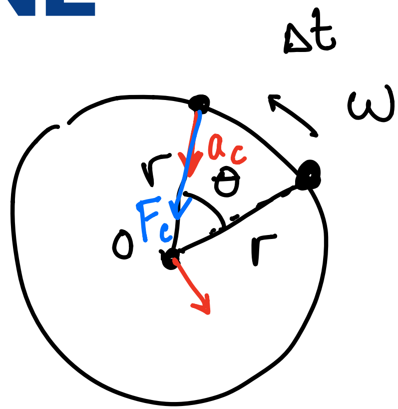
LE MOUVEMENT DE ROTATION

PGC-03

MOUVEMENT CURVILIGNE UNIFORME – RAPPEL



$$\omega = \frac{d\theta}{dt}$$

$v, \omega = \text{const.}$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$F_c = m a_c = m\frac{v^2}{r} = m\omega^2 r$$

$\vec{a}_c \parallel \vec{r}$

QUESTION

Une balle roule du sommet d'une colline avec une vitesse *v*. À ce moment:

$\Sigma F = 0 \Leftrightarrow \vec{v} = \text{const}$

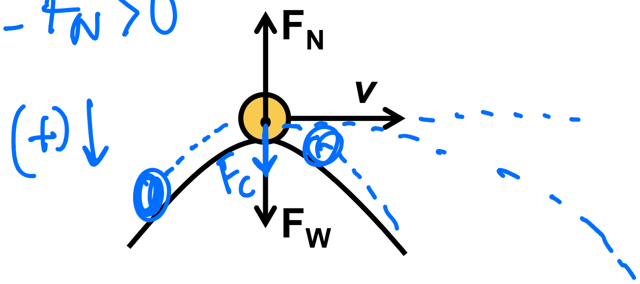
(a) $F_N > F_W$

(b) $F_N = F_W$

(c) $F_N < F_W$

(d) On ne peut pas dire si on ne connait pas *v*

$\Sigma F = F_c = F_W - F_N > 0$



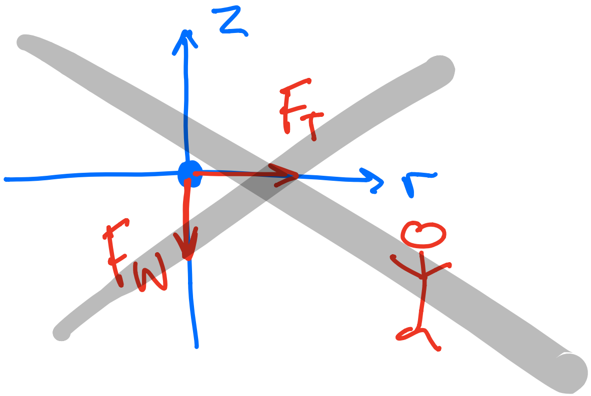
EXEMPLE – PIERRE SUR CORDE

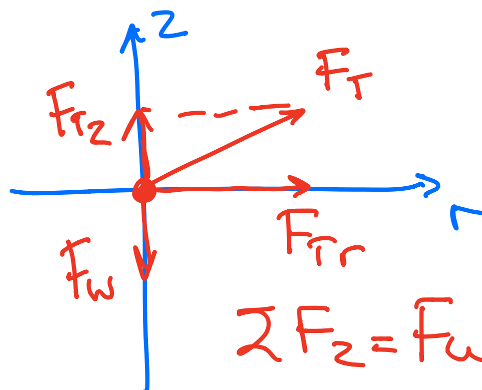
Un chasseur de l'âge de pierre fixe une pierre sur une corde de longueur d'1 m et la tourne dessus de sa tête 'horizontalement'. Si la corde se casse à une tension de 200 N, quelle est la vitesse angulaire maximale en rpm avec la quelle il peut faire tourner la pierre?

$m = 10\ kg.$

$$\cancel{\Sigma F_z = F_W = mg \neq 0}$$

$$\cancel{\Sigma F_r = F_T = F_c}$$

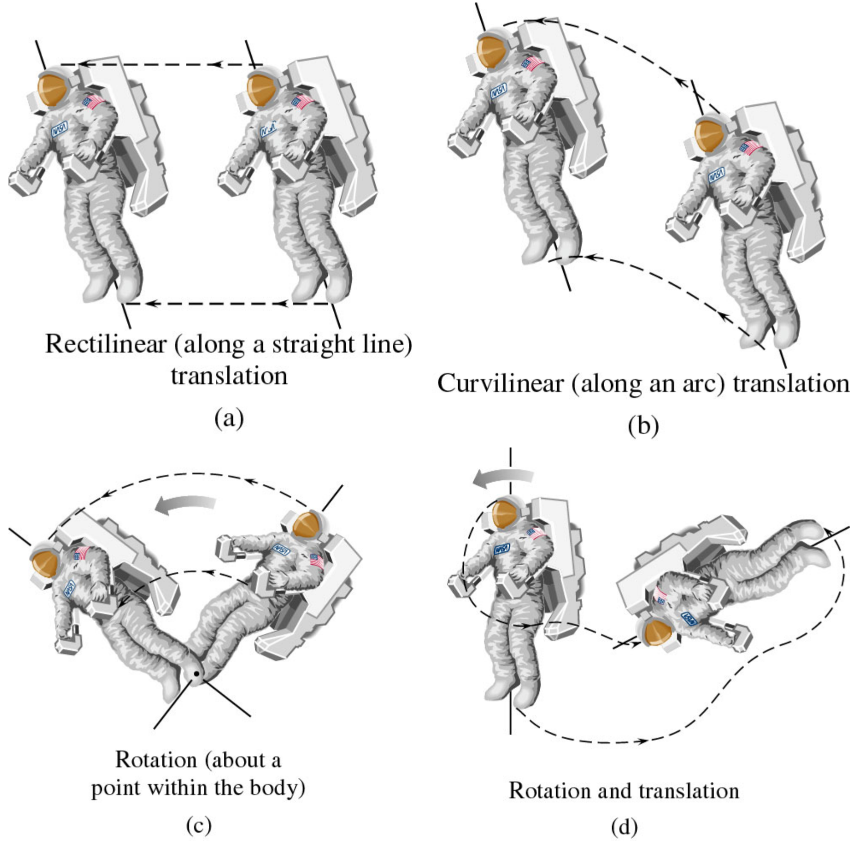




$$\Sigma F_z = F_W - F_{T_z} = 0$$

$$\Sigma F_r = F_{T_r} = F_c$$

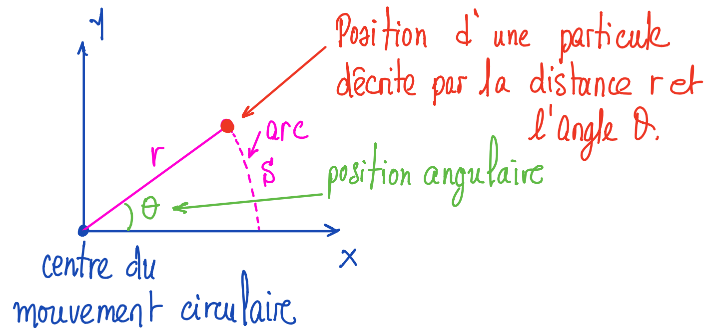
MOUVEMENT DE ROTATION

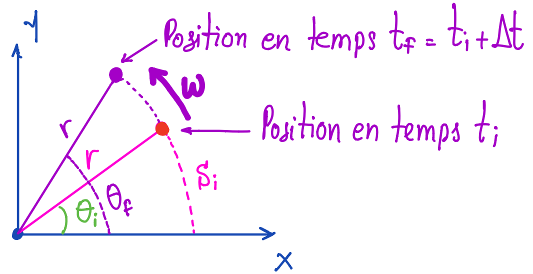


PARAMÉTRISATION DU MOUVEMENT DE ROTATION

θ : radiant

$1 \text{ rad} : \frac{360°}{2\pi} = 57.3°$



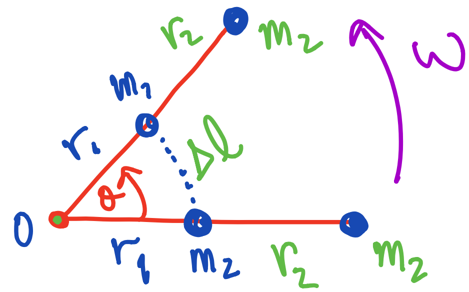


LA VITESSE ANGULAIRE

$$\Delta l = r\,\Delta\theta$$

$$v_m = \frac{\Delta l}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

$$v = r\omega$$



$$v_1 = r_1\,\omega$$

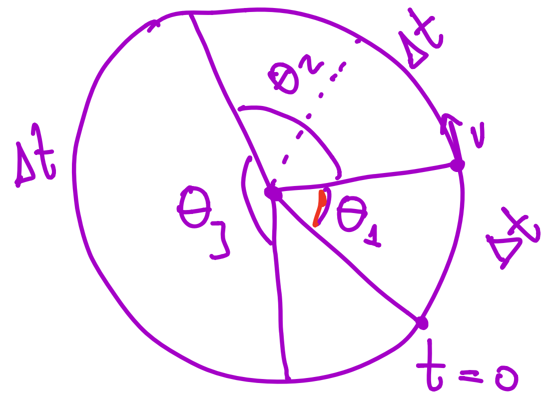
$$v_2 = r_2\,\omega$$

$$\omega = \text{const}$$

$$v \neq \text{const} \,!$$

ACCÉLÉRATION ANGULAIRE

$$\omega = \frac{\Delta\theta}{\Delta t} \neq \text{cont}$$



$$a_{ang}^{m} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$$

$$[a_{ang}] = \frac{[\omega]}{[t]} = \frac{\text{rad}}{s^2}$$

$\Delta t \to 0$

$$a_{ang} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

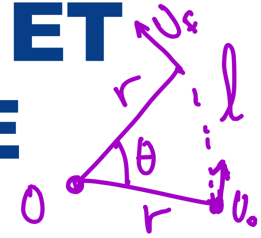
$$a_{ang} \neq a$$

$$v = r\omega \Rightarrow \frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow$$

$$a_T = r \cdot a_{ang}$$

$$\neq a_c$$

ACCÉLÉRATION ANGULAIRE ET ACCÉLÉRATION CENTRIPÈTE

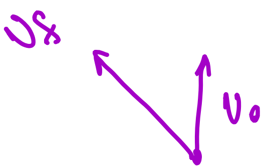


$l = r \cdot \theta$ $\quad [l, r] : m$ $\quad [\theta] = rad$

$\frac{dl}{dt} = r \frac{d\theta}{dt} \Rightarrow$ $\quad v = r \cdot \omega$ $\quad [v] = m/s$ $\quad [\omega] = rad/s$

$\frac{dv}{dt} = r \frac{d\omega}{dt} \Rightarrow$ $\quad a_T = r \cdot a_{ang}$ $\quad [a_T] = m/s^2$ $\quad [a_{ang}] = rad/s^2$

$a_c = \frac{v^2}{r}$ $\quad [a_c] = m/s^2$

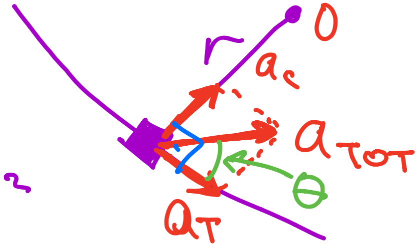


$[a_c] = \frac{[v^2]}{[r]} = \frac{m^2/s^2}{\cancel{m}} = m/s^2$

$r = 50m$

$\omega = 0.60\ rad/s$

$a_{ang} = 0.20\ rad/s^2$



EXEMPLE

Une voiture de Formule 1 prend un virage de 50m de rayon avec une vitesse angulaire de 0.60 rad/s et une accélération angulaire de 0.20 rad/s^2. Calculez sa vitesse linéaire au début du virage, son accélération centripète, ses accélérations tangentielle et totale.

$$v = r \cdot \omega = 50m \cdot 0.60\ rad/s = 30\ m/s$$

$$a_c = \frac{v^2}{r} = \frac{30^2}{50}\ m/s^2 = 18\ m/s^2$$

$$a_T = r \cdot a_{ang} = 50m \cdot 0.20\ \cancel{rad}/s^2 = 10\ m/s^2$$

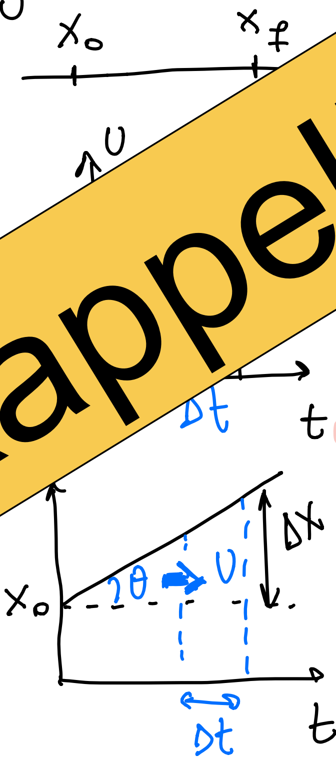
$$a_{TOT} = \sqrt{a_c^2 + a_T^2} = \cdots 21\ m/s^2$$

$$\tan\theta = \frac{a_c}{a_T} = \frac{18}{10} \Rightarrow \theta = \cdots$$

RESUMÉ

Rappel!

MRU $a=0$



$$v = \frac{dx}{dt} = \text{const}$$

$$x = v_0 t + x_0$$

$$[v] = \frac{m}{s}$$

MRVA

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = \text{constante}$$

$$[a] = m/s^2$$

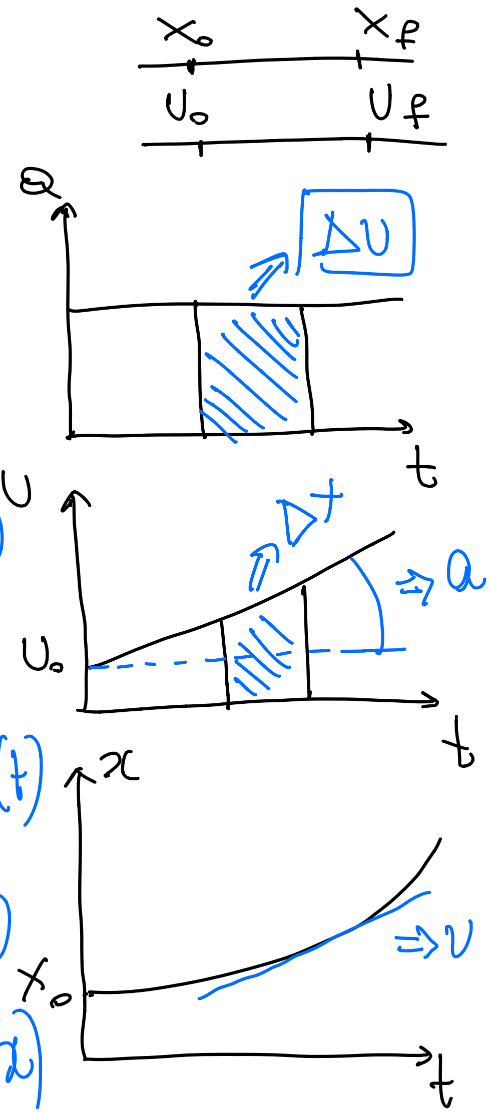
$$v = at + v_0 \quad (1) \quad v = f(t)$$

$$x = \frac{1}{2}at^2 + v_0 t + x_0 \quad (2) \quad x = f(t)$$

$$(1) \rightarrow t = \frac{v - v_0}{a}$$

$$(2) \Rightarrow v^2 = 2ax + v_0^2 \quad (3) \quad v = f(x)$$

$x_0 = 0$



MOUVEMENT CURVILIGNE UNIFORMÉMENT ACCÉLÉRÉ

MRUA

$$v = v_0 + at$$

$$v_m = \frac{1}{2}(v + v_0)$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 - v_0^2 = 2ax$$

MCUA

$$v = v_0 + a_T t$$

$$v_m = (v_0 + v)/2$$

$$l = v_0 t + \frac{1}{2}a_T t^2$$

arc

$$v^2 - v_0^2 = 2a_T l$$

$$l = r \cdot \theta$$

$$v = r \cdot \omega$$

$$a_T = r \cdot a_{ang}$$

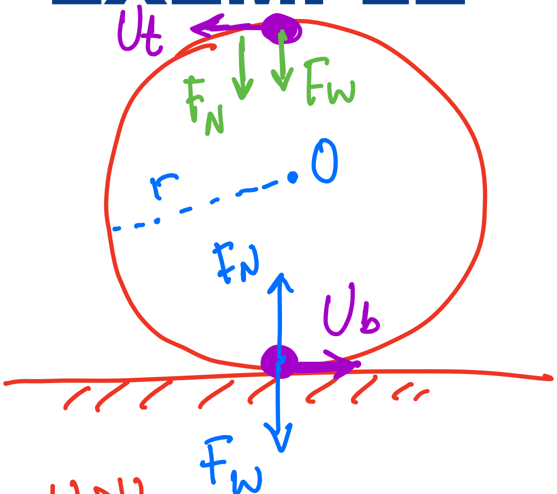
$$\omega = \omega_0 + a_{ang} t$$

$$\omega_m = \frac{1}{2}(\omega + \omega_0)$$

$$\theta = \omega_0 t + \frac{1}{2}a_{ang} t^2$$

$$\omega^2 - \omega_0^2 = 2a_{ang} \cdot t$$

EXEMPLE – LOOP VERTICAL



$$\sum F_r^b = F_N^b - F_w = F_c = \frac{m v_b^2}{r} \Rightarrow$$

$$\Rightarrow F_N^b = \frac{m v_b^2}{r} + mg > mg \quad !$$

$$\sum F_r^t = F_w + F_N^t = F_c^t = \frac{m v_t^2}{r} \Rightarrow$$

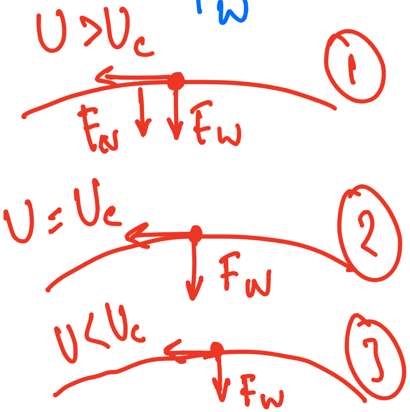
$$\Rightarrow \boxed{F_N^t = \frac{m v_t^2}{r} - mg} \Rightarrow \frac{m v_t^2}{r} = F_N^t + mg$$

Si $F_N^t = 0 \Rightarrow F_w = F_c \Rightarrow$

$$\Rightarrow mg = \frac{m v_t^2}{r} \Rightarrow \boxed{v_t = \sqrt{gr}}$$

$v_c = \sqrt{gr}$

"vitesse critique"



Tir Orizontal.

ROULEMENT SANS GLISSEMENT

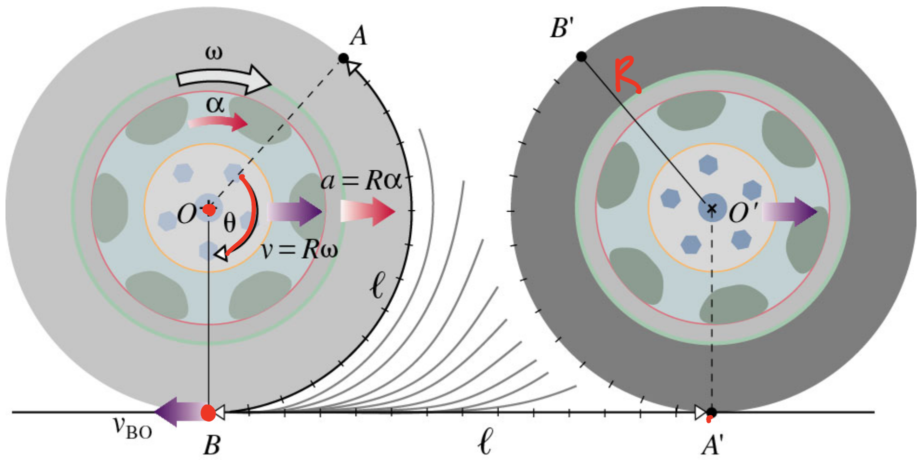
$AB = A'B' = \ell$

$\ell = R.\theta$

$a_T = R\, a_{ang}$

$v_{BO} = R\omega = v = v_{OS}$

$\vec{v}_{BS} = \vec{v}_{BO} + \vec{v}_{OS} \Rightarrow$

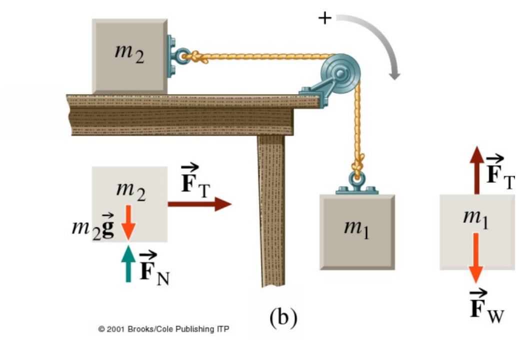


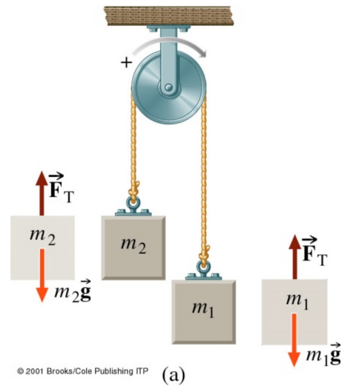
$v_{BS} = -\underbrace{v_{OB} + v_{OS}}_{\text{les deux ont le même module}} = 0 \Rightarrow \boxed{v_{BS} = 0} \Rightarrow$ En accord avec l'hypothèse : PAS DE GLISSEMENT!

ÉQUILIBRE – 1

PGC-02

RAPPEL – MOUVEMENT COUPLÉ





Au repos

$F_T = F_W$

$m_1 > m_2$

$a_1 = a_2$

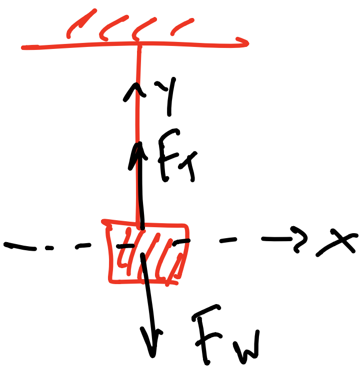
ÉQUILIBRE STATIQUE

$$\sum \vec{F} = 0 \Leftrightarrow \vec{v} = \text{const}$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

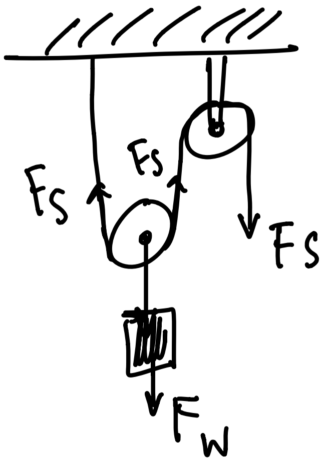
$$\sum F_y = F_T - F_W = 0$$

$$\Rightarrow F_T = F_W$$

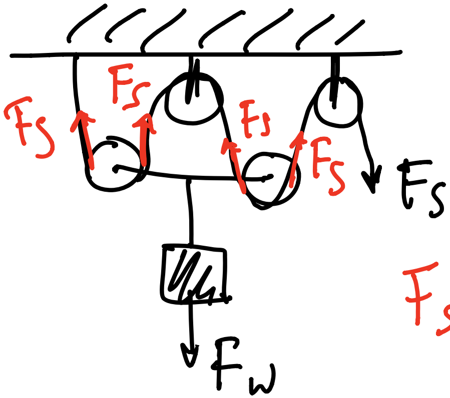


SYSTÈMES DE FORCES PARALLELES

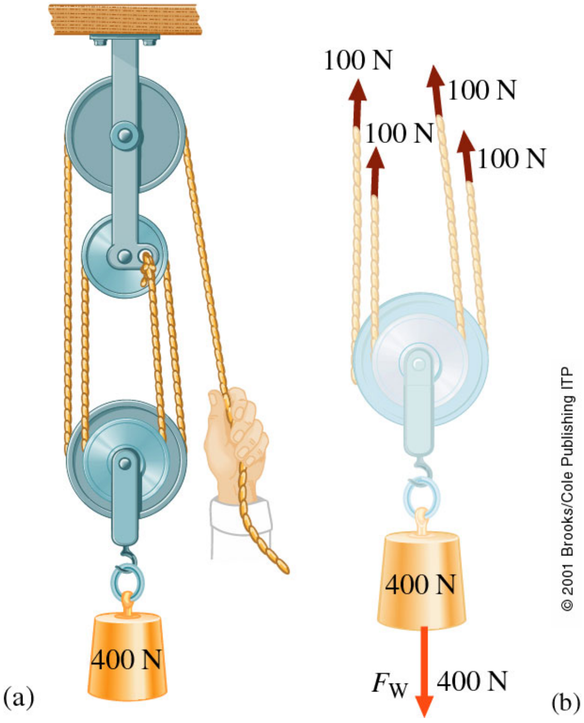
- DEMULTIPLIER FORCES
- REORIENTER FORCES

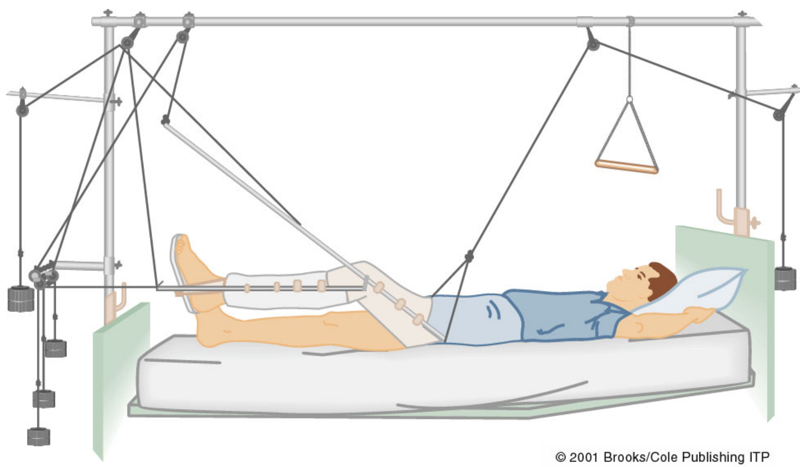


$F_S = \frac{1}{2} F_W$

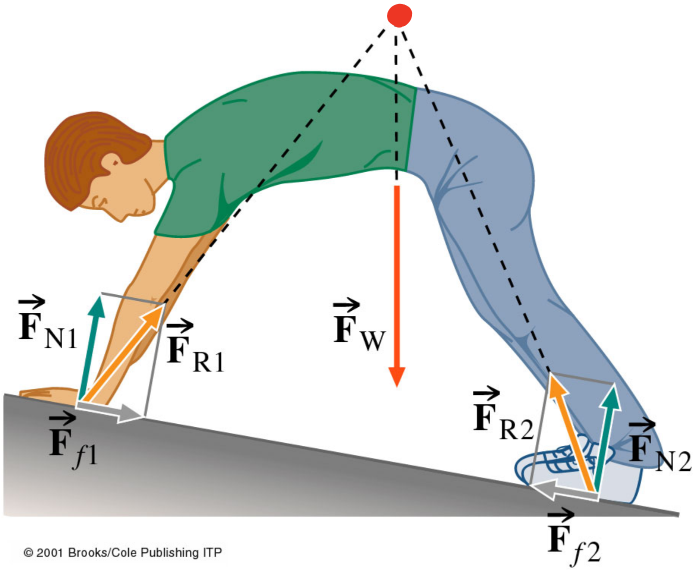


$F_S = \frac{1}{4} F_W$


100 N
100 N
100 N
100 N
400 N
400 N
F_W 400 N
(a)
(b)
© 2001 Brooks/Cole Publishing ITP



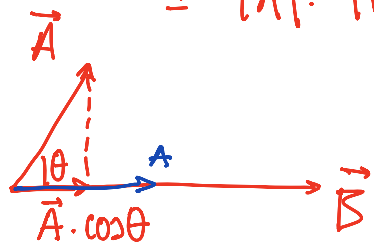
FORCES CONCOURANTES



Produit Scalaire

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$$
$$= |A|\cdot|B|\cdot\cos\theta$$



min : 90°

max = 0°

Produit VecToRIEL.

$$\vec{C} = \vec{A}\times\vec{B} \qquad |\vec{C}| = |\vec{A}|\cdot|\vec{B}|\cdot\sin\theta$$

$$\begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} A_y B_z - A_z B_y \\ -(A_x B_z - A_z B_x) \\ A_x B_y - A_y B_x \end{pmatrix}$$