A Muon Magnetic Moment Measurement

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Abstract

The muon had and will have an interesting role in particle physics and that is why it is important, in the context of a master, to experimentally infer some of its main intrinsic parameters. To achieve this goal, cosmic antimuons are stopped in a copper absorber placed between Helmholtz coils. First, the lifetime is calculated with the electromagnets switched off as a consistency check and yields, at $1\sigma$ CL,

$$\tau = (2.193 \pm 0.010) \mu s .$$

Subsequently, two measurements of the gyromagnetic ratio were performed for two different magnetic field settings, $B_+ = +(42.9 \pm 0.1) \text{ G}$ and $B_- = -(25.6 \pm 0.1) \text{ G}$. They yield similar and consistent values summarized in the table below for each of the two different fitting method A and B thoroughly described in the data analysis section.

<table>
<thead>
<tr>
<th>$B_+$</th>
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<tbody>
<tr>
<td>$g_A = 1.982 \pm 0.034$</td>
<td>$g_A = 2.013 \pm 0.061$</td>
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<td>$g_B = 1.991 \pm 0.025$</td>
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1 Introduction

Muons were discovered by Carl D. Anderson and Seth Neddermeyer at Caltech in 1936, while studying cosmic radiation. Anderson had noticed particles that curved differently from electrons and other known particles when passed through a magnetic field. Anderson initially called the new particle a mesotron, adopting the prefix meso- from the Greek word for "mid-". The existence of the muon was confirmed in 1937 by J. C. Street and E. C. Stevenson’s cloud chamber experiment [1].

It was first believed to be the particle in the muon’s mass range which existence had been predicted by Hideki Yukawa [2] but later proved to have the wrong properties. Yukawa’s predicted particle was finally identified in 1947. In order to differentiate between the two different types of mesons after the second meson was discovered, the initial mesotron particle was renamed the mu meson (the Greek letter \( \mu \) (mu) corresponds to m), and the new 1947 meson (Yukawa’s particle) was named the pi meson.

As more types of mesons were discovered in accelerator experiments later, it was eventually found that the mu meson significantly differed from all other types of mesons. The difference was that mu mesons did not interact with the nuclear force and that its decay products included both a neutrino and an antineutrino.

In the eventual Standard Model of particle physics codified in the 1970s, mu mesons had shown themselves to be fundamental particles (leptons) like electrons, with no quark structure. Thus, mu mesons were not mesons at all. With this change in definition, the term mu meson was abandoned, and replaced whenever possible with the modern term muon.

Muons have had and will have an important role in particle physics. In 1941 for instance, Rossi and Hall used muons to observe the time dilation (or alternately, length contraction) predicted by special relativity, for the first time [3]. Moreover, research and development on muon cooling is in progress (MICE [4]) and could lead to several applications. On the one hand, neutrino factory experiments such as \( \nu \)STORM [5] plan on using a muon storage ring to produce clean neutrino beams. On the other hand, muon colliders could be the future of \( e^+e^- \) synchrotron as muons experience much lower radiative losses than electrons.

The aim of this experiment is to measure the muon lifetime and magnetic moment. To achieve this goal, we use a copper plate which traps antimuons long enough for them to decay. The time measured between the capture of an antimuon and the detection of a positron (product of its decay along with undetectable neutrinos), gives enough information to measure the lifetime with good accuracy. Moreover, the addition of a uniform magnetic field parallel to the plate makes the antimuon spin precess around the magnetic field lines. The determination of the Larmor frequency in this magnetic field gives the last parameter needed to derive the muon magnetic moment.

This laboratory report will start with a theoretical reminder necessary to the understanding of this experiment, followed by a description of the apparatus and ending with results drawn from a one semester intermittent run.
2 Theoretical background

2.1 Muon characteristics

The muon is an elementary particle similar to the electron, with unitary negative electric charge (-1) and a spin of 1/2, but with much more mass. Together with the electron, the tau, and the three neutrinos, it is classified as a lepton. As is the case with other leptons, the muon is not believed to have any sub-structure at all. Like all elementary particles, the muon has a corresponding antiparticle of opposite charge (+1) but equal mass and spin: the antimuon.

The muon is an unstable subatomic particle with a mean lifetime of approximately 2.2 $\mu$s. This comparatively long decay lifetime (the second longest known after the neutron) is due to being mediated by the weak interaction.

Muons have a mass of $105.7 \text{MeV}/c^2$, which is about 200 times the mass of an electron [6]. Due to their greater mass, muons are not as sharply accelerated when they encounter electromagnetic fields, and do not emit as much bremsstrahlung (deceleration radiation). This allows muons of a given energy to penetrate far more deeply into matter than electrons, since the deceleration of electrons and muons is primarily due to energy loss by the bremsstrahlung mechanism. As an example, so-called "secondary muons", generated by cosmic rays hitting the atmosphere, can penetrate to the Earth’s surface, and even into deep mines.

In this report, we will focus on the positively charged muon and not its counterpart as the latter is trapped in the absorber [7] in a time much shorter than the lifetime of the muon ($\tau_{\text{trap}} \approx 0.163 \mu s << \tau$) through the process

$$\mu^- + Cu \rightarrow Ni + \nu_{\mu} + \gamma,$$  \hspace{1cm} (2.1)

$$\mu^- + u \rightarrow d + \nu_{\mu} + \gamma.$$ \hspace{1cm} (2.2)

This section will follow a chronological progression from the creation of the muons to their decay.

2.2 Cosmic rays

The cosmic muons are created by cosmic rays entering the Earth’s atmosphere. Strictly speaking, the cosmic rays are not only rays, but can as well be particles from outer space produced by our sun, other galactic sources or extragalactic sources (active galactic nuclei, black holes, quasars, supernovae, etc.). They consist mainly of protons, electrons, light nuclei and photons. Their energy ranges all the way close to the ZeV (Record energy: $3 \times 10^{20} \text{eV}$ at HiRes [8]).

When the cosmic rays enter the atmosphere, they can interact with the atoms composing it in different fashions. The electrons, positrons and photons will produce electromagnetic showers that don’t concern us in the scope of this experiment as they do not involve muons. Protons and light nuclei on the other hand will collide with nitrogen, oxygen and carbon nuclei and generate more complex hadronic showers which will further result in the production of our particle of interest [9].

2.3 Hadronic showers

Most of the hadronic showers will originate from protons as they are the most abundant particles in cosmic rays. The primary proton interacts with a nucleus to produce mainly pions (kaons can as well be produced). The phenomena which determine the development of the hadronic showers are: hadron production, nuclear deexcitation and pions and muon decays [9]. Neutral pions amount, on average, to 1/3 of the produced pions and their energy is dissipated in the form of electromagnetic showers. Charged pions on the other hand can further decay into muons.
For instance, the positive pion decays through the following weak process

\[ \pi^+ \to \mu^+ + \nu_\mu. \]  

(2.3)

Figure 2.1: Weak decay of the positive pion into an antimuon.

It is interesting to compute the total decay width of the pion into a lepton and a neutrino, considering any possible outgoing lepton. As the mass of the pion is way below the W boson’s mass, we can approximate the interaction by a point like 4 fields coupling. The computation is lengthly and non trivial as the pion is a bound state of the quarks and, as a result, cannot be described by perturbative QCD. Nevertheless, all the unknowns can be summarized into one structure constant \( f_\pi \). The final result is given by

\[ \Gamma(\pi^+ \to l^+\nu_l) = \frac{G_F^2 f_\pi^2 \cos^2 \theta_c}{8\pi} \frac{m_l^2 m_\pi}{m_\pi^2} \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2, \]  

(2.4)

with \( G_F \) the Fermi constant and \( \theta_c \) the Cabbibo mixing angle [10]. This result allows us to compare the branching ratios of the pion decay into a muon and into an electron:

\[ \frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{m_e^2 (m_e^2 - m_\pi^2)}{m_\mu^2 (m_\mu^2 - m_\pi^2)} = 1.28 \times 10^{-4}, \]  

(2.5)

with \( m_e = 0.511 \text{MeV} \), \( m_\pi = 139.57 \text{MeV} \) and \( m_\mu = 105.66 \text{MeV} \) [6], which shows that the muonic mode dominates by 4 orders of magnitude. If one applies radiative corrections to that result, one can find that this value is shifted a little towards \( 1.24 \times 10^{-4} \) [11]. Experimentally one measures \( (1.228 \pm 0.022) \times 10^{-4} \) which corresponds to the theoretical value within \( 1 \sigma \).

Before reaching the surface of the Earth, most of the protons, pions and electrons respectively are absorbed, decay or lose their energy such that muons is the dominant component alongside with neutrinos. We can observe on Fig. 2.2 that this predominance is important by 2 orders of magnitude over the protons and about 4 over electrons and pions. Moreover, as the stopping power for muons is much lower than the one for electrons\(^1\), the average energy of the formers is higher by a few orders of magnitude at sea level. Thus, taking into account the thickness of the laboratory walls and the lead layer on the top of the absorber, it is safe to say that the electron component in this experiment is negligible\(^2\). The same process can be applied to show that the proton part is equally negligible.

In conclusion, the only component of cosmic rays to be taken into account in this experiment is the muons.

\(^1\)Bremsstrahlung is the main process through which the electrons and muons lose energy in the atmosphere. The total radiated power in this case can be written: \( P_{\text{abs}} = \frac{e^2 \gamma^2}{6\pi \varepsilon_0 c^3} \). This is proportional to \( \gamma^6 \) and, since \( E = \gamma mc^2 \), to \( m^{-6} \), which accounts for the electrons losing their energy much more rapidly than muons [12].

\(^2\)The average energy of the electrons at sea level is in the range of 1 to 10MeV. The stopping power for electrons is of \( 1.585 \text{MeVcm}^2\text{g}^{-1} \) in concrete and \( 11.38 \text{MeVcm}^2\text{g}^{-1} \) in lead, leaving a thin chance for any electron to reach the absorber [13].
2.4 Antimuon decay

After being absorbed in the copper plate, the antimuon decays following the weak process:

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu. \]  \hfill (2.6)

The left Feynman diagram in Fig. 2.3 can once again be approximated by a point like interaction of four fields in the Fermi picture which gives the right diagram as the COM energy is far from the W boson rest mass. It is then possible to compute the invariant amplitude of this transition and, furthermore, the total decay width of this process. The invariant amplitude reads

\[
\frac{4G_F}{\sqrt{2}} [\bar{\mu} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \nu_\mu] \times [\bar{e} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \nu_e],
\]  \hfill (2.7)

with \( \mu, \nu_\mu, e \) and \( \nu_e \) the Dirac fields corresponding to the 4 fermions involved in the reaction. The decay rate for this process is given by

\[
d\Gamma = \frac{1}{2E} |M|^2 dQ.
\]  \hfill (2.8)
Averaging the invariant amplitude over the two initial spins of the muon, summing over the initial and final spins and introducing the phase space \(dQ\) into the equation, we eventually find

\[
\frac{d\Gamma}{dE'} = \frac{G_F^2 m_\mu^2 E'^2}{12\pi^3 m^2} \left(3 - \frac{4E'}{m_\mu}\right),
\]

(2.9)

where \(E'\) is the energy of the positron. This is called the Michel spectrum. The form of the decay rate is represented on Fig. 2.4 and shows that the positron tends to take the biggest amount of energy available, i.e. half of the rest mass of the muon. Finally, integrating over all possible energies, we find the total decay width of the muon

\[
\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3},
\]

(2.10)

which corresponds to the multiplicative inverse of the lifetime of the muon \(\tau\) \[10\], \[11\]. As the decay of a particle is a stochastic phenomenon, it obeys the Poisson statistics \[14\]. The number of muons that survive after a period of time \(t\) can be derived from the differential equation

\[
\frac{dN}{dt} = -\Gamma N = -\frac{N}{\tau},
\]

(2.11)

which solution trivially reads

\[
N(t) = N_0 e^{-t/\tau} + B.
\]

(2.12)

### 2.5 Decay Asymmetry

Since the pion has spin 0 \((J^P = 0^-)\), the products of its decay must have equal and opposite spins. In the context of the standard model, the neutrinos are considered massless\(^3\). As a result, they can only be left handed and, as a result, antimuons have to be left handed too in order to conserve the spin in the pion decay. They can, however, be emitted upwards or downwards in the referential of the pion. These two cases will result in two different helicities in the lab frame, as depicted in Fig. 2.5. A right-handed antimuon with small momentum and a left-handed antimuon with big momentum. Since the particles have to go through the atmosphere, a concrete wall and a lead layer before they reach apparatus, we can say that the experiment is mostly sensitive to left handed antimuons (with momenta pointing downwards in the lab frame).

---

\(^3\)Even though they have been proven to have a very small mass \((m_\nu < 2.2eV)\) as they oscillate between their different flavour states \[15\], \[16\]. The mixing of the different families of neutrinos is defined by the PMNS matrix.
Let us get back to the muon decay now. We now know that, provided with the lead buffer on top of the absorber, antimuons with helicity $-1/2$ in the lab frame are predominant. We can therefore go back to the computation of the decay rate but without summing and averaging over all initial spins. This process yields the formula

$$
\frac{d\Gamma}{dE'} = \frac{G_F^2 m^2 \mu E'^2}{12\pi^3 m^2 \mu} \left( 3 - \frac{4E'}{m\mu} \right) \left[ 1 - \frac{1 - 4E'/m\mu}{3 - 4E'/m\mu} \cos \theta \right] \left[ \frac{1 - \hat{n} \cdot \hat{s}_e}{2} \right] \frac{\sin \theta d\theta d\phi}{4\pi},
$$

(2.13)

with $\theta$ the angle between the antimuon and the positron and $\hat{s}_e$ the spin of the positron [11]. We now consider the most likely case of a very energetic positron and apply our formula to it. The factor (i) is nothing but the Michel spectrum that we obtained before and becomes $G_F^2 m^4 \mu / 64\pi^3$. The second factor (ii) simply becomes $[1 + \cos \theta]$. (iii) simplifies to 1 as, for a positron with negligible mass, $\hat{n} \cdot \hat{s}_e/2 = -1/2$ is the only possibility (left-handed). Then, the last factor (iv) can be integrated over $d\phi$ and gives $1/2 \sin \theta$. Finally the integration over $dE'$ yields an extra $m\mu$ and we can rewrite equation 2.13 as

$$
\frac{d\Gamma}{d\theta} = \frac{G_F^2 m^5 \mu}{128\pi^3} [1 + \cos \theta] .
$$

(2.14)

It is now very clear that there is a strong asymmetry between the forward and the backward region in the antimuon decay. Emission of the positron at angle $\theta \sim 0^\circ$ (parallel to the antimuon’s spin) is highly favoured whereas emission at $\theta \sim 180^\circ$ is highly suppressed. These two possibilities are represented on the Fig. 2.6.
If we now consider the more general case of a positron emitted with any given energy, we can still simplify expression 2.13 into a more condensed form

$$dΓ = C(1 + a \cos θ) \sin θ dθ,$$

with $C$ a constant accounting for all the parameters in 2.13 and $a$ the asymmetry coefficient which is simply

$$a = \frac{4E'/m_μ - 1}{3 - 4E'/m_μ} = \frac{2x - 1}{3 - 2x},$$

(2.16)

with the definition $x = E'/E_{max} = 2E'/m_μ$. The decay rate is thus bigger in the forward region than in the backward region, confirming our earlier observation. Note that in the case where $E' → E_{max}$, then $x → 1$ and $a → 1$, which is exactly what we had in Eq. 2.14 and shows that the asymmetry is stronger at high energies. Selecting higher energy antimuons will also enhance the precession of their spin. This asymmetry will show in the experiment through a bigger amount of events recorded by the scintillators on top of the absorber.

### 2.6 Spin precession

When we switch the external magnetic field on, the spin of the muon will precess around the magnetic field lines at the Larmor frequency $ω$ which reads

$$ω = \frac{egB}{2m_μ},$$

(2.17)

with $e = 1.602 \times 10^{-19}$ the charge of the muon [6], $g$ the Landé g-factor or gyromagnetic ratio and $B$ the intensity of the magnetic field. This rotation is represented on Fig. 2.7. Moreover, the muon magnetic moment is defined by

$$μ = \frac{geS}{2m_μ},$$

(2.18)

Figure 2.7: Precession of the spin around a magnetic field line $B_0$.

with $S = h\sqrt{s(s + 1)} = h\sqrt{3}/2$ the spin of the muon. It is then possible to rewrite the Larmor frequency in terms of the muon magnetic moment:

$$ω = \frac{gμB}{h},$$

(2.19)

with $h = 6.626 \times 10^{-34} m^2 kgs^{-1}$ the Planck constant [6]. Similarly the magnetic moment in terms of the Larmor frequency:

$$μ = \frac{ℏωS}{Bg}.$$

(2.20)
It is now interesting to express this magnetic moment in the natural unit of this quantity. The best choice in the context of atomic and particle magnetic moment is the Bohr Magneton\(^4\)

\[
\mu = \frac{2m\mu_0\omega S}{eBg} \left[ \frac{eh}{2m\mu} \right].
\] (2.21)

Comparing with Eq. 2.18, we can derive an expression for the gyromagnetic ratio

\[
g = \frac{2m\mu_0\omega}{eB}.\] (2.22)

We now see that if we manage to measure the Larmor frequency, we can directly infer the muon magnetic moment and its gyromagnetic ratio. Dirac’s theory predicted a value of 2 for that ratio. Nevertheless, modern quantum electrodynamics has found corrections to this ratio and called it the anomalous magnetic moment. This deviation from Dirac’s first prediction is often expressed by the parameter \(a\) defined by

\[
a = \frac{g - 2}{2}.\] (2.23)

The most accurate value of this ratio ever calculated included computation involving 4 loops and prediction on the 5 loops level [17]. This work yielded the following value

\[
a_{\mu}^{QED} = \frac{\alpha}{2\pi} + 0.765857410(27) \left( \frac{\alpha}{\pi} \right)^2 + 24.05050964(43) \left( \frac{\alpha}{\pi} \right)^3 + 130.8055(80) \left( \frac{\alpha}{\pi} \right)^4 + 663(20) \left( \frac{\alpha}{\pi} \right)^5 + \cdots \] (2.24)

\[
= 116584718(15) \times 10^{-11}.\] (2.25)

Experimentally, one finds that the tabulated value is

\[
a_{\mu}^{exp} = (11659209 \pm 6) \times 10^{-10}.\] (2.26)

In the presence of an external magnetic field, Eq 2.12 is modified and takes the form

\[
N = N_0 e^{-t/\tau} [1 + A \cos(\omega t + \delta)],\] (2.27)

with \(A\) depending on the asymmetry factor \(a\) defined in Eq. 2.16 and the detector’s acceptance. This relation can be derived, using perturbation theory, taking as an interaction lagrangian \(H_I = -\mu \cdot B\), considering that the magnetic field is uniform and has only one non-zero component [18].

\(^4\)It is the proportionality constant that links a particle’s magnetic moment to its kinetic moment: \(\vec{M} = \mu_B \vec{J}\). It is defined for the muon by \(\mu_B = \frac{eh}{2m\mu}\).
3 Experimental apparatus

3.1 Description

The experimental apparatus is composed of four plastic scintillators sandwiching a copper absorber ((2 ± 0.1) cm thick) surrounded by Helmholtz coils used to generate a uniform magnetic field. The upper scintillators will later be called A and B and the lower ones C and D. Each one of the scintillators is connected to a photomultiplier tube (PMT) through a light guide. A layer of radiation protection grade lead bricks (2 inches thick) is disposed on the top of the entire set-up in order to slow down the muons and shield the experiment from undesired particles. The four PMTs are connected to a set of NIM crates which allow communication between the apparatus, the logic module and the computer through the programmable CAMAC software.

Fig. 3.1 provides two photographs of the whole apparatus. All the different modules described in the previous paragraph and their interactions have been represented schematically in Fig. 3.2. In this section we will thoroughly describe each one of their characteristics and their role in this experiment.

![Figure 3.1: Left: electronics modules. Right: detector and Helmholtz coils.](image)

3.2 Detection principle

The aim of this experiment is, as mentioned before, to determine the muon lifetime and its magnetic moment. In terms of equations this comes down to fitting the experimental data to derive the parameters $\tau$ in Eq. 2.12 and $\omega$ in Eq. 2.27.

To achieve these goals we have to measure the time lapse between the absorption of an antimuon in the copper plate and its decay. The start signal is the coincidence of the scintillators A and B and corresponds to the arrival of a particle in the detector. The stop signal is similarly defined as the coincidence of the scintillators A and B (called up stop) or C and D (called down stop) and corresponds to a positron exiting respectively through the top or the bottom of the absorber. The succession of a start signal and a stop signal is called an event. If a stop signal is not detected within a reasonable amount of time, the system does not record the event, we call it a time out.
Figure 3.2: Schematics of the apparatus.
In this experiment, $20 \mu s$ has been chosen as the latency before resetting the logic, this corresponds to about 10 times the tabulated value of the muon lifetime. The use of two scintillators on each side of the absorber helps to avoid, as much as possible, false signals. Unfortunately, the experiment is designed in such a way that it is impossible to avoid them completely. Fig. 3.3 represents the different situations in which false events could be recorded and true events could be lost.

**Recorded false events**

A Two particles that cross one of the two pairs of scintillators without stopping in or crossing the absorber.

B A first particle stops in the copper absorber but a second one crosses the scintillators before the first one gets a chance to decay.

C A second start signal is mistaken for a stop signal, i.e. another particle is absorbed before the first one decays.

D A particle reaches the copper absorber without crossing any scintillators and, before it decays, another particle crosses the top scintillators to trigger the experiment. It then decays and stops the counting, giving a wrong value for its decay time.

**Lost true events**

A The positron, product of the decay of an antimuon, exits the detector without crossing any scintillator.

B A particle stops in the absorber without crossing the upper scintillators.

Due to the horizontal geometry of the experiment and the fact that absorbed antimuons come from mostly vertical hadronic showers, it is important to note that almost all these cases are very unlikely. As a matter of fact, the most probable case of false event is C. This has non negligible consequences is the measurement of the antimuon lifetime because these events, as they occur before the antimuon actually decays, will inevitably bias the lifetime towards a smaller value than the expected one. We shall come back on this bias during the data analysis.

Figure 3.3: Possible recorded false events and lost true events. The green arrows are the *start signals* and the red the *stop signals*. 
3.3 Scintillators

Each scintillation detector consists of a large flat scintillator connected to a photomultiplier through a light guide as represented in Fig. 3.4. Scintillators have a thickness of 0.8 cm, a length of 49 cm and a width of 42 cm\(^5\).

![Figure 3.4: Scintillation detector set-up.](image)

The physical principle behind the detector in question is luminescence. It consists in the emission of light by a substance not resulting from heat\(^6\). The radiations are consequently called cold and can be caused by chemical reactions, electrical energy, subatomic motions, or stress on a crystal. Combining this type of material with a photomultiplier allows the conversion of light in electrical impulses for further analyses.

When a particle crosses a scintillator, it excites some of its electrons and causes electronic transitions or electronic-vibrating transitions. Fig 3.5 shows a schematic of the electronic levels of a luminescent material. When energy is transferred to an electron in its ground state \((S_0)\), it will be able to access higher energy levels \((S_1, S_2\) or \(S_3)\). The level of these spin singlet states can vary for a few tenths of eV for electronic levels to a few eV for vibrational levels. Depending on the material, an excited electron can adopt different ways of returning to the state \(S_0\). In a fluorescent material, the electrons return to \(S_0\) within a few ns by emitting light which frequency corresponds to the energy level through the equation \(E = h\nu\). In a phosphorescent material, on the other hand, the electrons get trapped in one or several metastable spin triplet states before returning to the level \(S_0\). In this case, the time between the excitation and the emission can be much longer and considered to vary between 1 ms and 10 s for common organic materials [19].

![Figure 3.5: Energy levels diagram of a luminescent material.](image)

In this experiment which tries to get an accurate measurement of the hit time, it is necessary to have a good time resolution. In terms of scintillator material it comes down to selecting one with a vary fast response. As a result, fluorescent plastic scintillators have been selected as they offer the fastest response (2.4 ns for \(NE102A\) [20]).

\(^5\)All these measurements are subject to a ±0.1 cm error which is the level of accuracy achieved with a standard ruler.

\(^6\)The emission of light caused by heating is called incandescence and is thus a hot body radiation.
3.4 Light guides

A light guide is a device designed to transport light from a light source to a point at some distance with minimal loss. Light is transmitted through a light guide by means of total internal reflection as depicted in Fig. 3.6. In order to minimize light losses, the reflexion index of the guide is chosen to match the one of the scintillator. Furthermore, a gel with similar characteristics is applied at the junction to create an optically continuous medium. Light signals from the scintillators are transmitted through them to a photomultiplier.

Figure 3.6: Total reflexion in a light guide epoxied to the scintillator.

3.5 Photomultipliers

A photomultiplier is a vacuum tube consisting of an input window, a photocathode, focusing electrodes, an electron multiplier and an anode usually sealed into an evacuated glass tube. Fig. 3.7 shows a schematic construction of a PMT. The basic functioning principle can be summarized in 4 steps enumerated here that we will develop in this subsection.

1. Light excites the electrons in the photocathode so that photoelectrons are emitted.
2. Photoelectrons are accelerated and focused onto the first dynode.
3. Photoelectrons are multiplied at each of the successive dynodes.
4. The secondary electrons emitted from the last dynode are collected by the anode.

Figure 3.7: Typical photomultiplier tube.
3.5.1 Photoelectron emission

Photoelectric conversion is broadly classified into external photoelectric effects by which photoelectrons are emitted into the vacuum from a material and internal photoelectric effects by which photoelectrons are excited into the conductive band of a material. The photocathode has the former effect.

As a photocathode is a semiconductor, we can represent these energy levels using band models such as the ones presented in Fig. 3.8. In a semiconductor, there is a forbidden-band gap or energy gap \( E_g \) that cannot be occupied by electrons, the electron affinity \( E_A \) which is the interval between the conduction band, the vacuum level barrier \( E_v \) and a work function \( \psi \) which is the energy difference between the Fermi level and the vacuum level. When photons strike the photocathode, electrons in the valence band absorb photon energy \( h\nu \) and become excited, diffusing toward the photocathode surface. If the electrons have enough energy to overcome \( E_v \), they are emitted into the vacuum with kinetic energy \( E_p = h\nu - E_v \).

![Figure 3.8: Photocathode band model.](image)

This is a stochastic process and quantum efficiency, i.e. the ration between the output electrons and the incident photons is given by

\[
\eta(\nu) = (1 - R) \frac{P_v}{k} \left( \frac{1}{1 + 1/kL} \right) P_s ,
\]

where \( \nu \) is the frequency of light, \( R \) is the reflection coefficient, \( k \) the full absorption coefficient of photons, \( P_v \) the probability that electrons will be excited above \( E_v \), \( L \) the mean escape length and \( P_s \) the probability that the photoelectrons may be released into the vacuum [21].

![Figure 3.9: Hamamatsu PMT quantum efficiency vs. incoming light wavelength](image)
Without getting into technical details, this equation tells us that the response of a photomultiplier is non linear in the wavelength $\lambda = c/\nu$. If we look at the efficiency with respect to the incident radiation wavelength (Fig. 3.9), we observe that only a small range of it is efficiently converted. For this reason the PM has to be combined with a scintillator which luminescence wavelength is within this range.

### 3.5.2 Electron trajectory

The photoelectrons have to be collected and focused on the first dynode before they can start to be multiplied, this is the task of the focusing electrode. Through the application of an electric field in a suitable configuration, such as the one schematically represented in Fig 3.10, the electron paths are curved in order to reach the first dynode.

Two features are important for a good focusing electrode. The collection efficiency and the time taken by the electron to reach the first dynode must be independent on the emission point on the photocathode. The former ensures that two similar events will generate signals of the same intensity and the latter that the recorded time of an event won’t be biased, which is very important in the context of our experiment.

![Figure 3.10: Focusing electric field in a PMT.](image)

### 3.5.3 Electron multiplier

The dynode section is used to amplify the weak primary photocurrent through secondary emissions$^7$. The electrons are accelerated between each one of the dynodes by applying a voltage that rises from the first dynode to the last one as represented on Fig 3.11. When an electron with initial energy $E_p$ strikes the surface of a dynode, $\delta$ secondary electrons are emitted. $\delta$ is called the secondary emission ration and can vary from a material to another and from a operational voltage to another. The total gain of the amplifier simply is $G = \delta^n$.

### 3.5.4 Anode

The anode of a photomultiplier tube is an electrode that collects secondary electrons multiplied in the cascade process through multi-stage dynodes and outputs the electron current to an external circuit. The most important factor in designing an anode is that an adequate potential difference can be established between the last dynode and the anode in order to prevent space charge effects and obtain a large output current.

The current at the anode can be written as the exponential decay law of the luminescence ($\tau_s =$

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$^7$Major secondary emissive materials used for dynodes are alkali antimonide, BeO, MgO, GaP and GaAsP. These materials are coated onto a substrate electrode made of nickel, stainless steel or Cu-Be alloy.
2.4\,\text{ns} \text{ for plastic}) \text{ multiplied by the gain } G, \text{ the amount of electrons which reached the first dynode and the charge of one electron, that is}

\[ I(t) = (GNe)e^{-t/\tau_s}. \]  \hspace{1cm} (3.2)

Since the output signal is proportional to the initial number of photoelectrons, a PMT behaves like a current generator in parallel with a resistance \( R \) and a capacitance \( C \). The current given by an RC circuit is

\[ I(t) = \frac{V}{R} + C \frac{dV}{dt}. \]  \hspace{1cm} (3.3)

Substituting 3.2 into 3.3 we get

\[ V(t) = -(GNERe) \left( \frac{\tau}{\tau - \tau_s} \right) \left( e^{t/\tau_s} - e^{t/\tau} \right), \]  \hspace{1cm} (3.4)

with \( \tau = RC \). \( V(t) \) is plotted with respect to \( t \) in Fig. 3.12 for different values of \( \tau \) and for a plastic scintillator with \( \tau_s = 2.4\,\text{ns} \).

![Figure 3.12: Signal shape of a PMT for different values of \( \tau \).](image)
3.6 Discriminators

Discriminators are used to convert the analogical signals coming out the photomultipliers into digital ones (square pulses of fixed width and height) as depicted in Fig. 3.13. In order to do so, discriminators only let through signals with a voltage above a certain threshold. Such a threshold is chosen to minimize the noise produced by detectors self-excitation or by weak energy particles that would have gone through (e.g. cosmic protons). We initially fixed the threshold voltage at the discriminator on 50mV (see) and the output signal width on 50ns, wide enough for easy calibration in voltage and time. After delay corrections, the width was lowered down to 20ns to avoid false coincidences. These discriminators are essential to this experiment as digital signals are the necessary inputs of the coincidences that are the basic bricks to build the trigger logic described in the next section.

![Figure 3.13: Discriminator output for a given PMT analogical input.](image)

3.7 Trigger system

In our experiment, the four photomultipliers are divided in two groups of two, the top PMTs (A and B) and the bottom PMTs (C and D). Similarly, two different basic coincidences are used in the trigger:

- \( A \cap B \cap C \cap D \) which is generated by a particle crossing the upper scintillators without crossing the lower ones (UP signal),
- \( \overline{A} \cap \overline{B} \cap C \cap D \) which is generated by a particle crossing the lower scintillators without crossing the upper ones (DOWN signal).

The logic has been developed on Quartus II software and implemented on a DE2 development board from Altera®. In Fig. 3.7 is shown the annotated schematics of the logic drawn with Quartus II. The basic blocs to build it are AND operators, Flip-Flops and a counter. A Flip-Flop is a digital circuit which has two stable states and thus can store a signal for a long amount of time, acting like a bit of memory. These Flip-flops are implemented in an external LAM (Look At Me) box which allows the experimenter to monitor and reset their states through the CAMAC software. Each LAM \( L_i \) constitutes a bit of information labeled \( L_i \) and, when combined, a 4 bits number \( (L_1L_2L_3L_4) \).

The counter is coupled with a 50MHz clock and consists of 10 bits. It counts the clock’s rising edges until the MSB\(^8\) turns to 1. The exact duration of a complete cycle before time out will be

\[
\tau_{to} = N \times \frac{1}{\nu_{clk}} = 2^{10} \times \frac{1}{5 \times 10^6} = 20.48 \mu s,
\]

with \( N \) the amount of edges necessary to reach the MSB and \( \nu_{clk} \) the frequency of the clock. The counter is reset by a start signal using its aclr input bit and inhibited the rest of the time by the AND door CntStop.

\(^8\)Most Significant Bit.
Figure 3.14: Schematics of the logic drawn in Quartus II.
Let us describe what happens in a typical up stop event. At the beginning, the four LAMs are at rest and thus their configuration reads (0000). A cosmic antimuon entering the detector will cross the upper scintillators and stop in the absorber. This corresponds to the UP signal defined before and will be considered in this case as the start signal. As the signal is the same as the up stop signal, it is necessary to introduce two VETO signals defined as follows

\[ VETO_1 = L_1 \cap L_2 \cap L_3 \cap L_4, \]

\[ VETO_2 = L_1 \cap L_2 \cap L_3 \cap L_4. \]

In the initial LAM configuration, VETO\(_1\) is true and VETO\(_2\) is false. The start signal is the cue for the counter to start one 20\(\mu\)s cycle before activating \(L_2\). In this way, during the time of 20\(\mu\)s, the LAM configuration is (1000), which means that the start has been recorded and the logic is waiting for a stop signal. In case of an up stop, a new UP signal happens and faces VETO\(_1\) and VETO\(_2\). In the active configuration, the first one is false while the second one is true. In consequence, the impulse goes through the AND door before \(L_3\) and sets the Flip-Flop on 1. The LAM configuration becomes (1010). If we had a down stop event, the configuration would similarly be (1001).

At the end of the 20\(\mu\)s cycle, the LAM configuration can be: (1110) for an up stop event, (1101) for a down stop one. If no event happens throughout the cycle, the final configuration will be (1100).

### 3.8 CAMAC

The CAMAC\(^9\) system is an IEEE\(^{10}\) and ESONE\(^{11}\) standard for modular systems of computer-controlled data transfers defining modules fixed on crates and forming systems called branches. Each CAMAC cycle is specified by an address (Specifying the branch number, crate number and position of module in the crate and sub-address) and a function (operation that the module can perform). Up to 24 bytes of data can be transferred. The different CAMAC functions are reading, writing and controlling (initializing, inhibiting or clear functions).

The CAMAC system is used as an interface between the logic defined in the previous subsection and the computer. A LAM module and a counter module coupled with a 50MHz clock are used in the CAMAC. The clock starts to count when a start signal occurs (inhibit on \(L_1\)) and stops when either one of the other LAMs is switched on (inhibit on \(L_2 \cup L_3 \cup L_4\)). The CAMAC program waits until the second bit of the LAM configuration turns to 1 then reads the full value of the configuration and the amount of counts. Only if the value is (1110) (up stop) or (1101) (down stop) does the program records the amount of counts. After recording the counter value, the CAMAC program resets the LAM configuration to the initial (0000) and restarts the cycle.

A typical digital signal timing diagram is given for an up stop event in Fig. 3.15 and divided in 4 steps. LCnt. Trig. is the signal that starts the 20\(\mu\)s counting process in the logic and ECnt. Inh. is the inhibit of the external CAMAC counter. At I, a start signal is recorded, \(L_1\) is switched on and the CAMAC counter starts counting. Then, when an up stop is recorded (II), the counter stops and \(L_3\) activates, rendering VETO\(_2\) false and preventing any further signal to be taken into account. When 20\(\mu\)s have passed (III), the logic reaches the end of its cycle and \(L_2\) activates. Finally, at IV, the CAMAC program reads the LAM configuration, the amount of counts recorded and resets the both of them.

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\(^9\)Computer Automated Measurement And Control
\(^{10}\)Institute of Electrical and Electronics Engineers
\(^{11}\)European Standards On Nuclear Electronics

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3.9 Helmholtz coils

Two Helmholtz coils are used to generate a somewhat uniform magnetic field parallel to the copper absorber. Their purpose is to make the spin of the captured antimuons to precess around the field lines and thus determine the Larmor frequency necessary to the derivation of the antimuon magnetic moment. The two coils are parallel to each other and apart by a distance equal to their radius. Each of the coil is made of 450 turns of copper and have a radius of \((47 \pm 1)\) cm.

A voltmeter is connected to the CAMAC system in order to monitor the constance of the magnetic field intensity. Moreover, as the coils tend to heat up tremendously, they need to be shut down when they cross the \(70^\circ\)C limit. This limit is reached quite regularly and the voltmeter allows us to distinguish the events recorded with a magnetic field from the others.

In this particular experiment, two different configuration of the magnetic field have been used. One with a current supply of \(+(5 \pm 0.01)\) A and another one with \(-(3 \pm 0.01)\) A in order to observe the dependency of the frequency in the intensity and direction of the magnetic field. The strength of the magnetic field can be calculated theoretically [23] to get the formula for Helmholtz coils

\[
B_0 = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{R},
\]

with \(B_0\) the strength of the magnetic field in the middle of the two coils and \(N = 450\) the number of loops in one coil. This gives two different values of the field: \(B_0^{th} = +(43.05 \pm 0.13)\) G and \(B_0^{th} = -(25.83 \pm 0.08)\) G. Nevertheless, these values consider that the coils generate a perfectly uniform magnetic field and do not take into account the earth magnetic field strength (up to 0.475 G in Geneva depending on the orientation [24]). To assert this inaccuracy, a mapping of the two magnetic field has been realized using a Gaussmeter (or Hall probe) and are shown as white dots in Fig. 3.16. The magnetic field has been measured in 15 points with coordinates given in cm (see Fig. 3.16) to

Figure 3.15: Typical up stop event timing diagram.
cartography it properly and then fit by a 2nd degree polynome of the form.

\[ B(x, y) = a + bx + cy + dx^2 + fy^2 + gxy. \] (3.9)

Finally, the average of each magnetic field over the whole surface has been calculated using the integration formula

\[ \overline{B} = \frac{1}{S} \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{y_{\text{min}}}^{y_{\text{max}}} B(x, y) \, dx \, dy, \] (3.10)

with \( S = 50 \times 40 = 2000 \, \text{cm}^2 \) the surface covered by the copper absorber. The two values found are summarized in the table below. The sources of errors are the sensitivity of a magnetic field measurement to the orientation of the Hall probe and the inaccuracies in the probe placement. The subsequent error has been estimated to 0.1 G even though the probe gives a value to the thousandth. The values we obtained correspond staggeringly well with the theory as shown in the following table.

| \( B_+ \) | \(+ (42.9 \pm 0.1) \, \text{G} \) |
| \( B_- \) | \(- (25.6 \pm 0.1) \, \text{G} \) |

### 3.10 Photomultipliers calibration

It is important to find the right tension for each of the PMTs. A low voltage would result in missing some of the particles going through the scintillators but a high tension would tear electrons off the dynodes without any particles actually generating the signal. We can observe both behaviors on the graph in Fig. 3.17.

It is possible to get a first appreciation of the right tension by looking at the number of counts recorded by A PMT after a certain period of time. We can see that, at around 2000V, we reach a plateau and that we should try to look from there. In order to accurately determine the right tension, we use a discriminator, a coincidence NIM crate and a multiscaler.

In order to compute the efficiency of the PMT A with respect to the voltage, we adopt a simplifying
postulate according to which the number of counts recorded by two synchronized PMT (either A&B or C&D) equals the number of counts of either one of them. In other words, the number of counts corresponding to the coincidence of 4 PMTs must equal the one of 3 PMTs only. We can then define the efficiency of the PMT A as

$$Eff(A) = \frac{A \cap B \cap C \cap D}{B \cap C \cap D},$$

(3.11)

where A (resp. B, C, D) corresponds to an event in one of the so-called photomultiplier.

To determine the optimal tension for the photomultiplier A, we tune the tension on the power supply board from 1300 V to 2200 V with 50 V steps (some measurements, apart by only 25 V, were taken around the inflection point) while the tensions in the other PMTs are all fixed at 2000 V. We then measure the integrated amount of counts for a period of 100 s for each PM and finally we plot the efficiency with respect to the voltage, the curves are given in Fig. 3.18-3.21.

These curves exhibit the same pattern: before reaching the inflection point, the efficiency equals zero. After having reached this figure, the efficiency grows linearly as a function of the voltage before reaching maximum efficiency. The tensions are set 100 V beyond the intersection point of the regression line calculated around the inflection point and the one obtained beyond $Eff \approx 1$. The final values that have been chosen for the voltage are represented in the table below.

<table>
<thead>
<tr>
<th>PM ID</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1900 ± 1) V</td>
</tr>
<tr>
<td>B</td>
<td>(1900 ± 1) V</td>
</tr>
<tr>
<td>C</td>
<td>(2000 ± 1) V</td>
</tr>
<tr>
<td>D</td>
<td>(2000 ± 1) V</td>
</tr>
</tbody>
</table>
Figure 3.18: Calibration curve for PM A.

Figure 3.19: Calibration curve for PM B.
3.11 Delay

In order to thoroughly complete the preparation of this experiment, it is necessary to correct the systematic time lapse that would present itself between the top planes and the bottom planes. The
signals from the upper and lower scintillators have to match in time, which means that we have to
find the right delay between them in order to maximize the coincidence counting. First we will check
that there isn’t any delay between two planes within a same group by measuring the coincidences
\( A \cap B \) and \( C \cap D \) with respect to the delay between them. Then, we will measure the coincidence
\((A \cap B) \cap (C \cap D)\) which will give us the information we need to infer the systematic delay.

The process used in this calibration is the same for each of the coincidences. It involves delay
boxes, AND logic doors and a multiscaler. The width of the signals coming out of the discriminator
is fixed on \(20\) ns.

In each case, one of the two signals has a fixed offset of \(32\) ns while the other one is progressively
offset from 0 to \(64\) ns by \(4\) ns steps (or shorter close to the inflection points). For each step, the scaler
counts the number of coincidences for 100 s, this number is recorded and plotted along with the other
counts with respect to \(\Delta t\). These plots are represented in Fig. 3.22, Fig. 3.23 and Fig. 3.24. Finally,
the scatter plot is fitted with a customized\(^{12}\), arbitrary, hat-shaped function which expression is

\[
f(x) = \frac{A}{1 + \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}.
\]

with \(A\) the amplitude of the function, \(2B\) its total width, \(\mu\) the center of the hat and \(\sigma\) the width
of the fast varying parts. Although this process might seem a little artificial, it provides us with
important information about the delay plots: the parameter \(\mu\) gives us the best fit on the delay we
should correct.

The first two fits confirm that PMTs within the same group have an extremely low and negligible
delay between them. The last fit reveals a slight offset flirting with the \(2\) ns. In consequence, the
signals \(C\) and \(D\) will be shifted by \(2\) ns during the experiment in order to compensate that delay.

Figure 3.22: Delay curve between PMT \(A\) and PMT \(B\).

\(^{12}\)The formula given here has been derived from the expression of a sigmoid \(1/(1 + \exp(-x))\) and modified to include
the parameters necessary and faithfully describe the distribution.
Figure 3.23: Delay curve between PMT C and PMT D.

Figure 3.24: Delay curve between the top planes and the bottom planes.
4 Data analysis

In this chapter, the data analysis is divided into two categories. The determination of the muon lifetime, which allows us to perform a consistency check, and the study of the muon magnetic moment. In both cases we measure the exact same quantities but in the former the magnetic field is turned off whereas in the latter it is switched on.

4.1 Lifetime

The theoretical law which is expected to describe the rate of decay events as a function of time follows a Poisson distribution described in the theoretical background and reads

\[ N(t) = N_0 e^{-t/\tau} + B. \] (4.1)

The collected data was plotted in histograms covering a range up to 20 \( \mu s \). The length of each bin has been studied to conclude that a length of 0.2 \( \mu s \) (which corresponds to 100 bins for 20 \( \mu s \)) was the most suitable choice as it is a multiple of the time resolution, minimizes \( \chi^2/ndf \) and thus maximizes the probability of the fit being relevant and will be a source of systematic error.

The fit was performed using ROOT and its log likelihood function which is one of the TMinuit functions. Several fits with the same function were realised with different fit ranges in order to determine the most appropriate limits.

Another error to consider before starting to analyse the data is the systematic error due to the scaler’s limited time resolution. In this experiment, we use a clock with a frequency of 50 MHz which corresponds to a time resolution of \( \sigma_t = 0.01 \mu s \). Using Monte Carlo methods, one can perform a series of pseudo-experiments with a generator following a normal distribution. One can then derive the systematic error from the difference of lifetime measured between the data and the pseudo-experiments. This would give an incertitude one order of magnitude smaller than \( \sigma_{FR} \) and two orders of magnitude smaller than the statistical error. It will thus be neglected in the following subsections.

4.1.1 Up events

We first start by determining the limits of the best fitting range \([t_1, t_2]\). In Fig. 4.1 we plot the variation of the probability of the fit including a data point with respect to the fitting limits. We then choose the values of \( t_1 \) and \( t_2 \) to be the ones that maximize the probability. It is obvious that the choice of a fitting range will introduce a systematic error that can be estimated by taking the standard deviation of the samples. We find that

\[ \sigma_{FR} = \sqrt{\frac{1}{N - 1} \sum_{t_1,t_2} (\tau(t_1,t_2) - \bar{\tau})^2} = 0.003 \mu s, \] (4.2)

with \( \bar{\tau} = 1/N \sum_{t_1,t_2} \tau(t_1,t_2) = 2.181 \mu s \) the average lifetime obtained and \( N \) the amount of measured values taken into account. We note that this uncertainty will be correlated with the statistical uncertainty. Nevertheless, we choose to linearly add this error to the statistical one.

The amount of good events recorded in the upper scintillators is 191017 and include 6 weeks of data dedicated to determining the muon lifetime and the data retrieved during the dead times of the electromagnets of the second and third run. The fitting result is shown in Fig. 4.2 and the parameters values are summarized in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_0 )</td>
<td>16213 ± 84</td>
</tr>
<tr>
<td>( \tau [\mu s] )</td>
<td>2.180 ± 0.007</td>
</tr>
<tr>
<td>( B )</td>
<td>22.226±1.017</td>
</tr>
</tbody>
</table>
If we combine the result for $\tau$ with its errors, we get, for the $up$ data sample,

$$\tau_{up} = (2.180 \pm 0.010_{(\text{stat})} \pm 0.003_{(\text{syst})}) \mu s . \quad (4.3)$$

### 4.1.2 Down events

From the same sources mentioned in the previous subsection, this experiment has yielded 138411 good $down$ events. The fitting range analysis is shown in Fig. 4.3. The systematic error which stems
from that choice can be derived according to the formula in (4.2) with $\tau = 2.209 \, \mu{s}$ and gives

$$\sigma_{FR} = 0.006 \, \mu{s}. \quad (4.4)$$

The fitting result is shown in Fig. 4.4 and the parameters values are summarized in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td>11051 ± 104</td>
</tr>
<tr>
<td>$\tau [\mu{s}]$</td>
<td>2.215 ± 0.011</td>
</tr>
<tr>
<td>$B$</td>
<td>23.114 ± 0.988</td>
</tr>
</tbody>
</table>

Figure 4.3: Values of the probability for different fitting ranges $[t_1, t_2]$ for the down data sample.

If we combine the result for $\tau$ with its errors, we get, for the down data sample

$$\tau_{up} = (2.215 \pm 0.011_{(\text{stat})} \pm 0.006_{(\text{syst})}) \, \mu{s}. \quad (4.5)$$

4.1.3 Combined results

In order to have a final estimate of the antimuon lifetime, it is necessary to combine the two results obtained in the above fits. A naive way to do it would be to combine the data from the upper and the lower scintillators and perform a new fit. In this case, however, the data from the two sources are correlated as our PMTs were calibrated together and not separately. An alternate way of doing it can be found in the literature and suggest that, if we have a set of measurements $x_i = a_i \pm \sigma_i$, their combination reads

$$x_{\text{comb}} = \frac{\sum a_i \times w_i}{\sum w_i} \pm \frac{1}{\sqrt{\sum w_i}}, \quad (4.6)$$

where the weights are defined as $w_i = 1/\sigma_i^2$ [25]. This technique allows the values with the smallest incertitude to play the biggest role in the combination. Applying this formula to the problem in question yields our muon lifetime final estimation
4.1.4 Comments

The tabulated value of the muon lifetime is

\[ \tau = (2.193 \pm 0.010) \mu s. \]  

(4.7)

\[ \tau = (2.1969811 \pm 0.0000022) \mu s. \]  

(4.8)

The estimation that we have managed to obtain corresponds beautifully to what was expected. This shows that the log likelihood method works properly, which is not surprising as the obtained values of \( \chi^2/\text{ndf} \) were extremely close to 1 for both fits\(^\text{13}\). The down fits include the tabulated value within 1\( \sigma \) and the up is slightly low as it was predicted in the experimental section.

Moreover, the amount of events in the upper PMs is higher by 38\% than the lower ones, which was expected from the decay asymmetry described in the theoretical background section. The amount of statistics yielded in this part of the experiment is high enough to clearly confirm the decay asymmetry suggested in the theoretical section. Finally, we can conclude that, according to the consistency of our result, it is safe to move on to the next step.

4.2 Magnetic moment

To measure the muon magnetic moment, data has been taken with the presence of a magnetic field strong enough to appreciate its effects. For each current setting, the number of up and down stops have been recorded as a function of the elapsed time.

The data taken has been recorded with two different magnetic fields. One generated using a current of +5 A and the other one with a current of -3 A. The number of good events in the first case reached 211559 before spring. Then, the electromagnets started to overheat too often so that the choice to lower the intensity of the current to 3 A was made while the polarity was inverted to investigate its influence on the oscillations. The events collected after this adjustment added up to

\(^{13}\chi^2/\text{ndf} = 90.64/91 = 0.996 \) for the up data sample and \( \chi^2/\text{ndf} = 88.66/89 = 0.996 \) for the down data sample.
As described in the theoretical part of this paper, the magnetic moment is derived from the measurement of the Larmor frequency in a uniform (cfr. subsection 2.9) magnetic field. For all the data, two fitting methods have been used, we shall call them method A and B.

The first one, method A, involves fitting each data sample separately. The first step of this fit consists in fitting the sample with a simple decay law function of the form

\[ N(t) = N_0 e^{-t/\tau} + B. \] (4.9)

Then, we extract the parameters \( N_0, \tau \) and \( B \) from the fit and hold them fixed in a second fit performed using the oscillating function

\[ N(t) = N_0 e^{-t/\tau} \left[ 1 + A \cos(\omega t + \delta) \right] + B, \] (4.10)

from which we extract \( \omega \), the coveted Larmor frequency.

Method B, on the other hand, involves combining the up and down data from a given magnetic field setting into the ratio

\[ R_i = \frac{(N_{\text{down},i} - B_{\text{down},i})/(N_{\text{down}} - B_{\text{down}})}{(N_{\text{up},i} - B_{\text{up},i})/(N_{\text{up}} - B_{\text{up}})}, \] (4.11)

where \( N_{\text{down},i} \) and \( N_{\text{up},i} \) are the number of events recorded in a given bin \( i \) in respectively the down and up data samples, \( B_{\text{down},i} \) and \( B_{\text{up},i} \) the noises in these bins, \( N_{\text{down}} \) and \( N_{\text{up}} \) the total number of events and \( B_{\text{down}} \) and \( B_{\text{up}} \) the values of the constant \( B \) obtained in the fits in method A. This normalization of the ratio is primordial as the number of hits recorded are higher in the top pair of scintillators. Finally, this ratio, plotted for a wide range of bins is fitted with the function

\[ R(t) = \frac{1 - A \cos(\omega t)}{1 + A \cos(\omega t)} + C, \] (4.12)

which corresponds to the ratio between two equations of the form 4.10 with the same decay constant \( \lambda = 1/\tau \), normalized \( N_0 \) constants and a phase offset by \( \pi \) radians.

In each of these cases, the fitting range had to be chosen sensibly and was subjected to an analysis similar to the one in subsection 4.1.

4.2.1 +5A current data

Method A

In this analysis, the considered fitting ranges have been taken smaller as the oscillations become subject to high levels of noise at \( t > 13 \mu s \). Moreover, the values of the parameter \( \omega \) are much more dependant to the range choice than \( \tau \) was in the previous subsection. As a result, only the best fits (within 0.1 in probability from the best fit) were taken into account to calculate \( \sigma_{FR} \) in Eq. 4.2, the other fits returning aberrant values. Finally, a logarithmic scale is used to show the oscillations more clearly.

**Up DATA**

From the fit in Fig. 4.6, we get

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>0.039 ± 0.006</td>
</tr>
<tr>
<td>( \omega ) [MHz]</td>
<td>3.635 ± 0.086</td>
</tr>
<tr>
<td>( \delta )</td>
<td>−0.260 ± 0.373</td>
</tr>
</tbody>
</table>
Adding the systematic error $\sigma_{FR}$ found for this sample, we get

$$\omega_{up} = (3.635 \pm 0.086_{(stat)} \pm 0.026_{(syst)}) \text{ MHz}.$$  \hfill (4.13)

Taking into account the average value of the magnetic field measured in section 2.9 $B_+ = +(42.9 \pm 0.1) \text{ G}$, the mass of the muon $m_\mu = 1.884 \times 10^{-28} \text{ kg}$, the charge of the electron $e = 1.602 \times 10^{-19} \text{ C}$ and $g = 2$, the magnetic moment is

$$\mu_{up} = (0.996 \pm 0.031) \frac{e\hbar}{2m_\mu}.$$  \hfill (4.14)

If we are interested in the value of the Landé factor, we use Eq. 2.22 to get

$$g_{up} = 1.992 \pm 0.062.$$  \hfill (4.15)

![Up MM -5A boundaries](image)

Figure 4.5: Values of the probability for different fitting ranges $[t_1, t_2]$ for the +5A up data sample.

**Down DATA**

From the fit in Fig. 4.8, we get

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.058 ± 0.006</td>
</tr>
<tr>
<td>$\omega$</td>
<td>3.606 ± 0.056</td>
</tr>
<tr>
<td>$\delta$</td>
<td>−3.392 ± 0.190</td>
</tr>
</tbody>
</table>

Adding the systematic error $\sigma_{FR}$ found for this sample, we get

$$\omega_{down} = (3.606 \pm 0.056_{(stat)} \pm 0.019_{(syst)}) \text{ MHz},$$  \hfill (4.16)

which translates into the following magnetic moment and Landé factor

$$\mu_{down} = (0.989 \pm 0.021) \frac{e\hbar}{2m_\mu},$$  \hfill (4.17)

$$g_{down} = 1.977 \pm 0.040/.$$  \hfill (4.18)

34
It is interesting to observe that the two graphs are in phase opposition, as we would expect. This validates the theory and necessary hypothesis for method B. We calculate

$$\Delta \delta = 3.132 \pm 0.42 = (0.997 \pm 0.135) \pi .$$  

(4.19)

**Combined results**

We use Eq. 4.6 to combine the two values we just measured into one

$$\omega_A = (3.615 \pm 0.062) \text{ MHz},$$  

(4.20)
or, in terms of magnetic moment and Landé factor,

\[
\mu_A = (0.991 \pm 0.017) \frac{e\hbar}{2m_\mu},
\]

(4.21)

\[
g_A = 1.982 \pm 0.034.
\]

(4.22)

**Method B**

For this procedure, we plot the ratio in Eq. 4.11 with respect to the time and get Fig. 4.9. The fitting range and the binning choice has been subjected to the same analysis as the previous fits and we get \( t_1 = 0.6 \mu s \) and \( t_2 = 6.6 \mu s \). The fit is accurate and gives us the following values for the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>0.053 \pm 0.004</td>
</tr>
<tr>
<td>( \omega ) [MHz]</td>
<td>3.631 \pm 0.031</td>
</tr>
<tr>
<td>( C )</td>
<td>0.028 \pm 0.006</td>
</tr>
</tbody>
</table>

This fit, in contrast with method A, has proven to be little range dependant. The value of the systematics is thus lower and we get the following overall result for the frequency,

\[
\omega_B = (3.632 \pm 0.031_{(stat)} \pm 0.013_{(syst)}) \text{MHz},
\]

(4.23)

or, in terms of magnetic moment and Landé factor,

\[
\mu_B = (0.996 \pm 0.012) \frac{e\hbar}{2m_\mu},
\]

(4.24)

\[
g_B = 1.991 \pm 0.025.
\]

(4.25)
4.2.2 -3A current data

Method A
The aforementioned observations made for the +5A data sample are still valid for this one.

**Up DATA**
From the fit in Fig. 4.11, we get

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$0.05274 \pm 0.003978$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$3.631 \pm 0.031$</td>
</tr>
<tr>
<td>$C$</td>
<td>$0.02752 \pm 0.006081$</td>
</tr>
</tbody>
</table>

Adding the systematic error $\sigma_{FR}$ found for this sample, we get

$$\omega_{up} = (2.166 \pm 0.062_{\text{(stat)}} \pm 0.036_{\text{(syst)}}) \text{MHz}.$$  \hspace{1cm} (4.26)

Taking into account the average value of the magnetic field measured in section 2.9 $B_\perp = -(25.6 \pm 0.1) \text{G}$, the mass of the muon $m_\mu = 1.884 \times 10^{-28} \text{kg}$, the charge of the electron $e = 1.602 \times 10^{-19} \text{C}$ and $g = 2$, the magnetic moment is

$$\mu_{up} = (0.995 \pm 0.045) \frac{e\hbar}{2m_\mu}.$$ \hspace{1cm} (4.27)

If we are interested in the value of the Landé factor, we use Eq. 2.22 to get

$$g_{up} = 1.99 \pm 0.09.$$ \hspace{1cm} (4.28)
From the fit in Fig. 4.13, we get

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$0.058 \pm 0.006$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$2.213 \pm 0.056$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$-3.565 \pm 0.225$</td>
</tr>
</tbody>
</table>

Adding the systematic error $\sigma_{FR}$ found for this sample, we get

$$\omega_{down} = (2.213 \pm 0.056_{(stat)} \pm 0.037_{(syst)}) \text{MHz,}$$

(4.29)
which translates into the following magnetic moment and Landé factor

\[
\mu_{\text{down}} = (1.017 \pm 0.043) \frac{e\hbar}{2m_\mu}, \tag{4.30}
\]

\[
g_{\text{down}} = 2.033 \pm 0.086. \tag{4.31}
\]

It is interesting to observe that the two graphs are in phase opposition, as we would expect. This validates the theory and necessary hypothesis for method B. We calculate

\[
\Delta \delta = 9.787 \pm 0.322 = (3.115 \pm 0.102) \pi, \tag{4.32}
\]

which is an odd number of \( \pi \).

Figure 4.12: Values of the probability for different fitting ranges \([t_1, t_2]\) for the -3A down data sample.

**Combined results**

We use Eq. 4.6 to combine the two values we just measured into one

\[
\omega_A = (2.191 \pm 0.067) \text{ MHz}, \tag{4.33}
\]

or, in terms of magnetic moment and Landé factor,

\[
\mu_A = (1.006 \pm 0.031) \frac{e\hbar}{2m_\mu}; \tag{4.34}
\]

\[
g_A = 2.013 \pm 0.061. \tag{4.35}
\]

**Method B**

For this procedure, we now plot the ratio in Eq. 4.11 with respect to the time and we get Fig. 4.14. The fitting range and the binning choice has been subjected to the same analysis as the previous fit and we get \( t_1 = 0.8 \mu s \) and \( t_2 = 6.2 \mu s \). The fit is accurate and gives us the following values for the parameters
Figure 4.13: Magnetic moment fit for the -3A down data sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.058 ± 0.003</td>
</tr>
<tr>
<td>( \omega [\text{MHz}] )</td>
<td>2.162 ± 0.021</td>
</tr>
<tr>
<td>C</td>
<td>0.019 ± 0.005</td>
</tr>
</tbody>
</table>

Figure 4.14: Magnetic moment fit for the -3A data sample, using method B.

This fit, in contrast with method A, has proven to be little range dependant. The value of the systematics is thus lower and we get the following overall result for the frequency

\[
\omega_B = (2.162 \pm 0.021_{\text{(stat)}} \pm 0.014_{\text{(syst)}}) \text{ MHz}
\]  

(4.36)
or, in terms of magnetic moment and Landé factor,

\[ \mu_B = (0.993 \pm 0.017) \frac{e \hbar}{2m_\mu}, \]  

(4.37)

\[ g_B = 1.986 \pm 0.033. \]  

(4.38)

4.2.3 Comments

All the values found, regardless of the sample choice or the magnetic field settings, are in good accordance with the expected values for its magnetic moment and the Landé factor. If we take \( g \) to be equal to 2, then \( \mu \) must be equal to Bohr Magneton \( \frac{e \hbar}{2m_\mu} \), which we found it to be. If we now concentrate on \( g \), then, according to the Standard model (cfr. section 2.6), we should find something close to 2, which is what we found too. All the values measured during this experiment for \( \mu \) and \( g \) are summarized in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mu )</th>
<th>( \frac{e \hbar}{2m_\mu} )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>B Setting</td>
<td>+5A</td>
<td>-3A</td>
</tr>
<tr>
<td>Up</td>
<td></td>
<td>0.996 ± 0.031</td>
<td>0.995 ± 0.045</td>
</tr>
<tr>
<td>Down</td>
<td></td>
<td>0.989 ± 0.021</td>
<td>1.017 ± 0.043</td>
</tr>
<tr>
<td>Combined</td>
<td></td>
<td>0.991 ± 0.017</td>
<td>1.006 ± 0.031</td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
<td>0.996 ± 0.012</td>
<td>0.993 ± 0.017</td>
</tr>
</tbody>
</table>

Unfortunately, this experiment is not sensitive to the anomalous magnetic moment of the muon. The variable \( a = (g - 2)/g \) that characterizes the anomalous moment is of the order of magnitude \( a \sim O(10^{-2}) \) but in our experiment, the error on \( g \) is of the same order, \( \delta g \sim O(10^{-2}) \). As a result, the error on \( a \) would be bigger or equal to its value, rendering any attempt irrelevant.

4.2.4 Magnetic field dependency

In this section, we analyse the influence of the intensity and the direction of the magnetic field on the different parameters of this experiment. Note that the fit of the +5A up data sample, having the highest \( \chi^2/ndf \), gives the most inaccurate values and thus they might have to be taken with a grain of salt in the following comments.

First of all, the presence of a magnetic field (and by extension its characteristics) is unrelated to the value of the lifetime \( \tau \). The values obtained in the different fits are similar to one another and are in accordance with the lifetime calculated in the previous subsection.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{5u} [\mu s] )</td>
<td>2.172 ± 0.013</td>
</tr>
<tr>
<td>( \tau_{5d} [\mu s] )</td>
<td>2.184 ± 0.022</td>
</tr>
<tr>
<td>( \tau_{-3u} [\mu s] )</td>
<td>2.191 ± 0.014</td>
</tr>
<tr>
<td>( \tau_{-3d} [\mu s] )</td>
<td>2.229 ± 0.016</td>
</tr>
</tbody>
</table>

The asymmetry factor \( A \) is as well relatively independent on the sign and intensity of the current going through the coils. Indeed, the asymmetry factor is directly related to the average antimuon polarization before their decay but not to their decay direction.
The Larmor frequency \( \omega \) is obviously intensity dependant by definition. The frequency is much higher for the sample with the higher magnetic field as we can see comparing Fig. 4.9 to Fig. 4.14. The sign of the frequency, on the other hand stays unchanged when the direction of the magnetic field does. As reminded in the theoretical background, the spin of the antimuon will rotate in the presence of a magnetic field. If a particle precesses clockwise around a field line when the current is travelling one way through the coils, it should precess the other way around when the current is reversed. Nevertheless, our experiment is not sensitive to this phenomenon as it is one dimensional. In other words, the experiment is structured as a succession of planes, each giving one byte of usable information, the time of a hit. As a result, weather the spin turns one way or the other before decaying, it won’t translate into an opposite frequency.

Finally, we observe that there is no phase shift \( \cos(x + 2n\pi) = \cos(x) \) between the two magnetic field settings for a given decay direction. This was expected as, for \( t = 0 \), the factor \( 1 + \cos(\omega t + \delta) \) should be the same for a given set of scintillators. This confirms that the experiment is not sensitive to the sense of rotation as a phase shift of \( \{2n+1\pi \mid n \in \mathbb{N}\} \) would have indicated it to be \( \cos(x+n\pi) = \cos(-x) \).

\[
\begin{align*}
\delta_{5u} &= -0.26 \pm 0.373 \\
\delta_{-3u} &= 6.222 \pm 0.230 \quad \rightarrow \Delta \delta_u = 6.482 \pm 0.438 = (2.063 \pm 0.139) \pi, \\
\delta_{5d} &= -3.392 \pm 0.190 \\
\delta_{-3d} &= -3.564 \pm 0.225 \quad \rightarrow \Delta \delta_d = 0.172 \pm 0.294 = (0.055 \pm 0.094) \pi.
\end{align*}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Parameter} & \text{Value} \\
\hline
A_{5u} & 0.039 \pm 0.006 \\
A_{5d} & 0.058 \pm 0.006 \\
A_{5B} & 0.053 \pm 0.004 \\
A_{-3u} & 0.053 \pm 0.005 \\
A_{-3d} & 0.058 \pm 0.006 \\
A_{-3B} & 0.058 \pm 0.003 \\
\hline
\end{array}
\]
5 Conclusion

This experiment perfectly confirmed the current leading theory in particle physics, the standard model, and what is already known about the muon lifetime and magnetic moment. This experiment might not be like other state-of-the-art devices out there but it has allowed us, in a short amount of time and with a relatively simple apparatus, to infer some important parameters of the muon with a decent precision.

The sensitivity could obviously be improved in several ways. The two main sources of imprecision on the gyromagnetic ratio are the statistical error and the magnetic field irregularity. To tackle these limitations, on the one hand, one could increase the amount of statistics by running the experiment for longer periods of time. On the other hand, one could increase the dimensions of the Helmoltz pair and distance it from the absorber as the magnetic field spikes close to the coils.

Nevertheless, the bottom line is that this experiment is good enough to give a master student its first contact with the world of particle physics and its techniques. For more precision, one can always turn to major experiments such as FAST at PSI [26] for the muon lifetime or high end Muon Spin Rotation experiments (µSR) for its magnetic moment.
Appendix A, Maximum Likelihood Estimation

Suppose there is a sample \( x_1, x_2, \ldots, x_n \) of \( n \) independent and identically distributed observations, coming from a distribution with an unknown probability density function \( f_0(\cdot) \). It is however surmised that the function \( f_0 \) belongs to a certain family of distributions \( \{ f(\cdot | \theta) \mid \theta \in \Theta \} \) (where \( \theta \) is a vector of parameters for this family), called the parametric model, so that \( f_0 = f(\cdot | \theta_0) \). The value \( \theta_0 \) is unknown and is referred to as the true value of the parameter. It is desirable to find an estimator \( \hat{\theta} \) which would be as close to the true value \( \theta_0 \) as possible. Both the observed variables \( x_i \) and the parameter \( \theta \) can be vectors.

To use the method of maximum likelihood, one first specifies the joint density function for all observations. For an independent and identically distributed sample, this joint density function is

\[
f(x_1, x_2, \ldots, x_n | \theta) = f(x_1 | \theta) \times f(x_2 | \theta) \times \cdots \times f(x_n | \theta) .
\]

(5.1)

Now we look at this function from a different perspective by considering the observed values \( x_1, x_2, \ldots, x_n \) to be fixed "parameters" of this function, whereas \( \theta \) will be the function’s variable and allowed to vary freely; this function will be called the likelihood:

\[
L(\theta | x_1, \ldots, x_n) = f(x_1, x_2, \ldots, x_n | \theta) = \prod_{i=1}^{n} f(x_i | \theta) .
\]

(5.2)

In practice it is often more convenient to work with the logarithm of the likelihood function, called the log-likelihood:

\[
\ln L(\theta | x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln f(x_i | \theta) ,
\]

(5.3)

or the average log-likelihood:

\[
\hat{\ell} = \frac{1}{n} \ln L .
\]

(5.4)

The hat over \( \ell \) indicates that it is akin to some estimator. Indeed, \( \ell \) estimates the expected log-likelihood of a single observation in the model.

The method of maximum likelihood estimates \( \theta_0 \) by finding a value of \( \theta \) that maximizes \( \ell(\theta | x) \). This method of estimation defines a maximum-likelihood estimator (MLE) of \( \theta_0 \),

\[
\{ \hat{\theta}_{MLE} \} \subseteq \{ \arg \max_{\theta \in \Theta} \ell(\theta | x_1, \ldots, x_n) \} ,
\]

(5.5)

if any maximum exists. An MLE estimate is the same regardless of whether we maximize the likelihood or the log-likelihood function, since log is a monotonically increasing function [27].

In the case of the antimuon lifetime for instance, the random variables follow an exponential distribution function of 3 parameters. As a result, Eq. 5.2 becomes

\[
L(p | x_1, \ldots, x_n) = (p_1 e^{x_1/p_2} + p_3) \times \cdots \times (p_1 e^{x_n/p_2} + p_3) .
\]

(5.6)

To find the best set of parameters for this function, one has to find the maximum log-likelihood for the given set of parameters \( p \), that is solving the set of equations

\[
\frac{\partial}{\partial p_i} \left( \frac{1}{n} \ln L(p | x_1, \ldots, x_n) \right) = 0 .
\]

(5.7)
Appendix B, Error Propagation

The standard deviation of a parameter $f$ function of $n$ random variables is given by

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial x_i} \bigg|_{x_i} \frac{\partial f}{\partial x_j} \bigg|_{x_j} \sigma_{i,j}},$$

(5.8)

with $\sigma_{i,j} = E[(x_i - \bar{x}_i)(x_j - \bar{x}_j)]$ the covariance for the variables $x_i$ and $x_j$. If we now have a set of $n$ independent variables, Eq. 5.8 becomes

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \bigg|_{\bar{x}_i}\right)^2 \sigma_i^2},$$

(5.9)

where $\sigma_i$ is the standard deviation of the variable $x_i$ [28]. This corresponds to the quadratic sum

$$\sigma_f = \frac{\partial f}{\partial x_1} \bigg|_{\bar{x}_1} \sigma_1 \oplus \cdots \oplus \frac{\partial f}{\partial x_n} \bigg|_{\bar{x}_n} \sigma_n.$$

(5.10)

In the scope of this experiment, we need to derive at several steps of the process. First of all, the error on the theoretical value of the magnetic field,

$$\sigma_{B_{th}} = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 N}{R} \sigma_I \oplus \left(-\left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 N I}{R^2} \sigma_R\right),$$

(5.11)

as $\mu_0$ if fixed at $4\pi$ and $N$ is a given.

In the data analysis section, we measure for the magnetic moment $\mu$, the gyromagnetic ratio $g$ and the phase shift $\Delta \delta$ their respective incertitudes

$$\sigma_\mu = \frac{2\omega S}{eB} \sigma_{m_\mu} \oplus \frac{2m_\mu \omega S}{eB} \sigma_\omega \oplus \left(-\frac{2m_\mu \omega S}{e^2 h B} \sigma_e\right) \oplus \left(-\frac{2m_\mu \omega S}{e h B^2} \sigma_B\right),$$

(5.12)

$$\sigma_g = \frac{2\omega}{eB} \sigma_{m_\mu} \oplus \frac{2m_\mu \omega}{eB} \sigma_\omega \oplus \left(-\frac{2m_\mu \omega}{e^2 B} \sigma_e\right) \oplus \left(-\frac{2m_\mu \omega}{e B^2} \sigma_B\right),$$

(5.13)

$$\sigma_{\Delta \delta} = \sigma_{\delta_1} \oplus \sigma_{\delta_2}.$$

(5.14)

The standard deviation of $\omega$ is due to statistics and to systematics related to the choice of fitting range. It is clear the dominant sources of uncertainty are the frequency $\omega$ and the magnetic field $B$. The muon mass $m_\mu$ and the charge of the electron $e$ are very well known parameters in particle physics and have thus be neglected in the values given in the data analysis section.

$$m_\mu = 1.883531475(96) \times 10^{-28} \text{ kg},$$

(5.15)

$$e = 1.602176487(40) \times 10^{-19} \text{ C}.$$

(5.16)
References


