LA CINÉMATIQUE - MRU

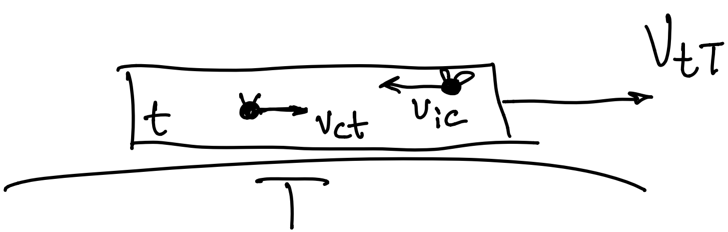
PGC-01

EXEMPLE

Dans un train (symbole t) qui se déplace par rapport à la Terre (symbole T) vers l'est à une vitesse v_{tT} = 10km/h, un grand chien (symbole c) se déplace lentement vers la tête du train à une vitesse v_{ct} = 5 km/h. Un insecte (symbole i) vole vers l'ouest à une vitesse v_{ic} = 0.01 km/h par rapport au chien. Quelle est la vitesse de l'insecte par rapport à la Terre (symbole T , v_{iT})?

$V_{iT} = ?$

$(+)$



$$\vec{V}_{iT} = \vec{V}_{ic} + \vec{V}_{cT}$$

$$= \vec{V}_{ic} + \vec{V}_{ct} + \vec{V}_{tT} \Rightarrow$$

$$V_{iT} = -V_{ic} + V_{ct} + V_{tT} \Rightarrow$$

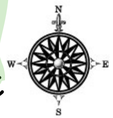
$$V_{iT} = +14.99 \text{ km/h}$$

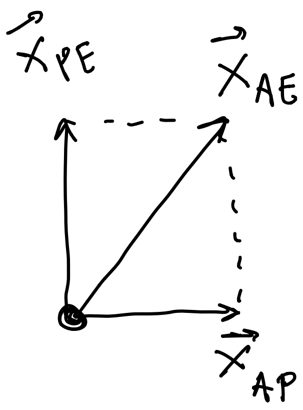
MOUVEMENT RELATIF – 2D

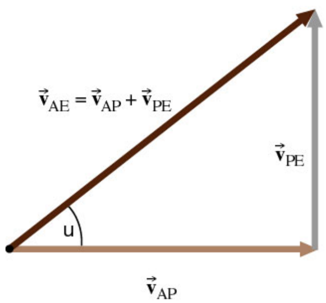
$$\vec{x}_{AE} = \vec{x}_{AP} + \vec{x}_{PE}$$

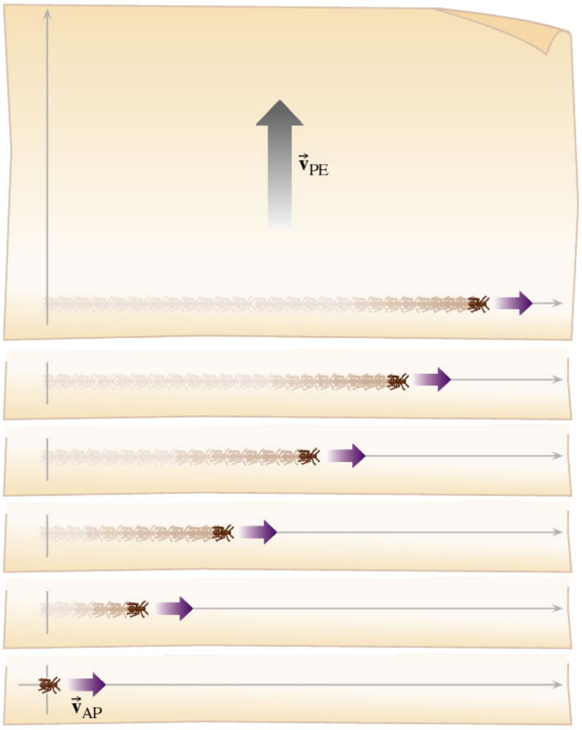
$$\frac{d}{dt}\vec{x}_{AE} = \frac{d}{dt}\vec{x}_{AP} + \frac{d}{dt}\vec{x}_{PE}$$

$$\vec{v}_{AE} = \vec{v}_{AP} + \vec{v}_{PE}$$

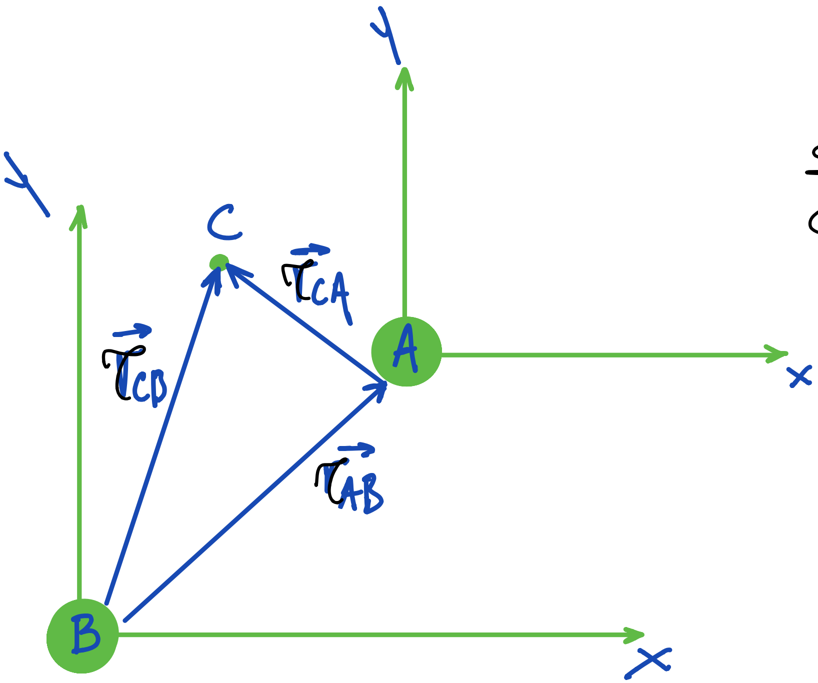








MOUVEMENT RELATIF – 2D



$$\vec{r}_{CB} = \underbrace{\vec{r}_{CA} + \vec{r}_{AB}}$$

$$\frac{d}{dt}\Big(\qquad \Big)$$

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

LA CINÉMATIQUE - MRUA

PGC-01

L'ACCÉLÉRATION

taux de variation

$$\vec{v} = \frac{d\vec{x}}{dt}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

$\vec{a} = 0 \Rightarrow v = $ constante en direction & module

$\vec{a}$ et $\vec{v}$ // $\Rightarrow$ $\vec{v}$ ne change pas de direction

accélération moyenne

$$\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

instantanée

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

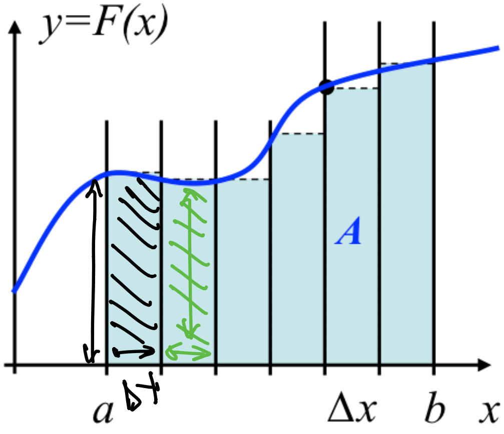
$$[a] = \frac{[v]}{[t]} = \frac{m}{s^2} \text{ (en SI)}$$

INTEGRAL

$$F(x) = \frac{df}{dx} \Rightarrow f(x) = \int F(x)\,dx$$

surface !

$$\int_a^b F(x)\,dx = \lim_{\Delta x \to 0} \sum_a^b F(x)\,\Delta x$$

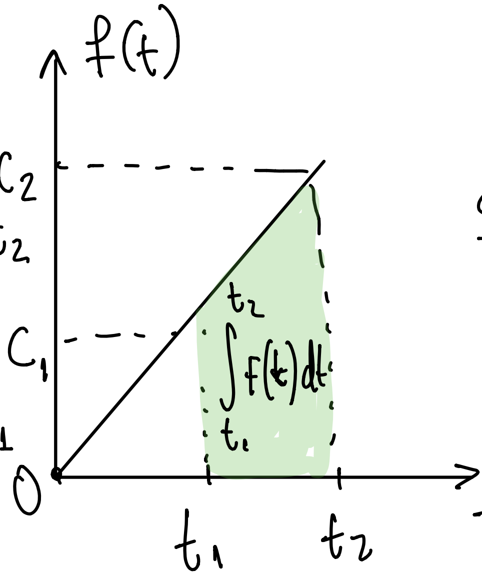


INTEGRAL

Exemple

$f(t_2) = C_2 \to a t_2$

$f(t_1) = C_1 \to a t_1$

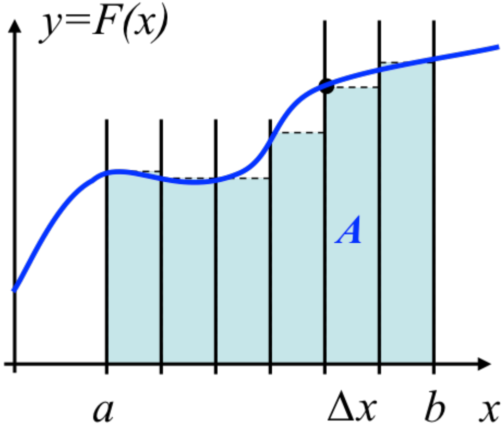


$f(t) = a \cdot t$

$\frac{df(t)}{dt} = a$

$\int_{t_1}^{t_2} f(t)\,dt = \frac{1}{2} c_2 \cdot t_2 - \frac{1}{2} c_1 t_1$

$= \frac{1}{2} a t_2^2 - \frac{1}{2} a t_1^2$



A SAVOIR!

Fonction $f(t)$	Dérivée df/dt	Primitive $F = \int f(t)dt + c$
$f_1 + f_2$	$df_1/dt + df_2/dt$	$F_1 + F_2$
$a\ f_1 + b\ f_2$	$a\ df_1/dt + b\ df_2/dt$	$a\ F_1 + b\ F_2$
$f_1 \cdot f_2$	$f_1 \cdot df_2/dt + f_2 \cdot df_1/dt$	
$f(g(t))$	$dg/dt \cdot df(g)/dg$	
$a,\ a = \text{const.}$	0	$at + c$
$at,\ a = \text{const.}$	a	$at^2/2 + c$
$at + b,\ a, b = \text{const.}$	a	$at^2/2 + bt + c$
$at^2,\ a = \text{const.}$	$2at$	$at^3/3 + c$
Ae^{at+b}	$Aae^{at+b} = a\ f(t)$	$(A/a)\ e^{at+b} = f(t)/a$
x^n	$n\ x^{n-1}$	$x^{n+1}/(n+1) + c$

MRUA

mouvement 1-D

$\vec{S}, \vec{V}, \vec{a}$ //

$\vec{a}$ = constante

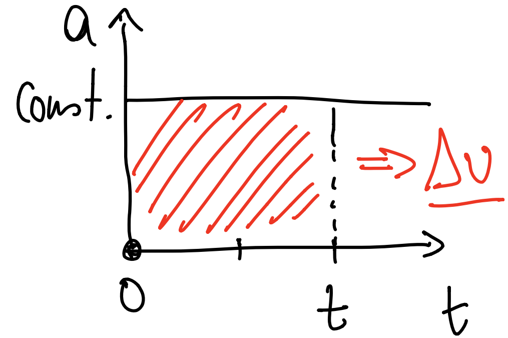
$$a = \frac{dv}{dt} \Rightarrow v = \int_0^t a\,dt = at + v_0 \Rightarrow \Delta v = at$$

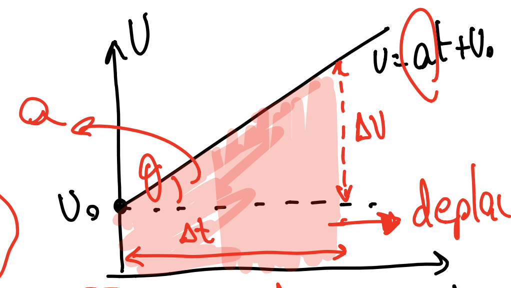
$(v - v_0)$

$$v = \frac{dS}{dt} \Rightarrow S = \int_0^t v\,dt = \int_0^t (at+v_0)\,dt = \frac{1}{2}at^2 + v_0 t + S_0$$

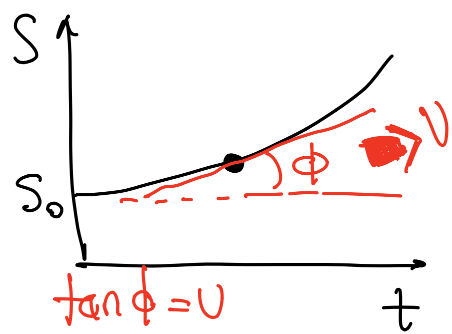
$\vec{a}$ et $\vec{v}$ // $\quad |\vec{a}| > 0 \leadsto v > v_0$ accel.

$|\vec{a}| < 0 \leadsto v < v_0$ decder.





$a = \tan\theta = \frac{\Delta v}{\Delta t} = a$

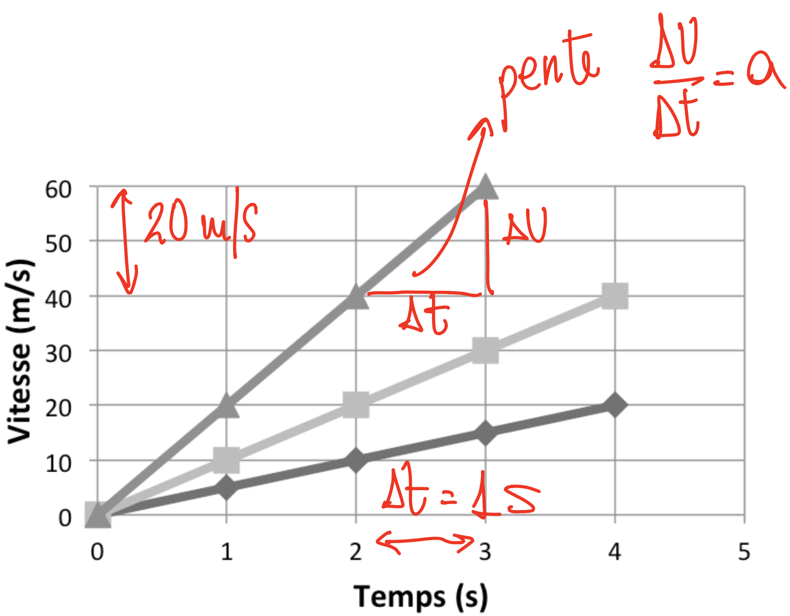


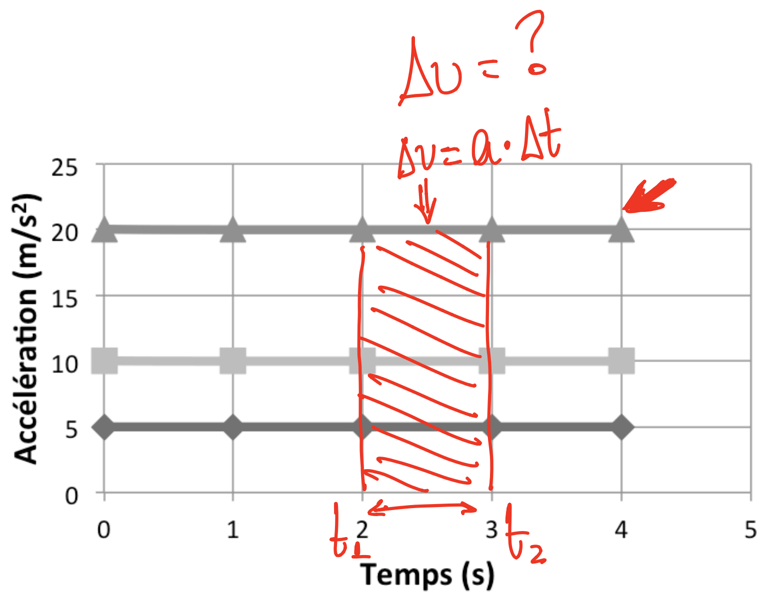
$\tan\phi = v$

MRUA

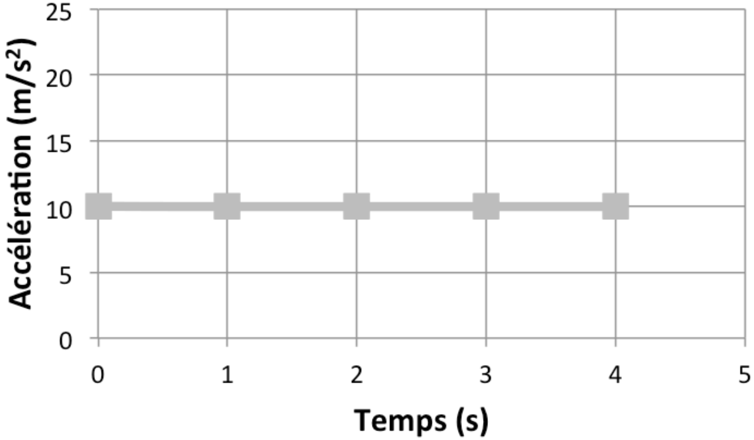
$$a = \frac{dv}{dt} \Rightarrow$$

$$\Rightarrow \Delta v = \int_{t_1}^{t_2} a\,dt$$

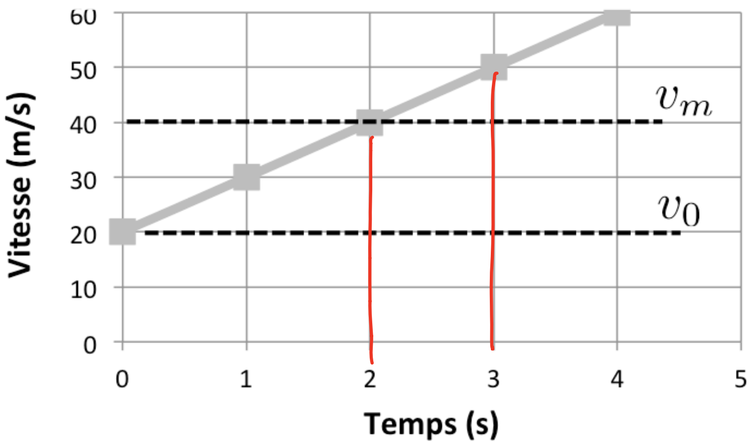




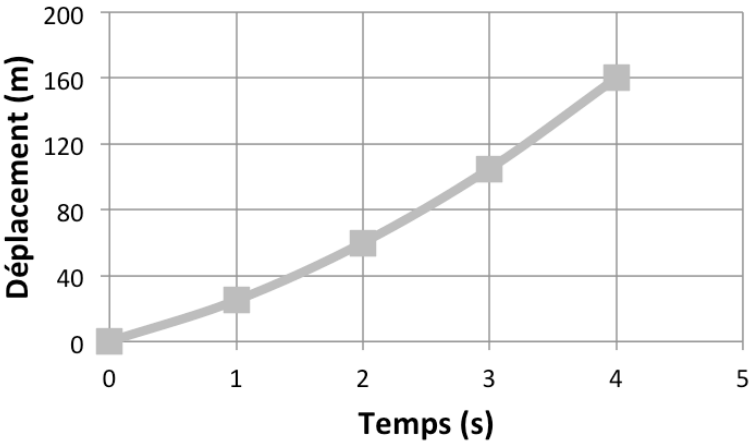
MRUA



Accélération (m/s²)
25
20
15
10
5
0
0
1
2
3
4
5
Temps (s)



Vitesse (m/s)
60
50
40
30
20
10
0
v_m
v_0
0
1
2
3
4
5
Temps (s)



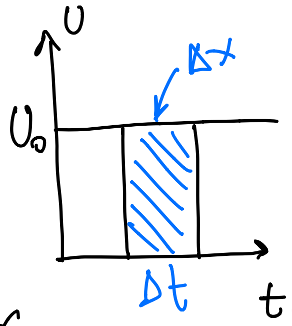
Déplacement (m)
200
160
120
80
40
0
0
1
2
3
4
5
Temps (s)

RESUMÉ

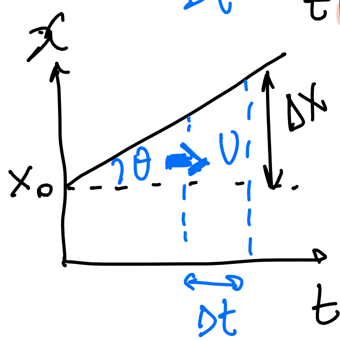
MRU

$a = 0$

$v = \frac{dx}{dt} = \text{const}$

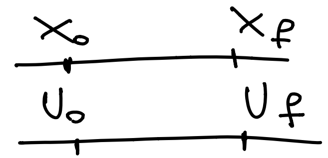


$x = v_0 t + x_0$



$[v] = \frac{m}{s}$

MRUA



$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$a = \text{constante}$

$[a] = m/s^2$

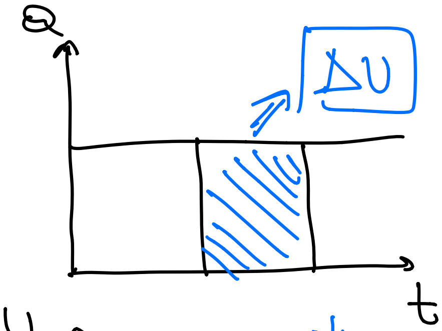
$v = at + v_0$ (1) $\quad v = f(t)$

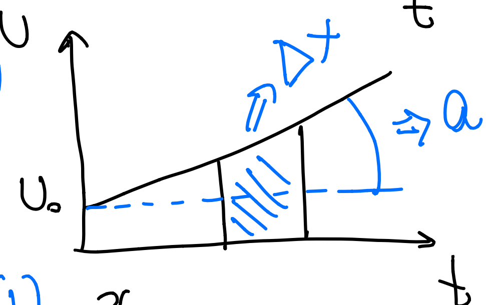
$x = \frac{1}{2}at^2 + v_0 t + x_0$ (2) $\quad x = f(t)$

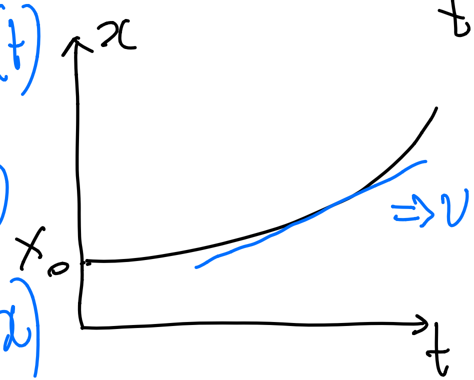
(1) $\Rightarrow t = \frac{v - v_0}{a}$

(2) $\Rightarrow v^2 = 2ax + v_0^2$ (3) $\quad v = f(x)$

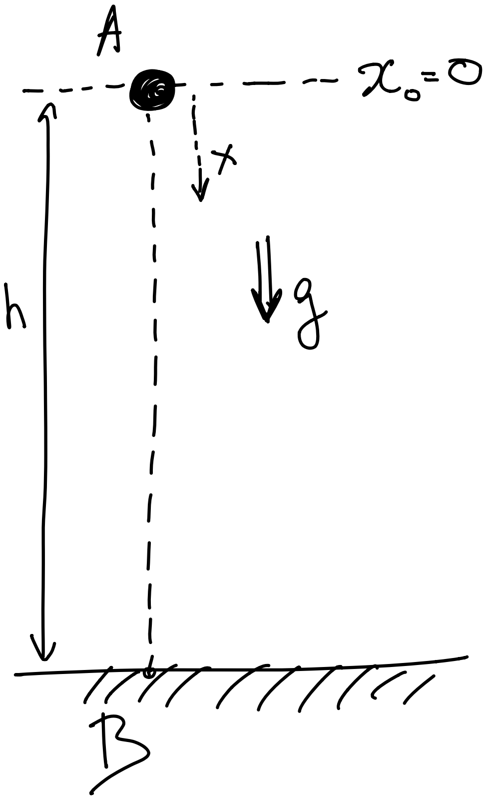
$x_0 = 0$







LA CHUTE LIBRE



$A:\ t=0 \qquad v_0=0 \qquad x_0=0$

$a=g$

Par ②: $x=\frac{1}{2}gt^2 \Rightarrow t=\sqrt{\frac{2x}{g}}$

$$t_{AB}=\sqrt{\frac{2h}{g}}$$

pas de masse !

L'EXTRATERRESTRE

Un extraterrestre explorant la Terre raconte que son pistolet, lâché d'une falaise, est tombé d'une distance de 1 "glong" pendant un temps de 1 "tock". Sa chute pendant 2 "tocks" serait de:

a. 1.5 "glongs"

b. 2 "glongs"

c. 3 "glongs"

d. 4 "glongs"

$x_1 = 1$ glong $\qquad x_2$

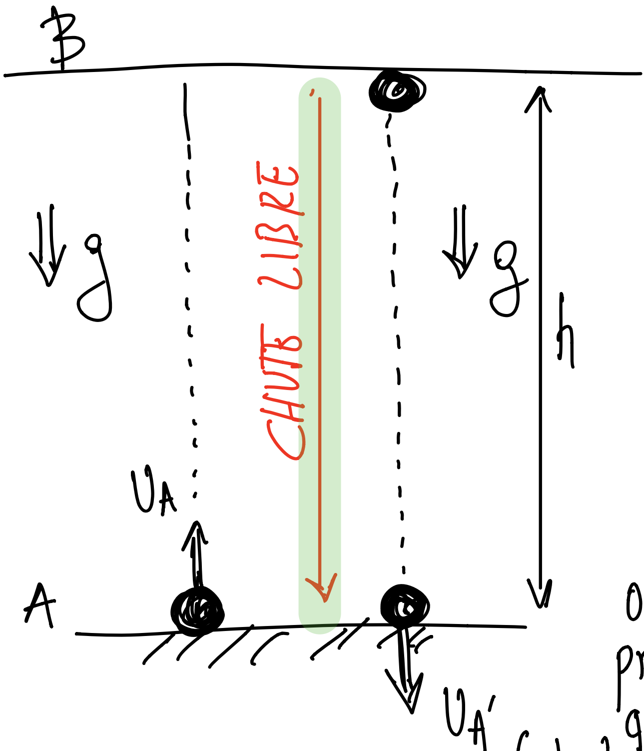
$t_1 = 1$ tock $\qquad t_2 = 2$ tocks $= 2t_1$

CHUTE LIBRE $\quad x = \frac{1}{2} g t^2$

$$\left.\begin{aligned} x_1 &= \frac{1}{2} g t_1^2 \\ x_2 &= \frac{1}{2} g t_2^2 \end{aligned}\right\} \Rightarrow \frac{x_1}{x_2} = \frac{t_1^2}{t_2^2} = \frac{t_1^2}{(2t_1)^2} = \frac{\cancel{t_1^2}}{4\cancel{t_1^2}} \Rightarrow$$

$$\Rightarrow \frac{x_1}{x_2} = \frac{1}{4} \Rightarrow x_2 = 4x_1$$

MOUVEMENT VERTICAL



$\bar{a}$ A: $t=0$ $\quad x_0=0 \quad v_A \quad a=-g$

$\bar{a}$ B: t $\quad x=h \quad v_B=0 \quad a=g$

② $x_{AB}(t) = v_A t - \frac{1}{2} g t^2$

$\underbrace{v_{AB}(t) = v_A - g t}$ $< v_A$

$\bar{a}$ B: $v_B = 0 \Rightarrow 0 = v_A - g t_M \Rightarrow$

$\Rightarrow t_M = v_A / g$

③: $v^2 = 2ax + v_0^2 \Rightarrow$

$0 = -2gh + v_A^2 \Rightarrow h = \dfrac{v_A^2}{2g}$

On peut prouver que:
(chute libre B→A)

$v_A' = v_A$ $\qquad t_D = t_M$