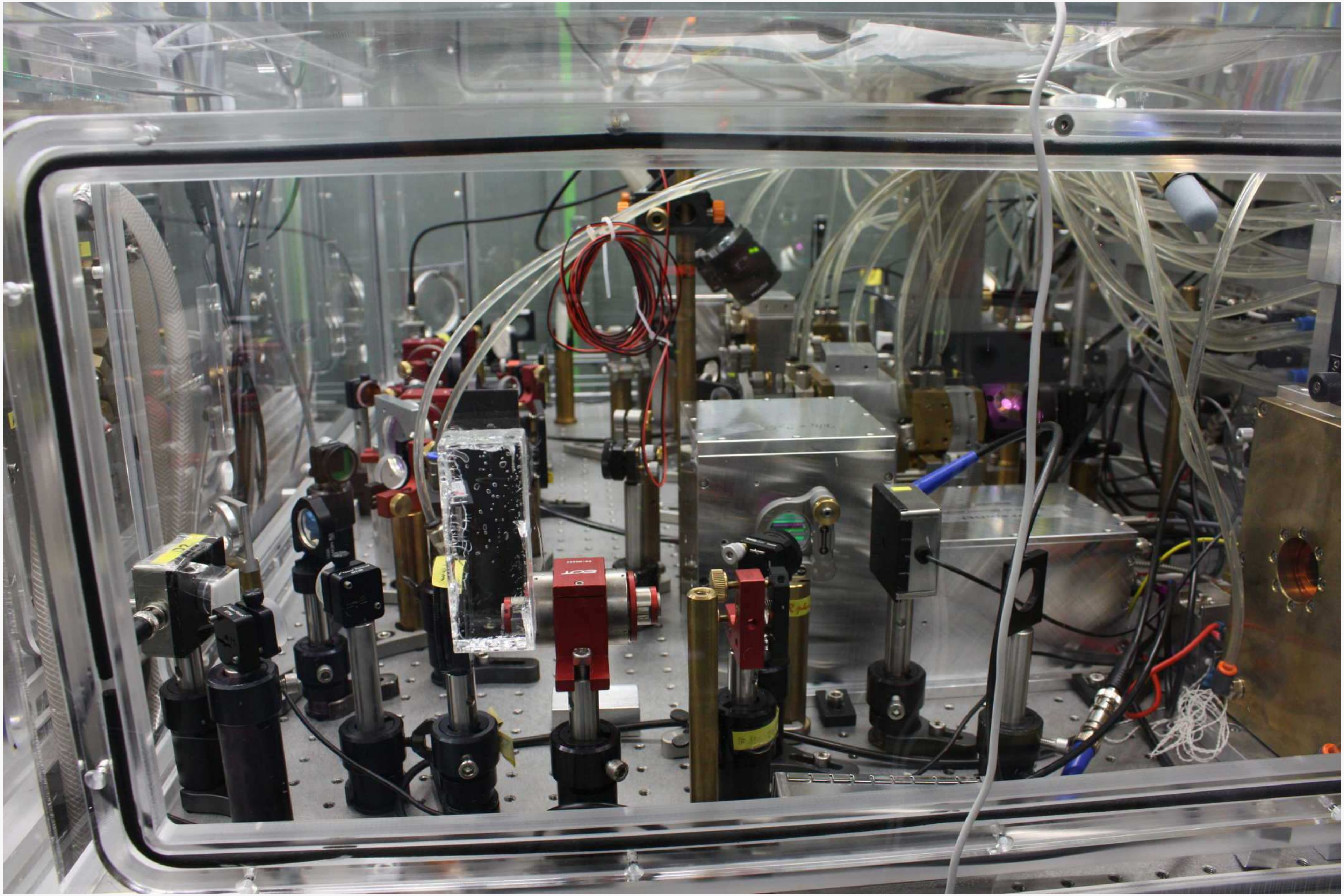




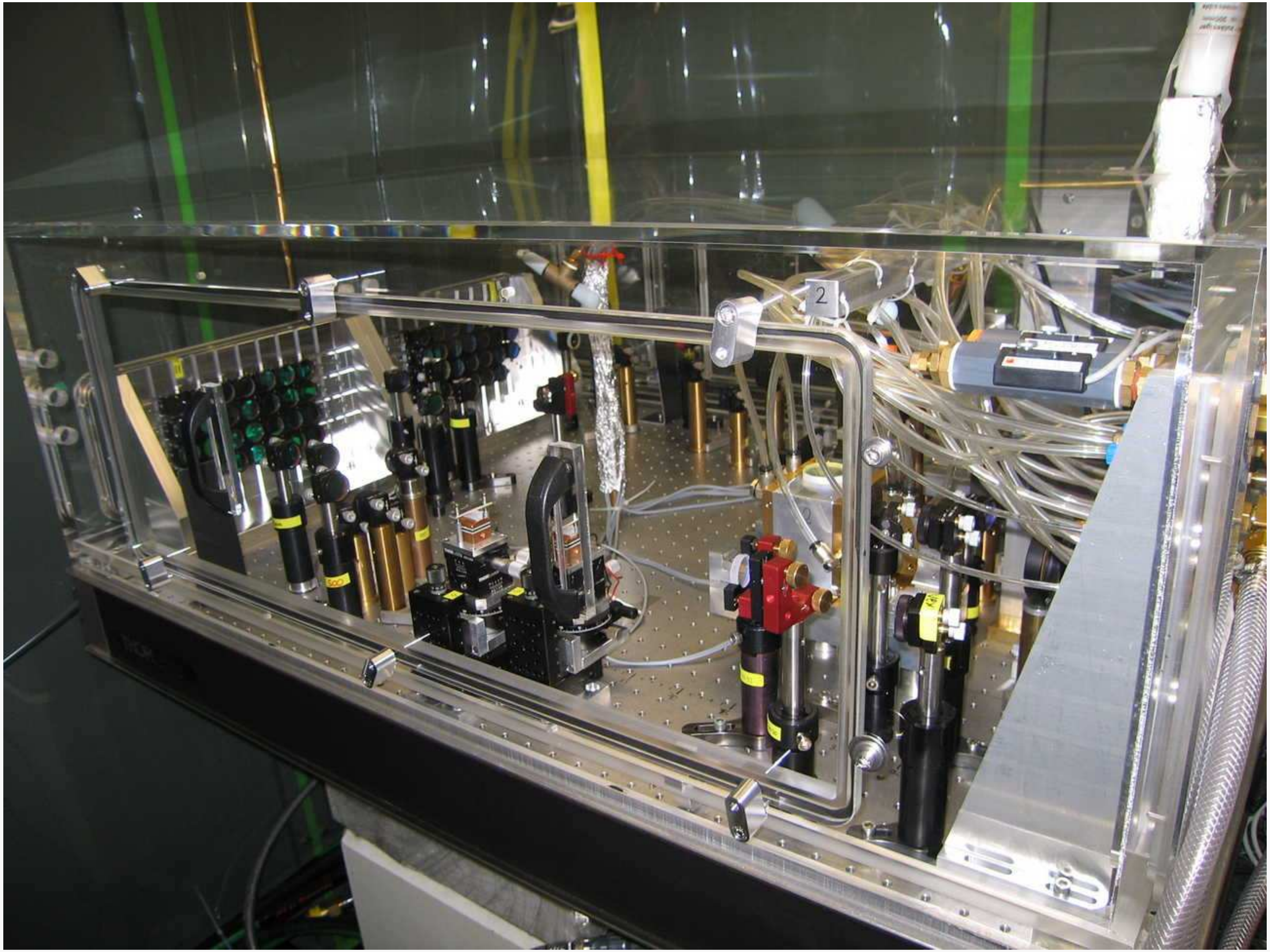
- Vacuum tube for 6 μm beam transport.
- Direct frequency calibration at 6 μm.

Impressions from the laser hut

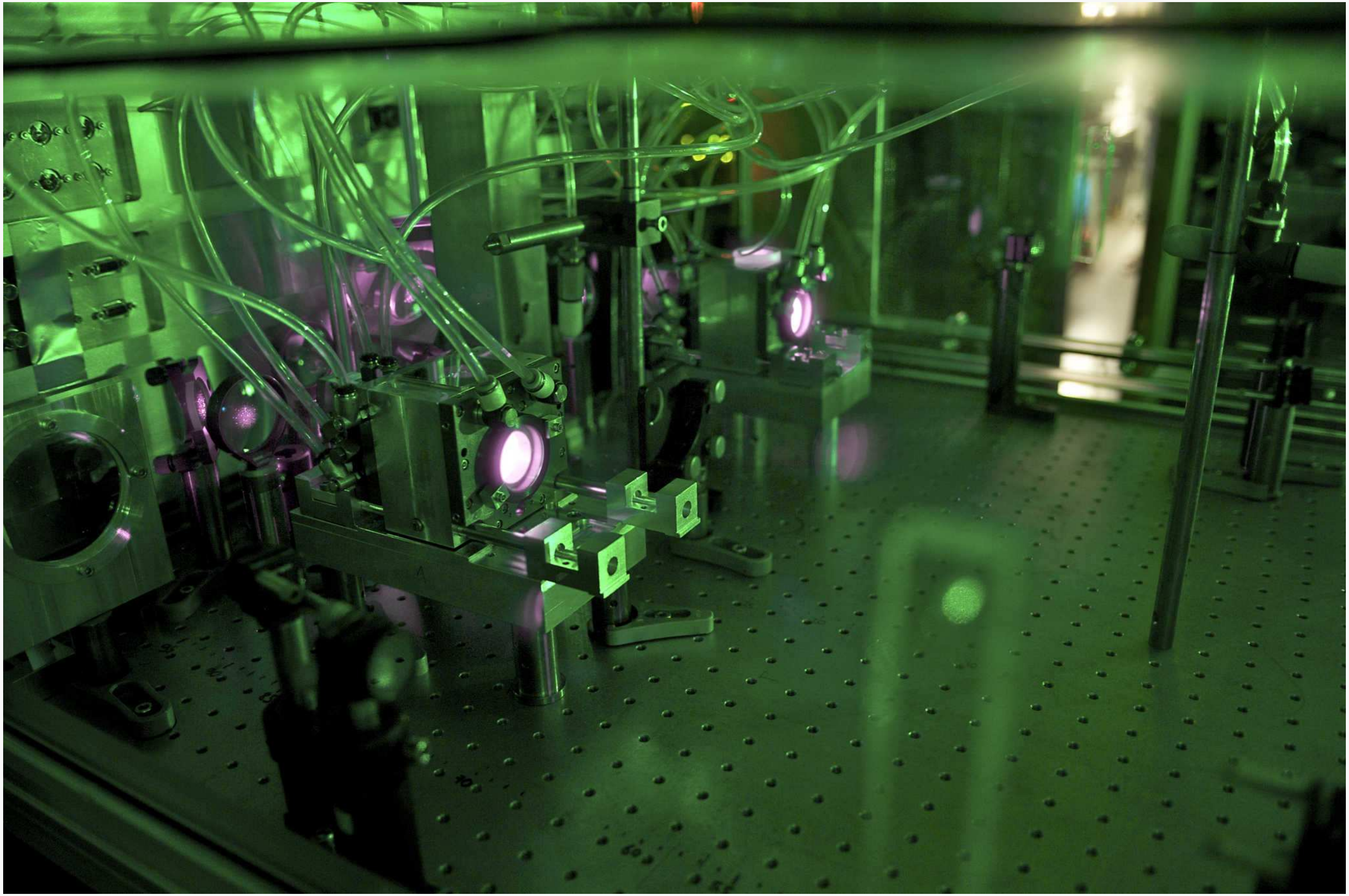
Disk laser oscillators



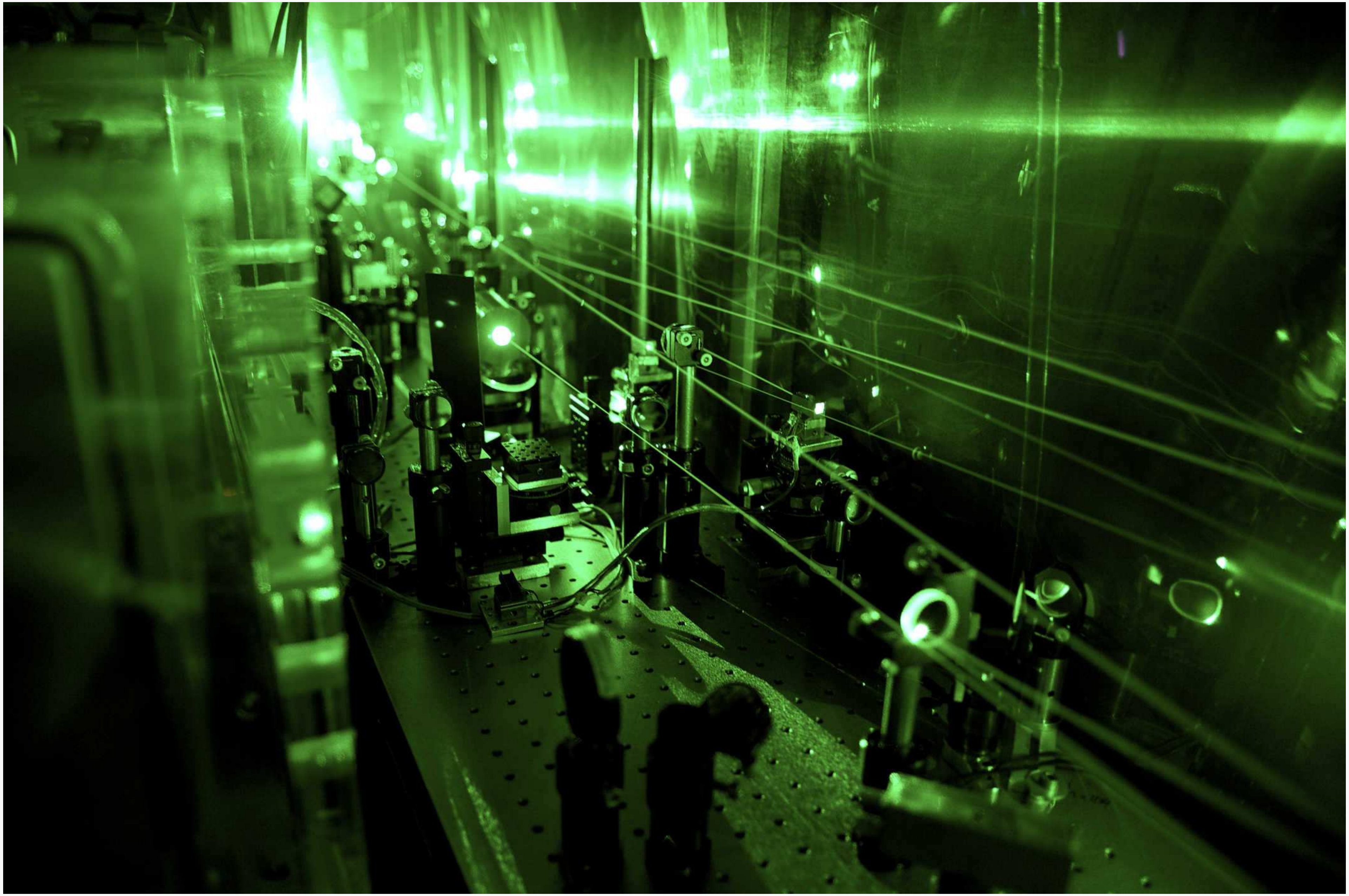
Disk laser amplifiers



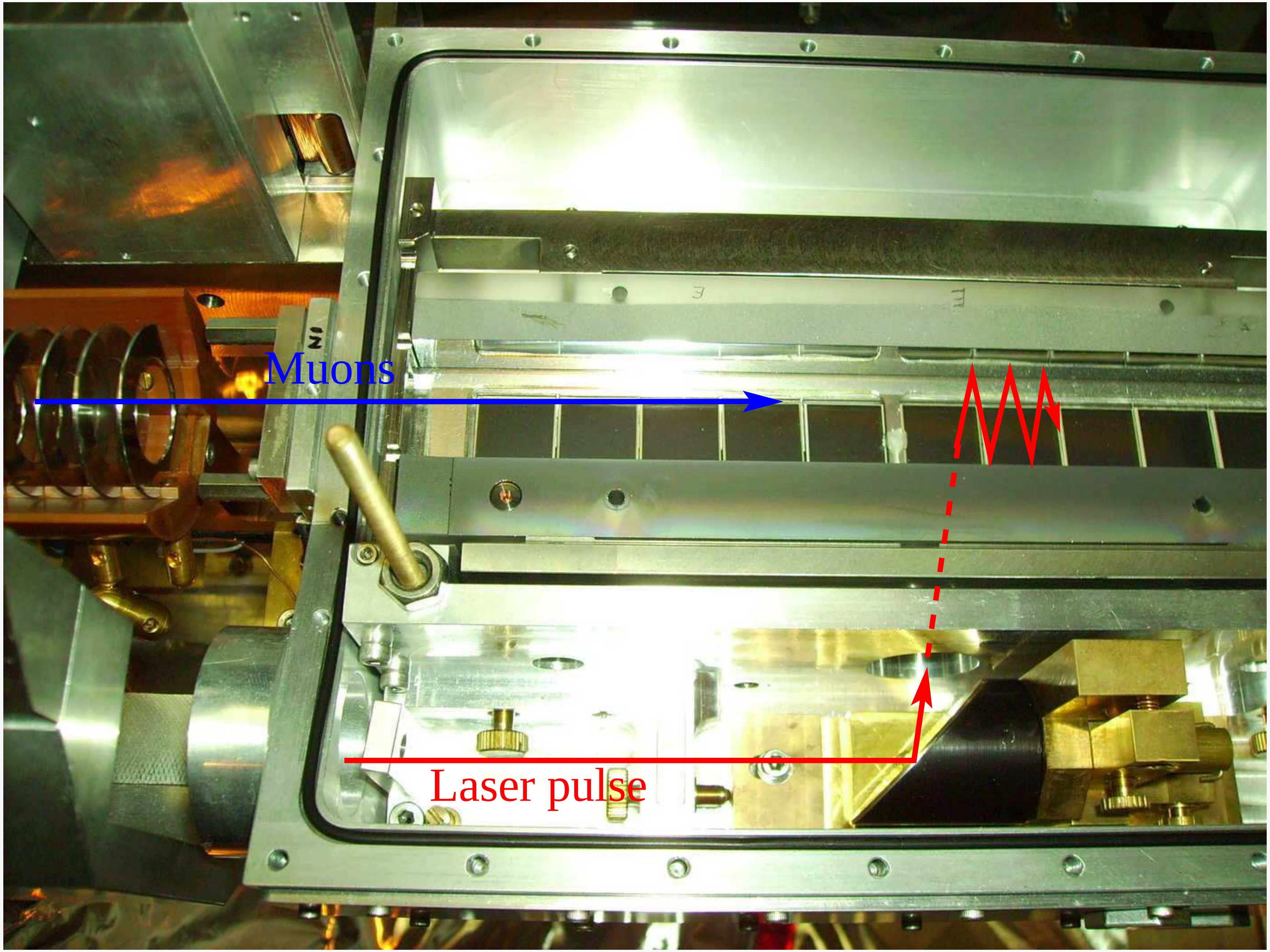
Disk amplifier laser heads



Disk laser doubling stages



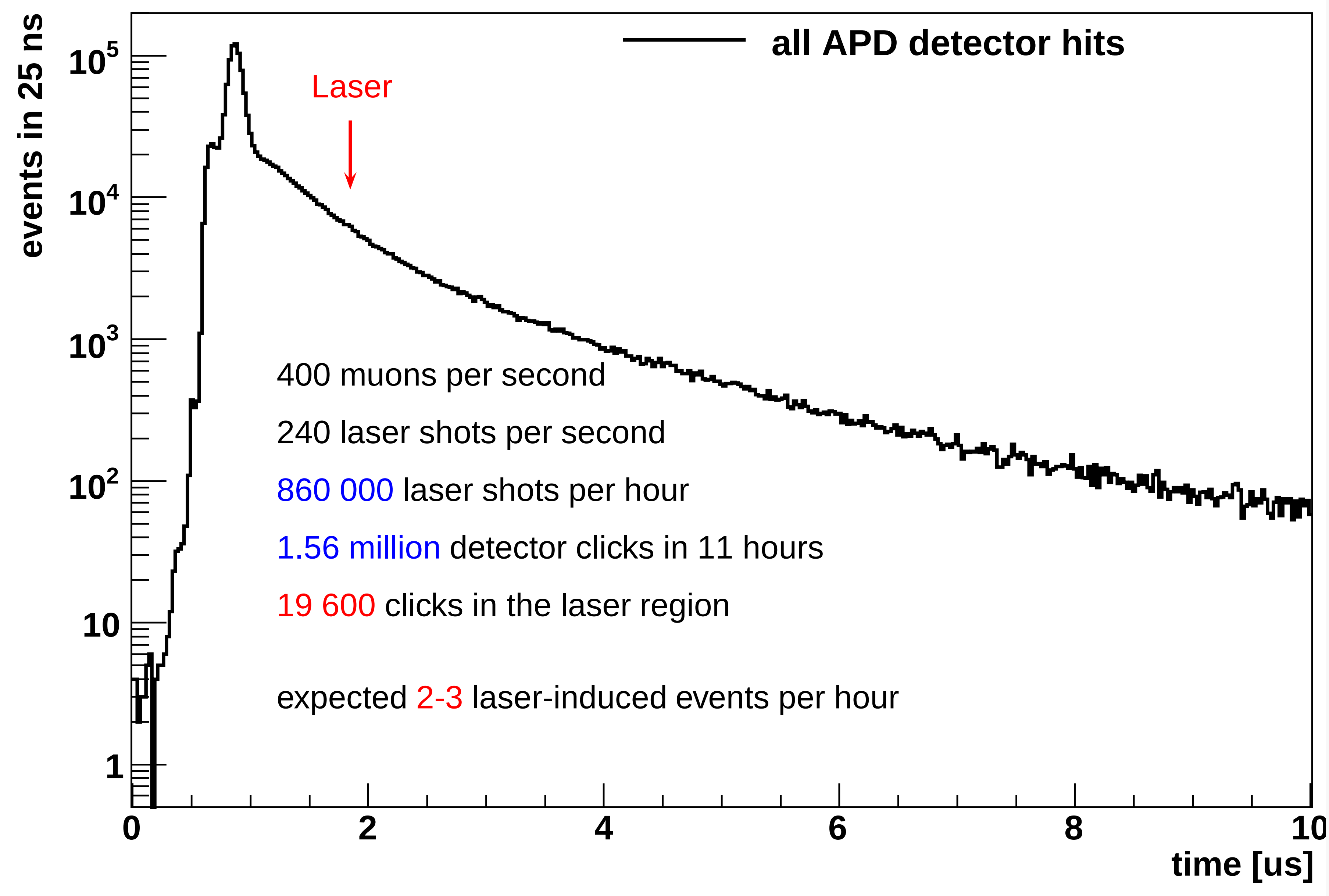
Open target



Measurements

Data analysis: time spectra

FP 900, 11 hours measurement



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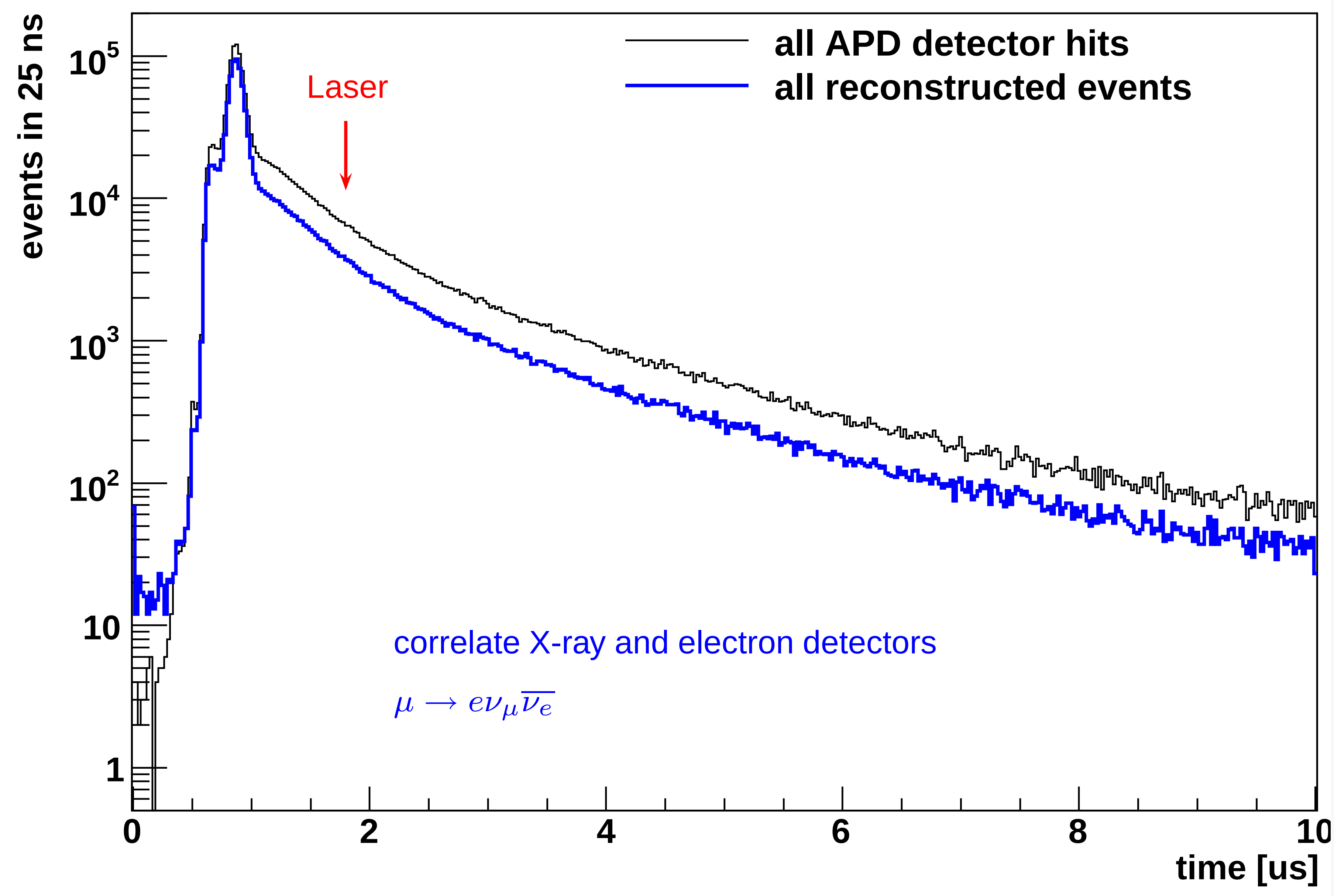
FP 900, 11 hours measurement



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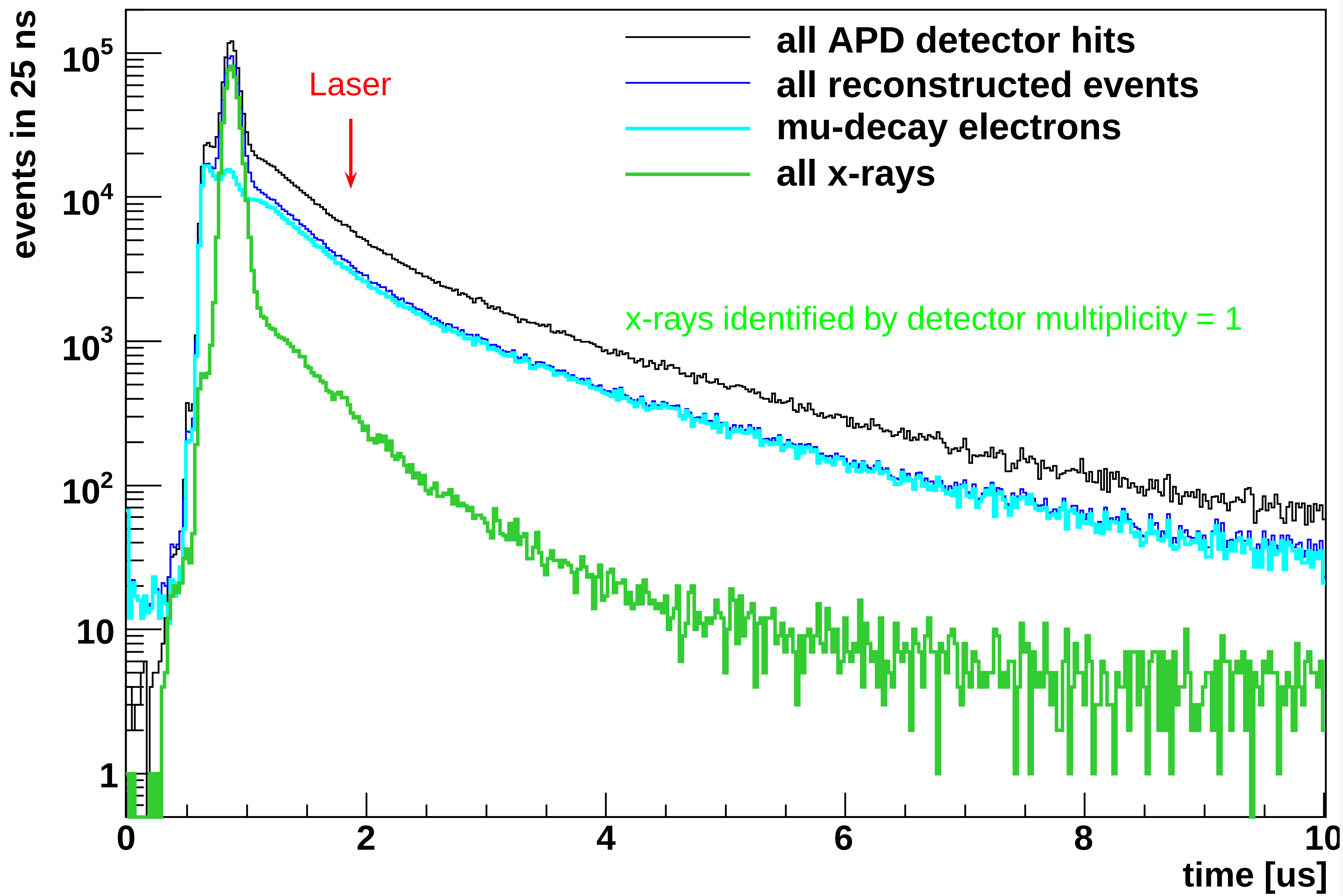

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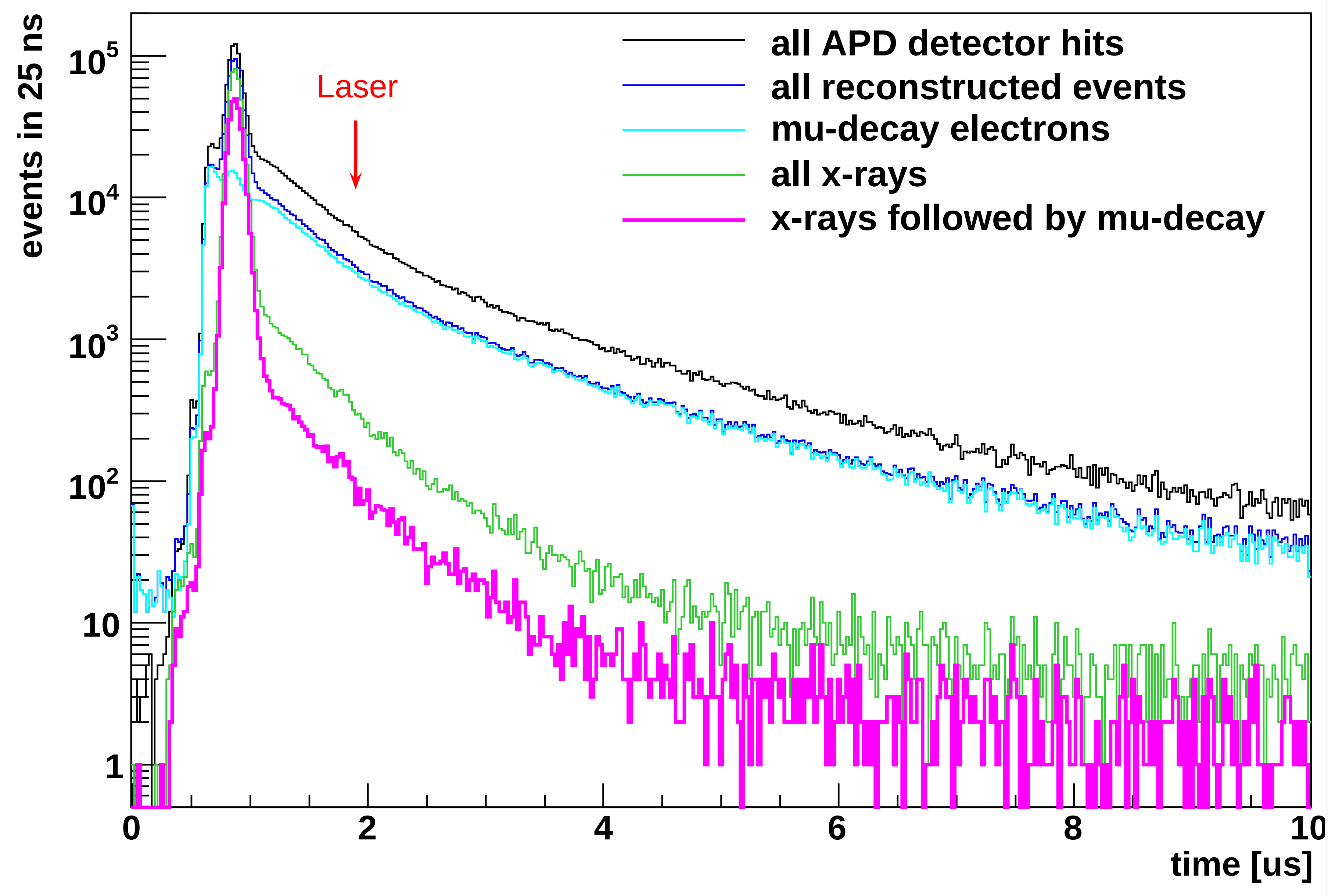
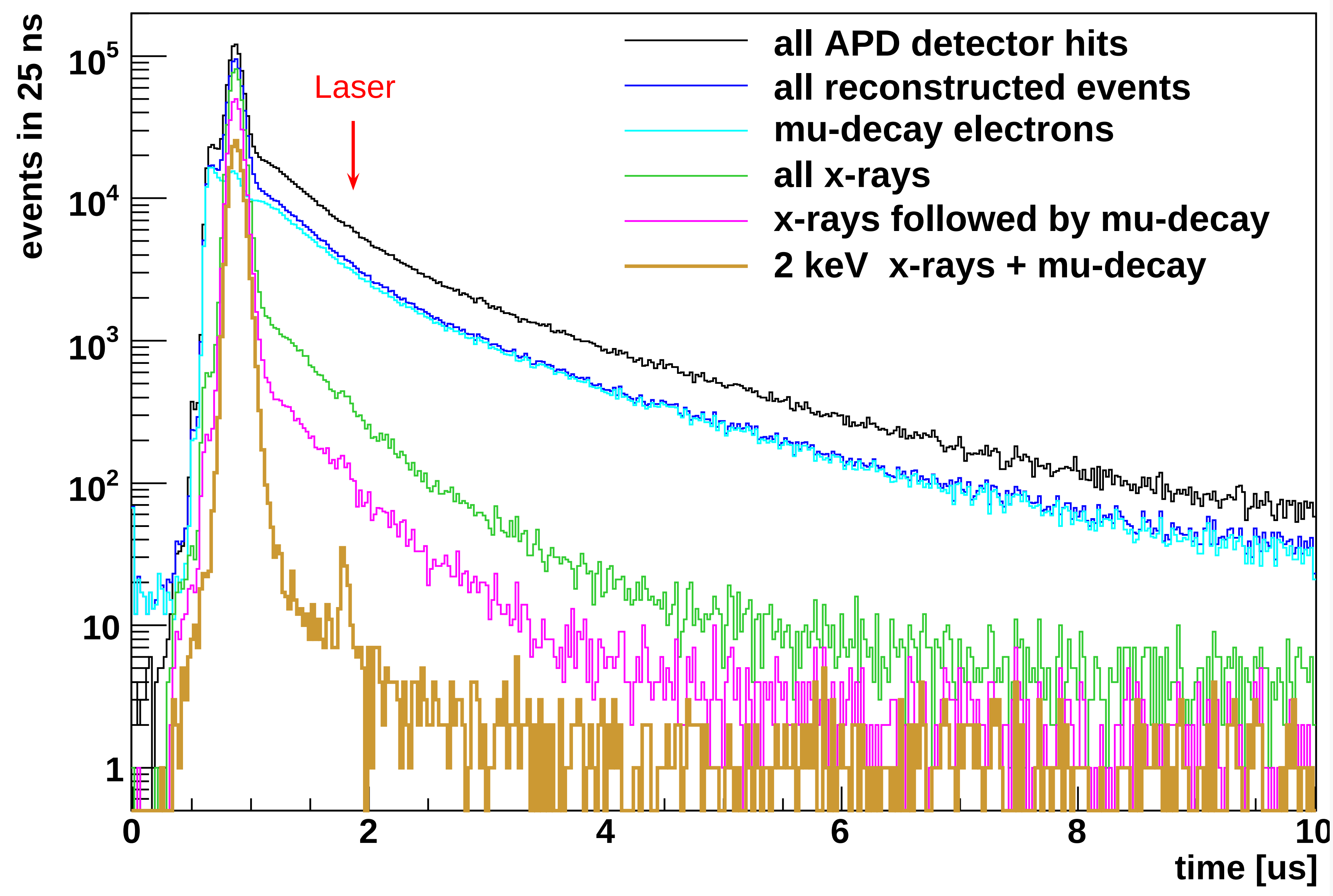


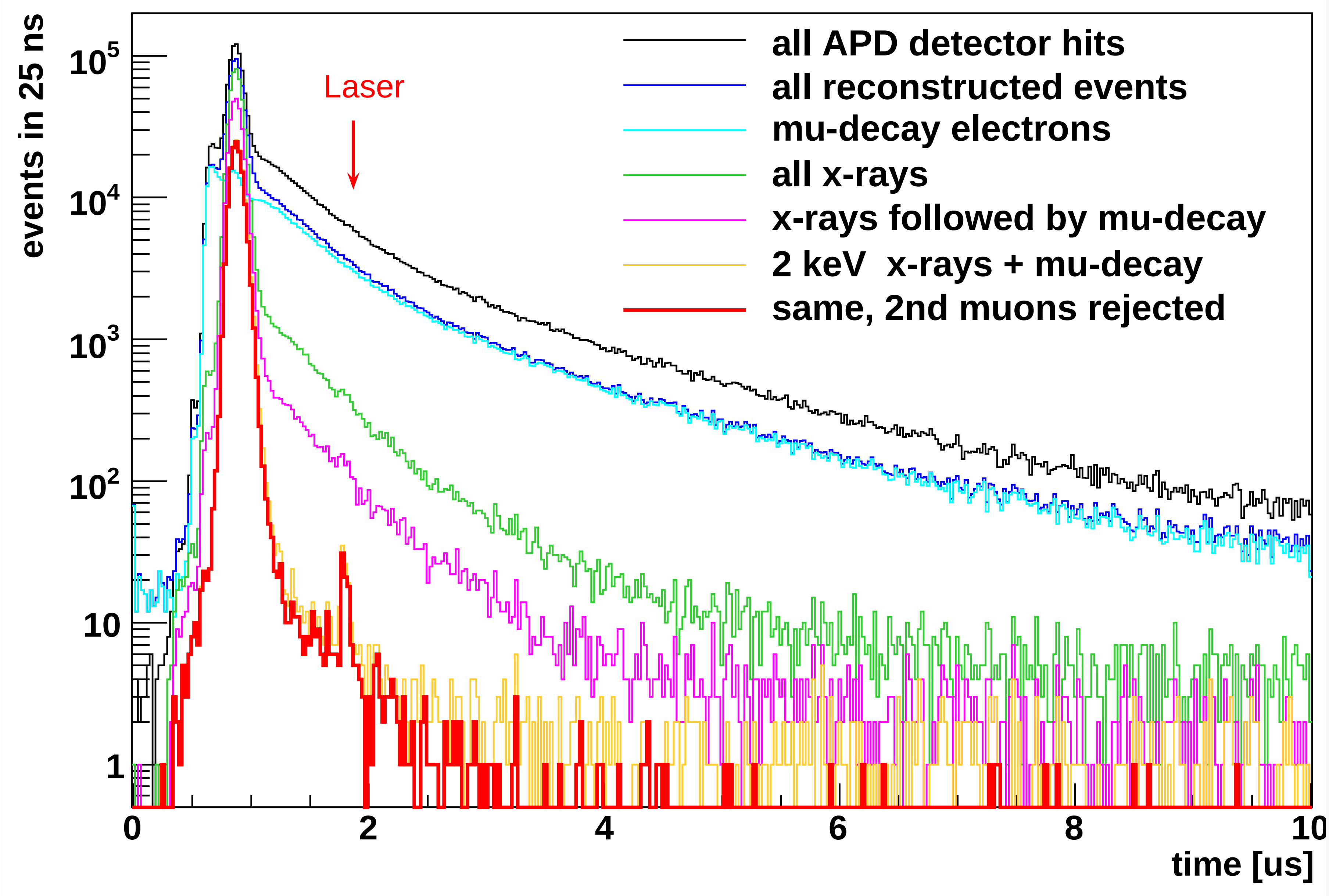
Data analysis: time spectra

FP 900, 11 hours measurement



Data analysis: time spectra

FP 900, 11 hours measurement 7 events per hour! 1 bgr. event/hour



Resonance search animation

Like in HOLLYWOOD movies

everything goes bad till five minutes before THE END

BUT.......

Uncertainty budget and sensitivity

• Statistics	
Centroid position uncertainty ($\sim 4\%$ of Γ))	700 MHz
• Systematics	
Laser frequency (H20 calibration)	300 MHz
AC and DC stark shift	< 1 MHz
Zeemanshift (5 Telsa)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz
• Total uncertainty of the line determination	760 MHz
• Discrepancy with prediction	80 GHz

Systematic effects are small since they scale like $1/m$

Finite size effect scales like m^3

μp Lamb shift theoretical prediction

Contributions to the μp Lamb shift

Contribution		E [meV]	ΔE $[10^{-4}\text{meV}]$
Vacuum polarization (VP)		205.0282	
Källen–Sabry + VP iteration +Sixth order VP		1.66671	
Mixed $\mu - e$ VP		0.00007	
Hadronic VP		0.011	20
Whichmann–Kroll		-0.00103	
Virtual Delbrück		0.00135	
Light–by–light		—	10
Muon self–energy and muonic VP		-0.66788	
Fourth order electron loops		-0.00169	
VP insertion in self energy		-0.0055	10
Proton self–energy		-0.0099	
Recoil		0.0575	
Recoil correction to VP (one–photon)		-0.0041	
Recoil (two–photon)+Recoil higher order		-0.05457	
Recoil finite size		0.013	10
Finite size of order $(Z\alpha)^4$	$(-5.1975(1)\, r_{\text{p}}^2)$	-3.995	(630)
Finite size of order $(Z\alpha)^5$	$(0.0347(30)\, r_{\text{p}}^3)$	0.0234	(20)
Finite size of order $(Z\alpha)^6$		-0.0005	
Finite size corrections to VP	$(-0.0273\, r_{\text{p}}^2)$	-0.0211	
Proton polarizability		0.015	40
Fine+hyperfine structure $(2P_{3/2}^{\text{F}=2} - 2S_{1/2}^{\text{F}=1})$		3.9207	22
Sum of corrections to Lamb shift		$209.978(5) - 5.226\, r_{\text{p}}^2 + 0.0347\, r_{\text{p}}^3$	

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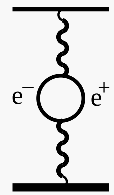

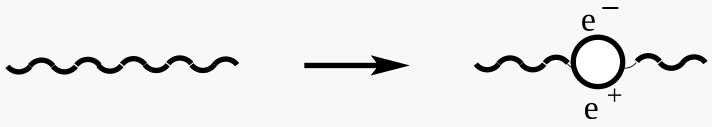

$$\frac{1}{\mathbf{q}^2} \longrightarrow \frac{1}{\mathbf{q}^2}\Pi(q^2)\frac{1}{\mathbf{q}^2}$$

$$V_{\mathrm{VP}}(q) = -4\pi Z\alpha \frac{1}{\mathbf{q}^2}\,\frac{\Pi(q^2)}{\mathbf{q}^2}$$

$V_{\mathrm{VP}}(r) \sim \delta(r)$ since $\Pi(q^2) \sim q^4$

$$\Delta E^{VP} = \langle \bar\Psi(r)|V(r)|\Psi(r)\rangle =$$

$$\int \frac{d^3q}{(2\pi)^3}\,\frac{-4\pi Z\alpha F(q^2)}{\mathbf{q}^2}\,\frac{\Pi(q^2)}{\mathbf{q}^2}\int d^3r\,|\Psi|^2 e^{-i\mathbf{q}\mathbf{r}}$$

Contributions to the μp Lamb shift

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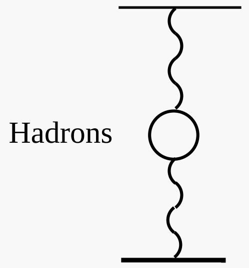

$$\frac{1}{q^2} \longrightarrow \frac{1}{q^2}\Pi(q^2)\frac{1}{q^2}$$

$(e^+ e^-)\ (\mu^+ \mu^-)$	Hadrons
QED	QCD
“ab initio” calculable	NOT calculable

$$\Pi(q^2) \sim \int_{4m_\pi^2}^{\infty} dt\, \frac{\sigma_{e^+e^-\to \mathrm{had}}(t)}{(t+|\vec{q}|^2)}$$

Experimental input required

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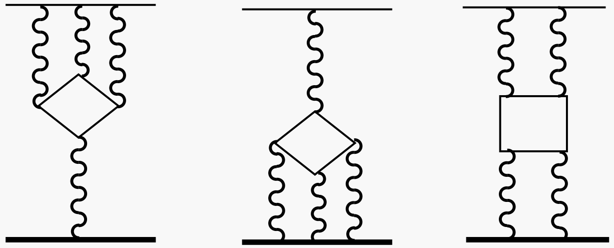
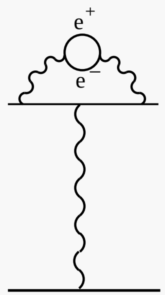
Main pure QED uncertainties





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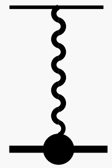
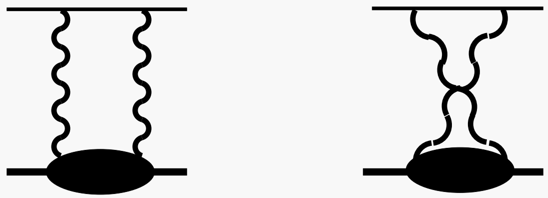


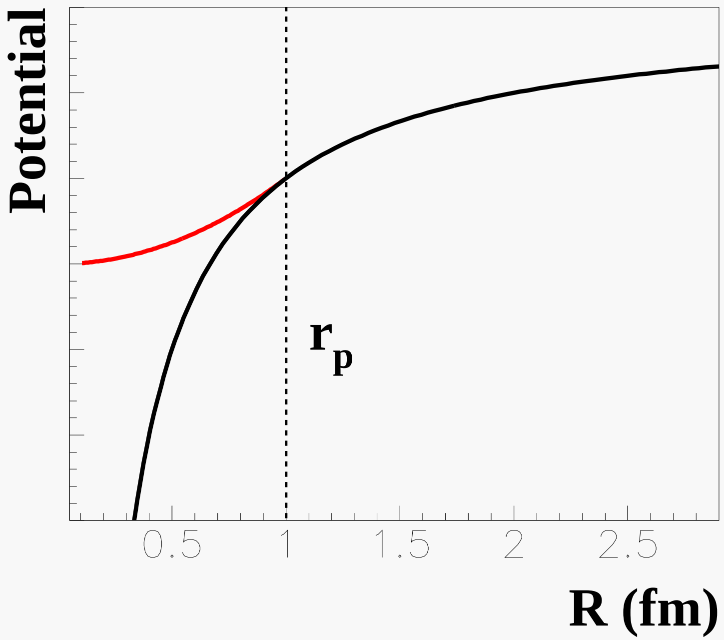
$$\Delta E = \frac{2(Z\alpha)^4}{3n^3} m_r^3\, r_\mathrm{p}^2\, \delta_{l0}$$

$$R_\mathrm{Z} = \int dr_1\, dr_2\, \rho_E(r_1)\rho_M(r_2)\, |\mathbf{r}_1 - \mathbf{r}_2|$$



Experimental input required

The leading proton finite size contribution



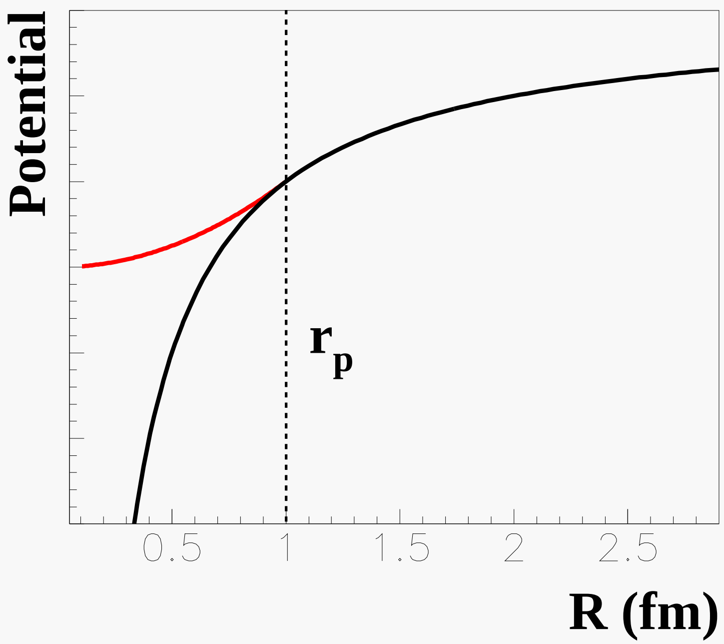
Maxwell equation: $\nabla E = 4\pi\rho$

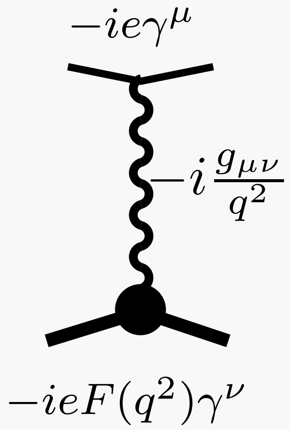
$$V = \begin{cases} -\frac{Z\alpha}{2r_{\rm p}}\left(3-(\frac{r}{r_{\rm p}})^2\right) & (r < r_{\rm p}) \\ -\frac{Z\alpha}{r} & (r > r_{\rm p}) \end{cases}$$

$$\Delta V = \begin{cases} -\frac{Ze^2}{2r_{\rm p}}\left(3-(\frac{r}{r_{\rm p}})^2 - \frac{2r_{\rm p}}{r}\right) \\ 0 \end{cases}$$

$$\Delta E^{FS} = \langle\bar{\Psi}|\Delta V|\Psi\rangle$$

The leading proton finite size contribution





$$\frac{1}{q^2} \longrightarrow \frac{F(q^2)}{q^2}$$

$$r_\mathrm{p}^2 \equiv \int d^3r\, \rho(\mathbf{r}) r^2$$

$$F(\mathbf{q^2}) = \int d^3r\, \rho(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \simeq Z(1 - \frac{\mathbf{q^2}}{6} r_\mathrm{p}^2 + \cdots)$$

Maxwell equation: $\nabla E = 4\pi\rho$

$$V = \begin{cases} -\frac{Z\alpha}{2r_\mathrm{p}}\left(3 - (\frac{r}{r_\mathrm{p}})^2\right) & (r < r_\mathrm{p}) \\ -\frac{Z\alpha}{r} & (r > r_\mathrm{p}) \end{cases}$$

$$\Delta V = \begin{cases} -\frac{Ze^2}{2r_\mathrm{p}}\left(3 - (\frac{r}{r_\mathrm{p}})^2 - \frac{2r_\mathrm{p}}{r}\right) \\ 0 \end{cases}$$

$$\Delta E^{FS} = \langle \bar{\Psi} | \Delta V | \Psi \rangle$$

$$\Delta V(r) = V(r) - \left(-\frac{Z\alpha}{r}\right)$$

$$\Delta V(\mathbf{q}) = \frac{4\pi Z\alpha}{\mathbf{q}^2}(1 - F(\mathbf{q})) \simeq \frac{2\pi(Z\alpha)}{3} r_\mathrm{p}^2$$

$$\Delta V(r) = \frac{2\pi(Z\alpha)}{3} r_\mathrm{p}^2\, \delta(r)$$

$$\begin{aligned} \Delta E^{FS} &= \frac{2\pi(Z\alpha)}{3} r_\mathrm{p}^2 |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3\, r_\mathrm{p}^2\, \delta_{l0} \end{aligned}$$

How large is the proton?

What may be wrong?

$$L^{\text{th.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - L^{\text{exp.}}_{\mu p} = \begin{cases} 80 \text{ GHz} \\ 0.32 \text{ meV} \\ 0.15 \text{ \%} \end{cases}$$

μp th. wrong?

μp exp. wrong?

H th. wrong?

H experiments wrong? $\rightarrow R_\infty$ wrong?

What may be wrong?

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μp th. wrong?

H th. wrong?

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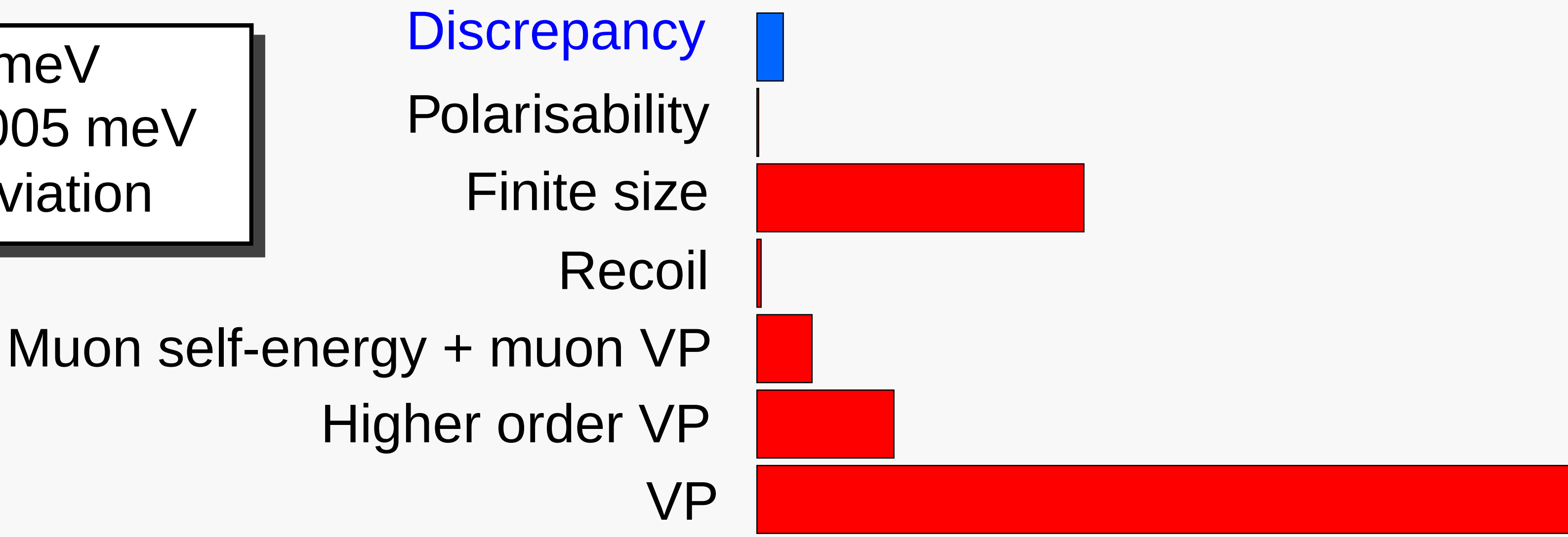
μp exp. wrong?

μp theory wrong?

Discrepancy=0.32 meV
Th. uncertainty=0.005 meV
$\Longrightarrow 60\delta(\text{theory})$ deviation



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μp th. wrong?

H th. wrong?
H experiments wrong? $\rightarrow R_\infty$ wrong?

μp exp. wrong?

μp experiment wrong?

Frequency mistake by 80 GHz ($\Leftrightarrow 0.15\%$)?

(in optics people measure frequency with $u_r \sim 10^{-14}$ and Hz precison)

What may be wrong?

$$L^{\text{th.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - L^{\text{exp.}}_{\mu p} = \begin{cases} 80 \text{ GHz} \\ 0.32 \text{ meV} \\ 0.15\ \% \end{cases}$$

μp th. wrong?

H th. wrong?

H experiments wrong? $\rightarrow R_\infty$ wrong?

μp exp. wrong?

H experiments wrong?

H theory wrong?

Hydrogen energy levels

and the proton radius

Hydrogen energy levels and spectroscopy

- Hydrogen energy levels and definition of the Lamb shift L

$$E_{n,l,j,F} = E^{\text{Dirac}}_{n,j} + E^{\text{Recoil(0)}}_{n} + L_{n,l,j} + E^{HF}_{n,l,j,F}$$

Higher order recoil QED Nuclear structure effects

Hydrogen energy levels and spectroscopy

- Hydrogen energy levels and definition of the Lamb shift L

$$E_n \simeq \frac{R_\infty}{n^2} + L_{n,l,j}(r_{\mathrm{p}}, \mathrm{QED}, \alpha, m_e \ldots)$$

Hydrogen energy levels and spectroscopy

- Hydrogen energy levels and definition of the Lamb shift L

$$E_n \simeq \frac{R_\infty}{n^2} + L_{n,l,j}(r_{\mathrm{p}}, \mathrm{QED}, \alpha, m_e \dots)$$

- Hydrogen transition frequencies

$$\nu(1S-2S) \simeq (1-\frac{1}{4})R_\infty + L_{1S} - L_{2S}$$

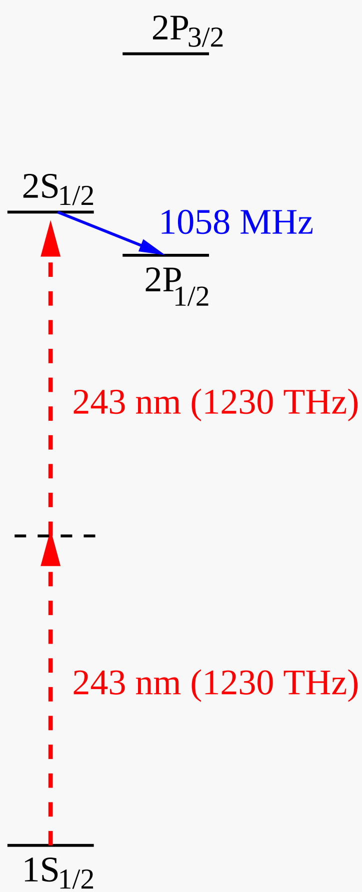
$$\nu(2S-8S) \simeq (\frac{1}{4}-\frac{1}{64})R_\infty + L_{2S} - L_{8S}$$

$$L_{nS} = \frac{1}{n^3}L_{1S} + \epsilon$$

Hydrogen energy levels and spectroscopy

- Hydrogen energy levels and definition of the Lamb shift L

$$E_n \simeq \frac{R_\infty}{n^2} + L_{n,l,j}(r_\mathrm{p}, \mathrm{QED}, \alpha, m_e \dots)$$



- Hydrogen transition frequencies

$$\nu(1S-2S) \simeq (1-\frac{1}{4})R_\infty + L_{1S} - L_{2S}$$

$$\nu(2S-8S) \simeq (\frac{1}{4}-\frac{1}{64})R_\infty + L_{2S} - L_{8S}$$

$$L_{nS} = \frac{1}{n^3}L_{1S} + \epsilon$$

- Measured transitions

$2S-2P$ (classical Lamb shift, one-photon transition)

$1S-2S, \quad 2S-8S/D, \quad 2S-12S/D, \quad 2S-4S, \quad 1S-3S \quad 2S-6S \dots$

$\dots$ Rydberg states (two-photon transitions)

Bound-state QED in H and r_p

- Lamb shift from measurements in H

$$\left.\begin{array}{rl} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array}\right\} \Rightarrow \quad L_{1S}^{\text{exp}} = 8172.840(19)\,\text{MHz}$$

R_∞

Bound-state QED in H and r_p

- Lamb shift from measurements in H

$$\left.\begin{array}{rl} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array}\right\} \Rightarrow \quad L_{1S}^{\text{exp}} = 8172.840(19)\text{MHz}$$

R_∞

- Lamb shift theoretical prediction

$$\left.\begin{array}{r} \text{QED} \\ r_\text{p} \\ \alpha,\ m_e,\ m_p\ c\ldots \end{array}\right\} \Rightarrow \quad L_{1S}^{\text{th}}(r_\text{p}) = 8171.636(4) + 1.5645\, r_\text{p}^2 \quad \text{MHz}$$

Bound-state QED in H and r_p

- Lamb shift from measurements in H

$$\left.\begin{array}{rl} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array}\right\} \Rightarrow \quad L_{1S}^{\mathrm{exp}} = 8172.840(19)\mathrm{MHz}$$

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- Proton radius from hydrogen spectroscopy

$L_{1S}^{\mathrm{th}}(r_\mathrm{p}) = L_{1S}^{\mathrm{exp}} \Longrightarrow r_\mathrm{p} = 0.8760(78)$ fm with $u_r = 1\%$

Bound-state QED in H and r_p

- Lamb shift from measurements in H

$$\left.\begin{array}{rl} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array}\right\} \Rightarrow \quad L_{1S}^{\text{exp}} = 8172.840(19)\text{MHz}$$

R_∞

- Lamb shift theoretical prediction

$$\left.\begin{array}{r} \text{QED} \\ r_\text{p} \\ \alpha,\ m_e,\ m_p\ c\dots \end{array}\right\} \Rightarrow \quad L_{1S}^{\text{th}}(r_\text{p}) = 8171.636(4) + 1.5645\, r_\text{p}^2 \quad \text{MHz}$$

- Bound-state QED test in hydrogen

$$\left.\begin{array}{r} L_{1S}^{\text{th}}(r_\text{p}) \\ r_\text{p} = 0.895(18)\ \text{fm (from scatt.)} \\ r_\text{p} = 0.84184(76)\ \text{fm (from } \mu\text{p)} \end{array}\right\} \Rightarrow \quad \begin{array}{ll} L_{1S}^{\text{th}} = 8172.889(4)(50)\text{MHz} & \text{(scatt.)} \\ L_{1S}^{\text{th}} = 8172.742(4)(2)\text{MHz} & (\mu\text{p}) \end{array}$$

Discrepancy with L_{1S}^{exp}

Bound-state QED in H and $\mathrm{r_p}$

- Lamb shift from measurements in H

$$\left.\begin{array}{rl} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array}\right\} \Rightarrow \quad L_{1S}^{\mathrm{exp}} = 8172.840(19)\mathrm{MHz}$$

R_∞

- Lamb shift theoretical prediction

$$\left.\begin{array}{r} \mathrm{QED} \\ r_\mathrm{p} \\ \alpha,\ m_e,\ m_p\ c\ldots \end{array}\right\} \Rightarrow \quad L_{1S}^{\mathrm{th}}(r_\mathrm{p}) = 8171.636(4) + 1.5645\, r_\mathrm{p}^2 \quad \mathrm{MHz}$$

- Discrepancy between theory and experiment

$$\boxed{L_{1S}^{\mathrm{th}}(r_\mathrm{p}^{\mu p}) - L_{1S}^{\mathrm{exp}} = 96(19)(4)(2)\ \text{kHz}}$$

δR_∞ $\delta\mathrm{QED}$ δr_p

Are experiments in hydrogen wrong?

Discrepancy between theory and experiment

$$L^{\rm th}_{1S}(r_{\rm p}^{\mu p}) - L^{\rm exp}_{1S} = 96(19)(4)(2)\ \text{kHz}$$

δR_∞ (2S-8S) δQED $\delta r_{\rm p}$

$L^{\rm exp}_{1S}$ extracted from $1S-2S$ and $2S-8S$ transition

- $1S-2S$ has to be corrected by thousands of σ to explain the discrepancy
- $2S-8/12S$ has to be corrected by 5σ to explain the discrepancy
  this corresponds to a small relative change $d\nu/\nu = 5\times 10^{-11}$
  $\rightarrow$ need to control the systematics at this level of precison
  (systematics $\sim n^3$, measurements accuracy $\sim 1/100\ \Gamma$)

Free and bound-state QED

- Free QED

$g-2 \quad \rightarrow \quad$ electron anomaly: test of QED, determination of α, NP

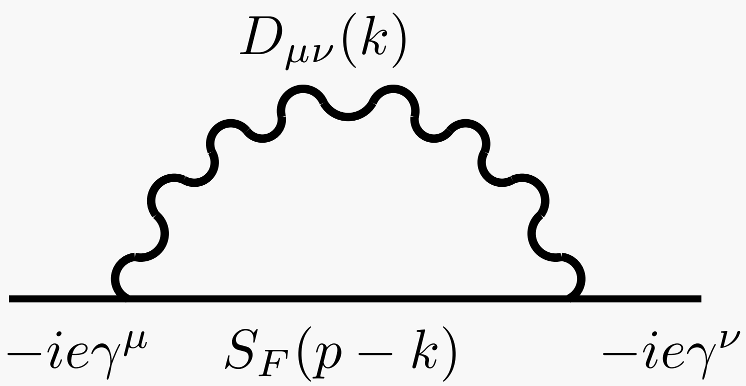
$$a_e = C_1 \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \Delta(\text{had.}, \text{NP})$$

$$u[a_e^{\text{exp}}] = 6.6 \times 10^{-10}, \quad u[a_e^{\text{th}}] = 2.4 \times 10^{-10}, \qquad u[\text{QED test}] = 7 \times 10^{-9}$$

- Bound-state QED in Hydrogen

  - Binding effects ($Z\alpha$) — bad convergence, all-order approach/expansion
  - Radiative corrections (α and $Z\alpha$)
  - Recoil corrections (m/M and $Z\alpha$) — relativity $\not\Leftrightarrow$ two-body system
  - Radiative–recoil corrections (α, m/M and $Z\alpha$)
  - Proton structure corrections (r_{p}, r_{Zemach} and $Z\alpha$)

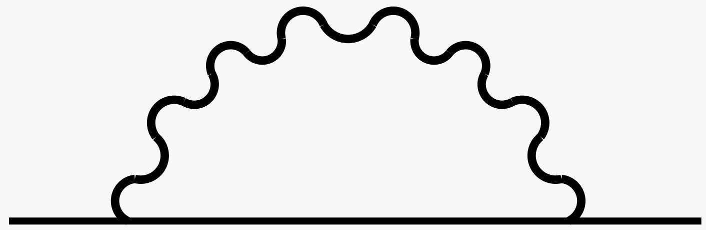
Free electron propagator



$\alpha = e^2/4\pi$

$$-i\Sigma^{(1)}(p) = \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(k)(-ie\gamma^\mu)S_F(p-k)(-ie\gamma^\nu)$$

Free electron propagator



$\alpha = e^2/4\pi$

$$-i\Sigma^{(1)}(p) = \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(k)(-ie\gamma^\mu)S_F(p-k)(-ie\gamma^\nu)$$

Photon propagator

$$D_{\mu\nu}(k) = \frac{ig_{\mu\nu}}{k^2}$$

Electron propagator

$$S_F(q) = \frac{i}{\not{q}-m}$$

Propagator

$$G(E) = \frac{i}{H-E}$$

Maxwell equation

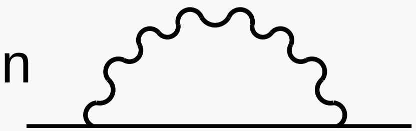
$$g_{\mu\lambda}(\partial_\mu\partial^\mu)A^\lambda = 0$$

Dirac equation

$$(\not{\partial} - m)\phi = 0$$

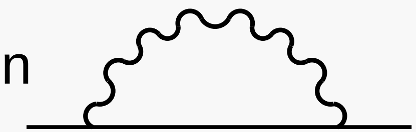
Bound (Coulomb) electron propagator

Free electron



Bound electron

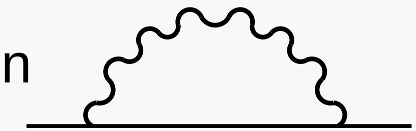
Bound (Coulomb) electron propagator

Free electron   Bound electron

Dirac equation in Coulomb potential $\rightarrow$ Bound electron propagator

$$(\not q - m)\phi = \gamma^0 V \phi \qquad S_B(q) = \frac{i}{\not q - m - \gamma^0 V}$$

Bound (Coulomb) electron propagator

Free electron   Bound electron

Dirac equation in Coulomb potential $\rightarrow$ Bound electron propagator

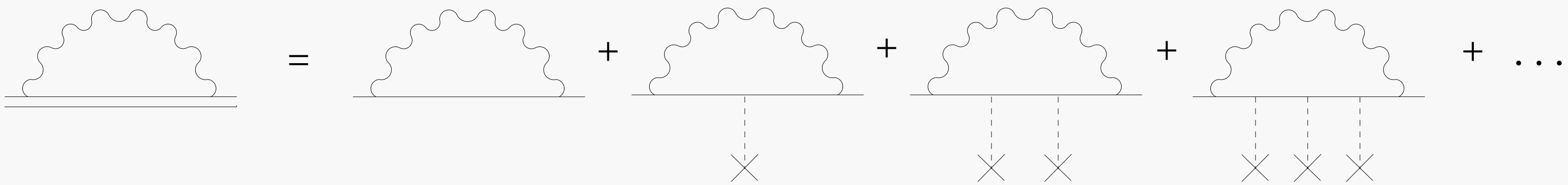
$$(\not q - m)\phi = \gamma^0 V \phi \qquad\qquad S_B(q) = \frac{i}{\not q - m - \gamma^0 V}$$

$$\frac{1}{X-Y} = \frac{1}{X} + \frac{1}{X}Y\frac{1}{X} + \frac{1}{X}Y\frac{1}{X}Y\frac{1}{X} + \frac{1}{X}Y\frac{1}{X}Y\frac{1}{X}Y\frac{1}{X} + \cdots$$

[U. Jentschura]

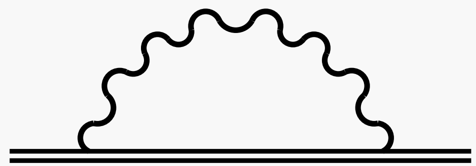
$$\frac{1}{\not q - m - \gamma^0 V} = \frac{1}{\not q - m} + \frac{1}{\not q - m}\gamma^0 V \frac{1}{\not q - m} + \frac{1}{\not q - m}\gamma^0 V \frac{1}{\not q - m}\gamma^0 V \frac{1}{\not q - m} + \cdots$$

Expansion in $(Z\alpha)^2$ since $V = \frac{Z\alpha}{r}$, r $\sim 1/(Z\alpha)$



One-loop self-energy in hydrogen

$$\Delta E_{SE}^{(1)} = \langle\bar{\Psi}|\Sigma_{\text{bound}}^{(1)}|\Psi\rangle$$



$$= ie^2 \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(k) \langle\bar{\Psi}|\gamma^\mu S_B(p)\gamma^\nu|\Psi\rangle - \langle\bar{\Psi}|\delta m|\Psi\rangle$$

$$= m \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F_n(Z\alpha)$$

$(Z\alpha)$ expansion *vs.* all-order approach

- Perturbative expansion of the Dirac-Coulomb propagator in $(Z\alpha)$

$$F_n = A_{40} + A_{41} \ln (Z\alpha)^{-2} + (Z\alpha)A_{50} + (Z\alpha)^2 \left[A_{62} \ln^2 (Z\alpha)^{-2} + A_{61} \ln (Z\alpha)^{-2} + G\right]$$

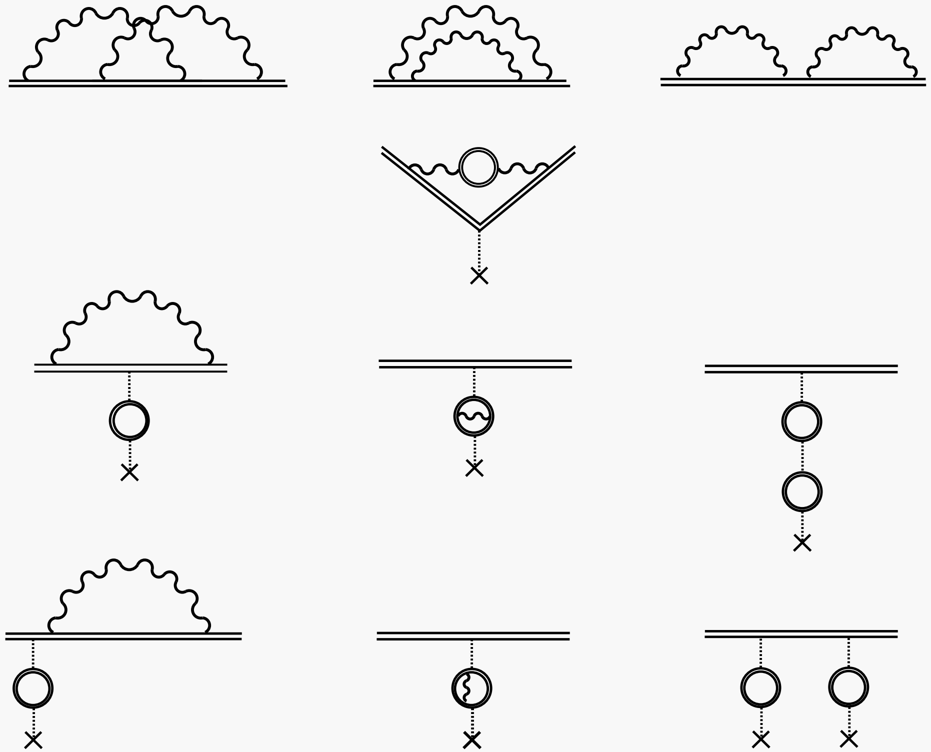
$$G = A_{60} + (Z\alpha)\left[A_{71} \ln (Z\alpha)^{-2} + A_{70}\right] + (Z\alpha)^2 \left[A_{83} \ln^3 (Z\alpha)^{-2} + A_{82} \ln^2 (Z\alpha)^{-2} + A_{81} \ln (Z\alpha)^{-2} + A_{80}\right]$$

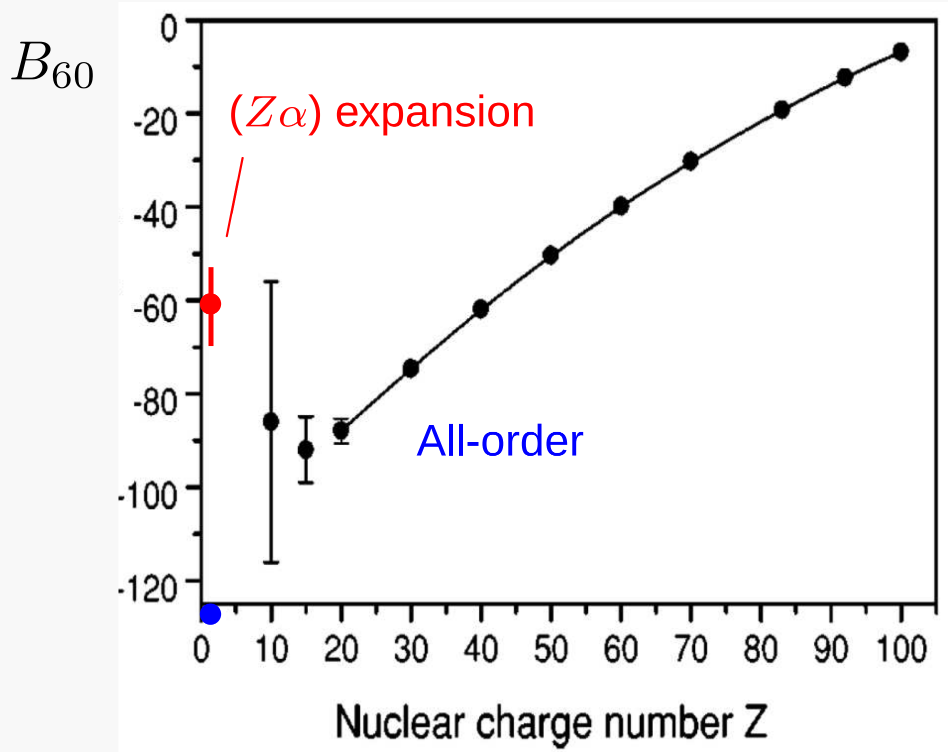
Uncertainty related to truncation of $(Z\alpha)$ expansion: 28 kHz

- All-order numerical exact treatment of the Dirac-Coulomb propagator

Uncertainty related to the numerical approach: 0.8 Hz

Two-loop self-energy in hydrogen





$$\Delta E_{SE}^{(2)} = m\left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} G_n(Z\alpha)$$

$$G_n = B_{40} + (Z\alpha)B_{50} + (Z\alpha)^2 \left[B_{63}\ln^3(Z\alpha)^{-2} + B_{62}\ln^2(Z\alpha)^{-2} + B_{61}\ln(Z\alpha)^{-2} + G_{h.o}\right] + \cdots$$

$$G_n = 1.409 - 0.177 \quad + [-0.015 - 0.003 + 0.026 - 0.003 + \cdots] \quad + \cdots$$

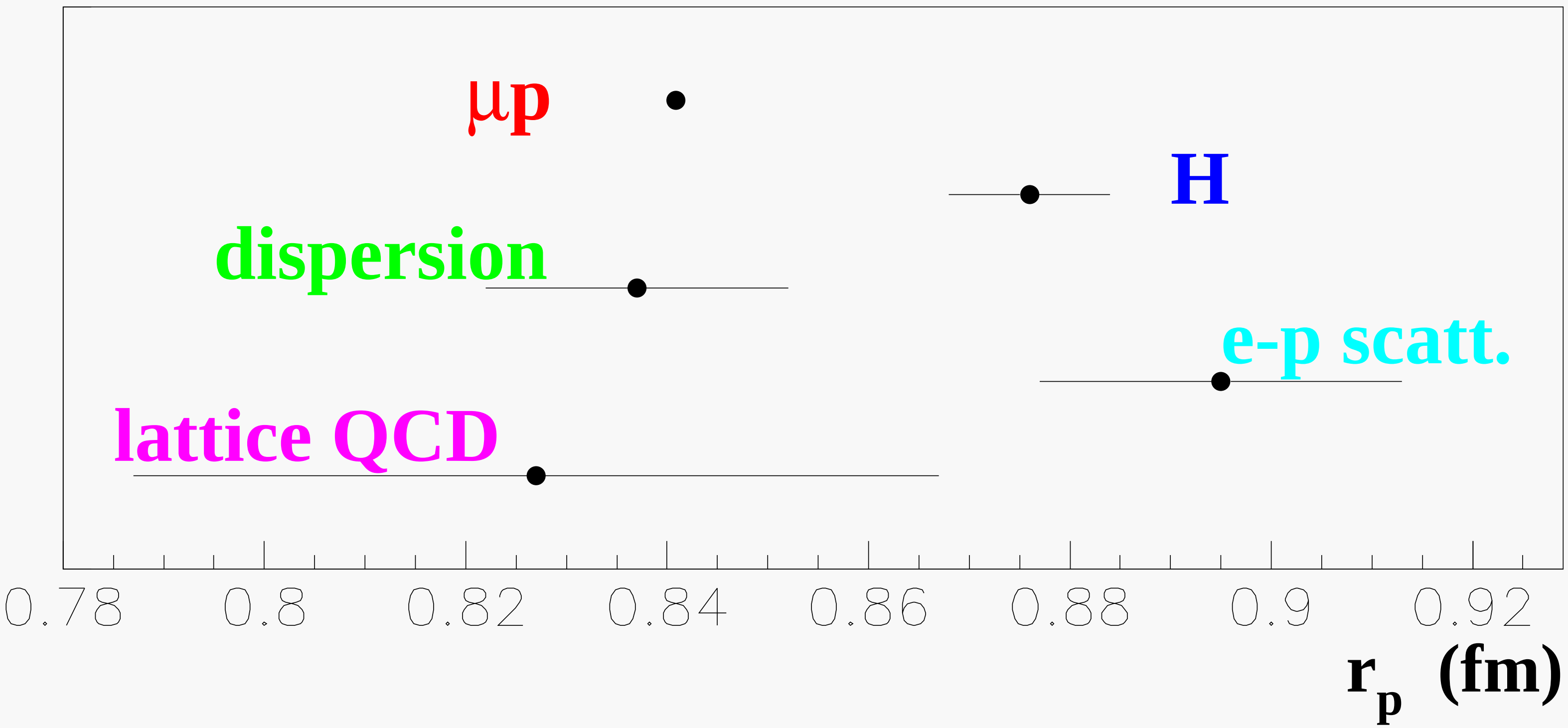
Bad convergence of the ($Z\alpha$) expansion

Is the theory in hydrogen wrong?

- According to theorists the only critical terms are coming from the two-loop self-energy.
- The difference between all-order and expansion approaches (for the two-loop self-energy) shifts the L_{1S} by 7 kHz
- The most crucial terms $B_{60} + B_{71} < 15$ kHz
- Compare this th. uncertainties with: $L_{1S}^{\rm th}(r_{\rm p}^{\rm CODATA}) - L_{1S}^{\rm th}(r_{\rm p}^{\mu{\rm p}})$=96 kHz

The theory should be corrected by $25 \times \delta(\text{theory})$ to bring the value of $r_{\rm p}$ extracted from H-spectroscopy in agreement with our value

Proton radius puzzle



- Pohl et al., submitted to Nature
- Mohr at al., Rev. Mod. Phys. **80** 633 (2008)
- Belushkin et al., Phys. Rev. C **75** 035202 (2007)
- Sick, Phys. Lett. B **576** 62 (2003)
- Wang et al., Phys. Rev. D **79** 094001 (2009)

The origin of the discrepancy?

- QED th. in μp: 60 δ(theory)
- QED th. in H: 25 δ(theory)
- 1S-2S in H: thousands of σ
- 2S-8S in H: 5 σ
- new physics?

Interesting confrontation with

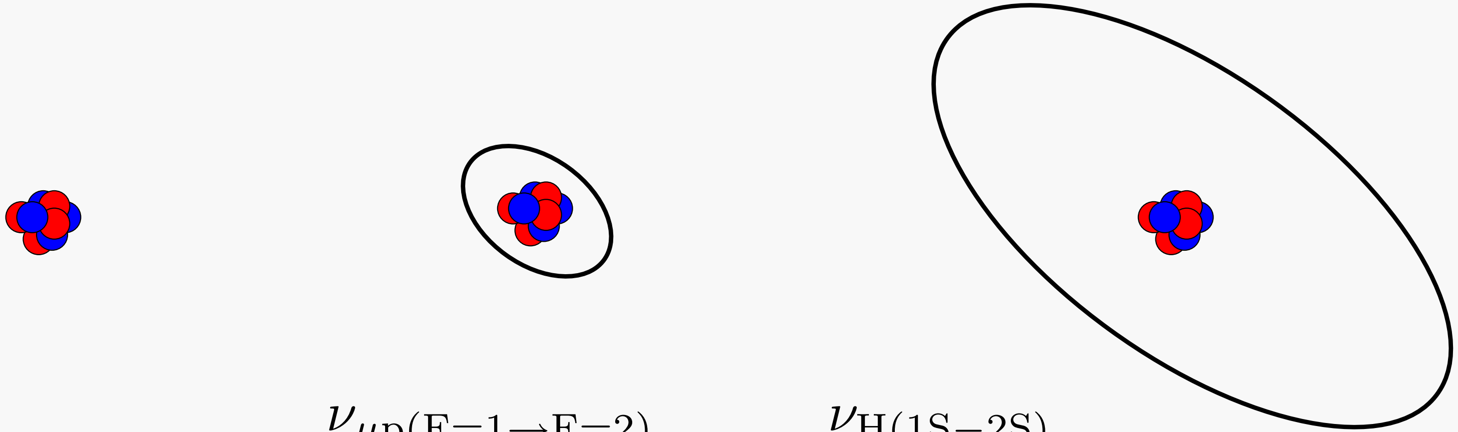
- e-p scattering
- dispersion analysis

Other measurements

Atomic and nuclear physics interplay

Atomic physics ⟷ nuclear physics

Nucleus | Muonic atoms | "Normal" atoms



$\nu_{\mu p(F=1 \rightarrow F=2)}$ ↓ $\nu_{H(1S-2S)}$ ↓

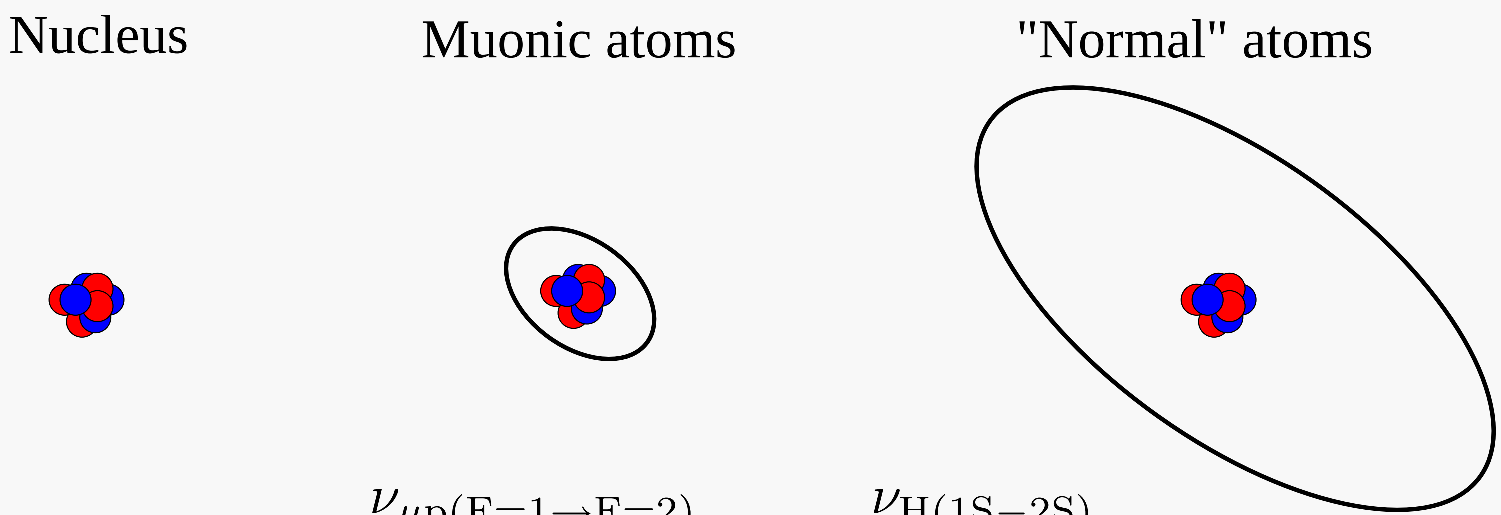
p. pol. → r_p → Bound-state QED test

Rydberg constant

Check p. model ← r_p

Bound-state th. ← Bound-state QED test

Atomic physics $\longleftrightarrow$ nuclear physics



$\nu_{\mu p(F=1 \rightarrow F=2)}$ → r_p

$\nu_{H(1S-2S)}$ → Bound-state QED test

p. pol. → r_p → Bound-state QED test, Rydberg constant

r_p → Check p. model

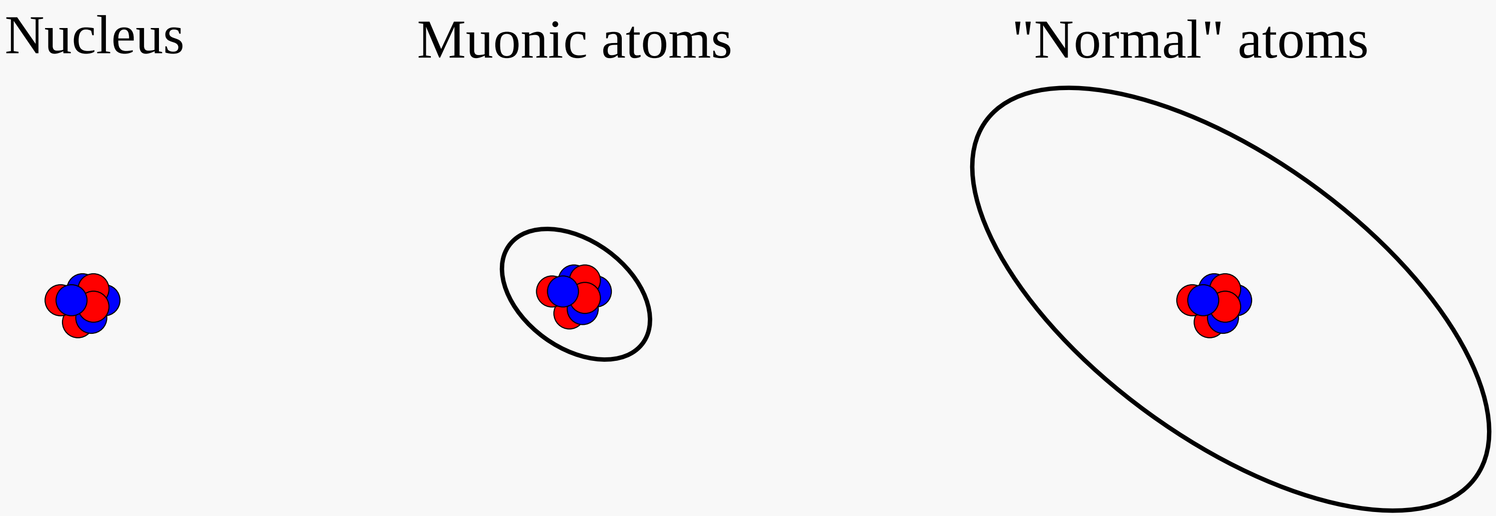
Bound-state QED test / Rydberg constant → Bound-state th.

p. pol. (HFS) → r_{Zemach}

r_{Zemach} → Check p. spin th.

HFS=$\nu_{\mu p(F=1 \rightarrow F=2)} - \nu_{\mu p(F=0 \rightarrow F=1)}$ → r_{Zemach}

Isotope shift, r_d and deuteron pol.



d. pol.

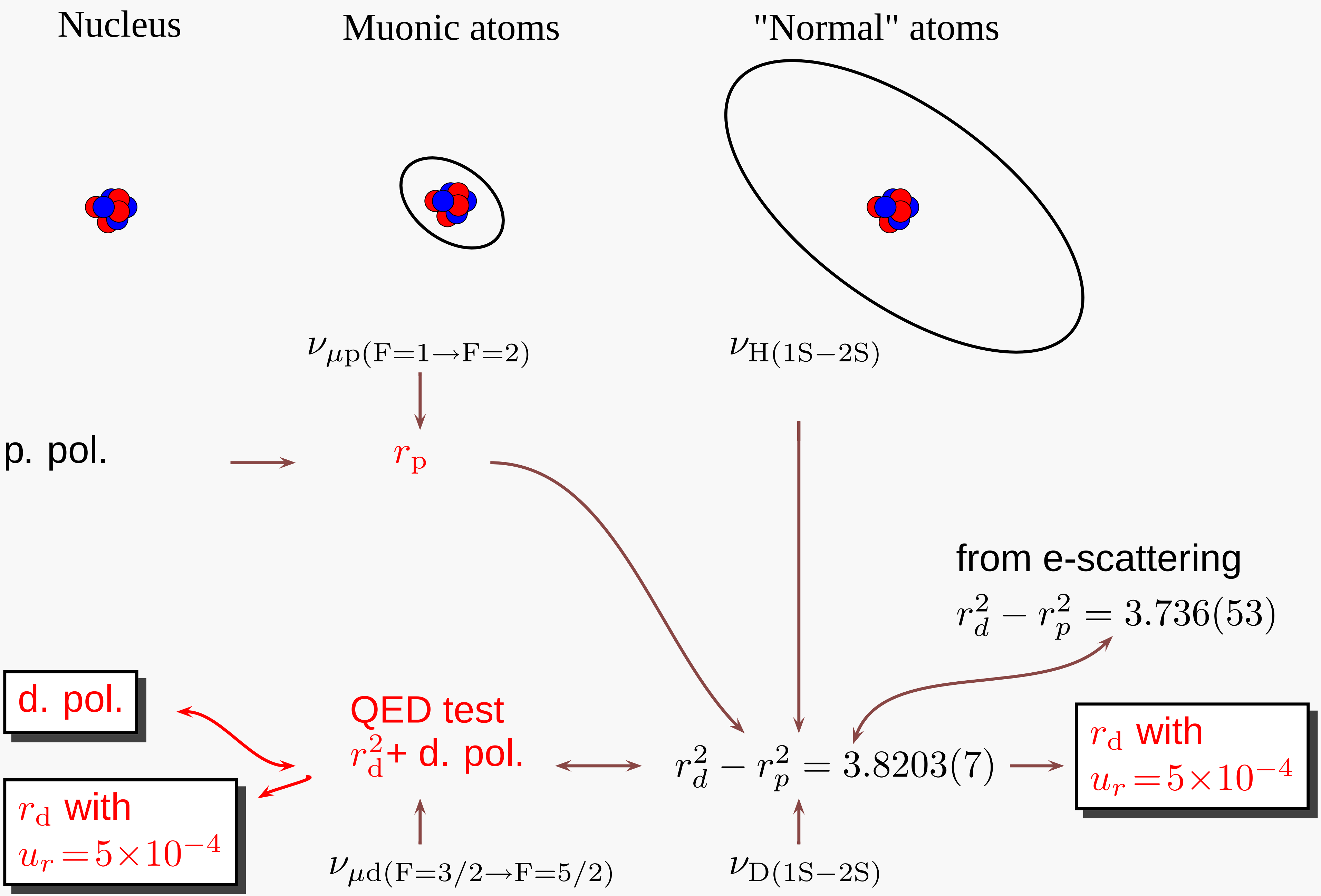
r_d with
$u_r = 5\times 10^{-4}$

QED test
r_d^2+ d. pol.

$\uparrow$

$\nu_{\mu\mathrm{d}(\mathrm{F}=3/2\rightarrow \mathrm{F}=5/2)}$

Isotope shift, r_d and deuteron pol.



Summary

- We have measured 5 transitions in μp and μd
- Discrepancy may originate from:
  - QED theory in μp (has to shift by 60$\times\delta$(theory) $\Leftrightarrow 1.6 \times 10^{-3}$)
  - QED theory in H (has to shift by $25 \times \delta$(theory)$\Leftrightarrow$ 96 kHz)
  - R_∞ (has to change by 5σ which is a 5×10^{-11} rel. effect)
  - New physics?
- These measurements lead to
  - r_{p}, r_{d} determination ($10\times$ better)
  - r_{Zemach} determination
  - Deuteron polarizability
  - QED test in hydrogen/deuterium and muonic hydrogen/deuterium
  - R_∞ determination ($6\times$ better)

Proton and deuteron models

Bound-state theories
Fundamental constants

- New experiment: μHe$^+$
  - Solve discrepancy and enhance sensitivity to QED effects, few-nucleon th.

Truth is ever to be found in the simplicity, and not in the multi-plicity and confusion of things.

Sir Isaac Newton (1642-1727)

The spectrum of hydrogen atom has proved to be the Rosetta stone of modern physics.

T.W. Hänsch