DYNAMIQUE DE ROTATION

PGC-04

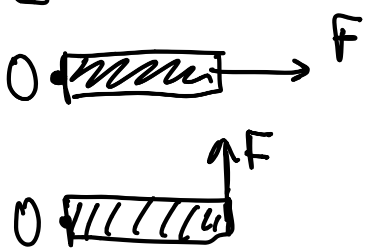
LE MOMENT DE FORCE

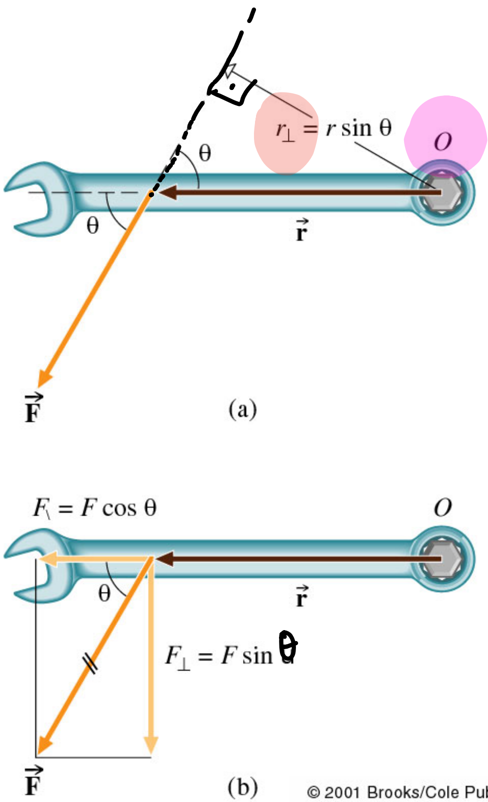
$\vec{\tau}_O = \vec{r} \times \vec{F}$

$$\tau_O = r_\perp \cdot F = r \cdot \sin\theta \cdot F$$
$$= r \cdot F_\perp = r \cdot F \cdot \sin\theta$$

bras de levier

$$[\tau] = [F][r] = N\,m$$





DYNAMIQUE DE ROTATION - INTRO

$a \Leftrightarrow F$

$F = ma$

m : inertie qui s' oppose au changement de son état. "Resistance"

$a_{ang} \Leftrightarrow \tau$

$\tau = \boxed{I} \cdot a_{ang}$

$I = \int r^2 \, dm = \sum_i m_i r_i^2$

I : inertie

masse et comment elle est repartie autour de l'axe de rotation.

$\Rightarrow$ "moment d' inertie"

MOMENT D'INERTIE DES CORPS SIMPLES

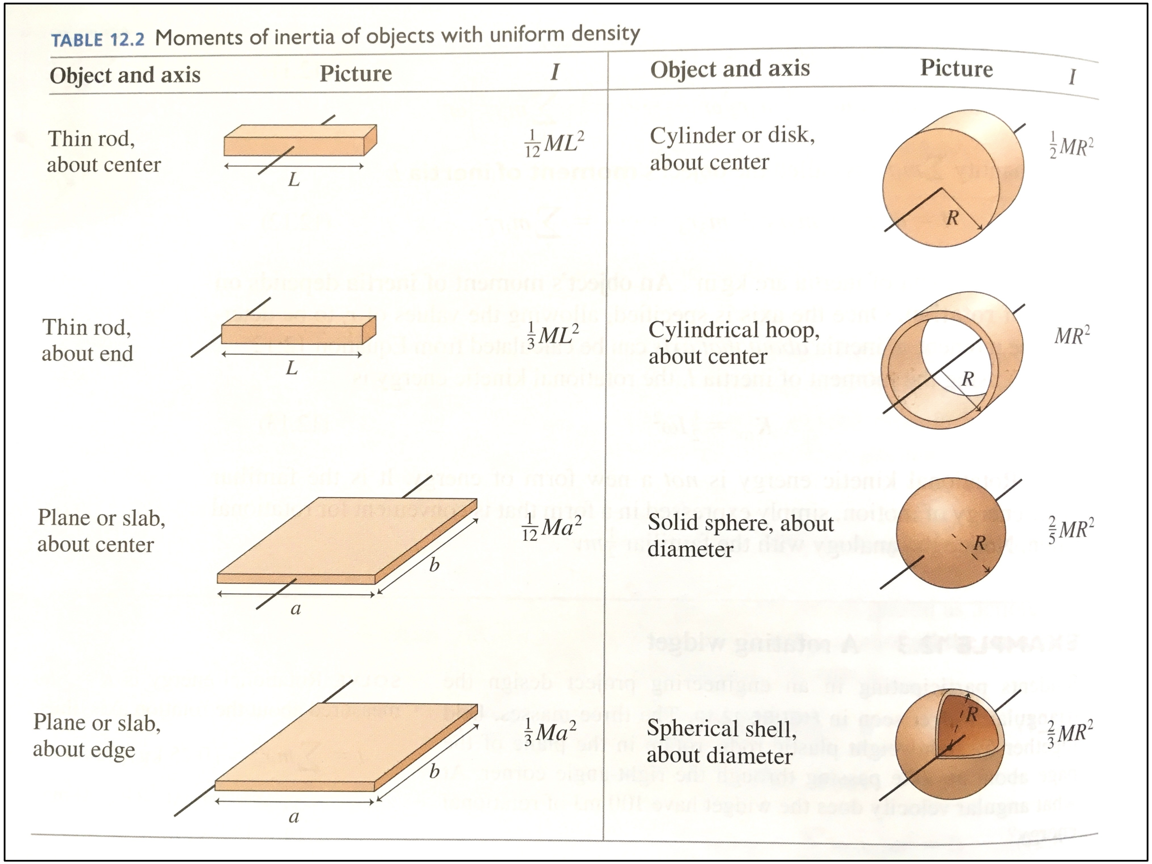


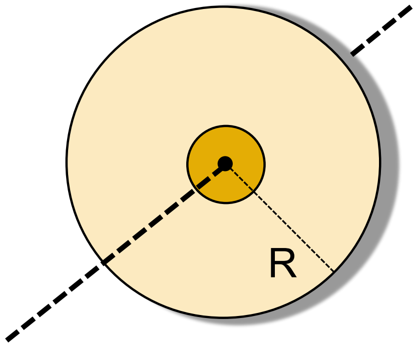
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Image prise du livre de Randall Kight

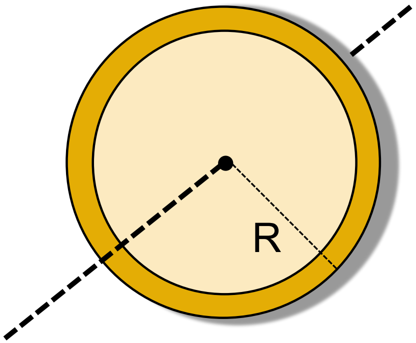
MOMENT D'INERTIE

Masse M



Objet 1

Masse M



Objet 2

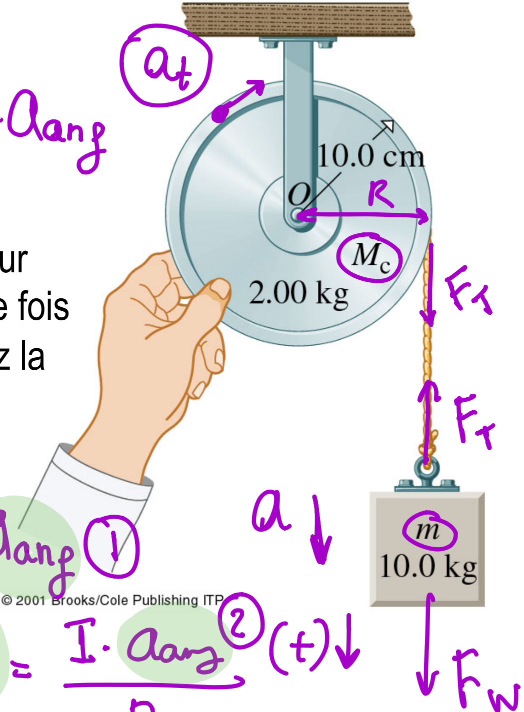
(a) $I_1 > I_2$

(b) $I_1 = I_2$

(c) $I_1 < I_2$

EXEMPLE

$$a_t = a = R \cdot a_{ang}$$



Une masse $m = 10.0$ kg est suspendue à une corde enroulée autour d'un cylindre de rayon $R = 10.0$ cm et de masse $M_c = 2.00$ kg. Une fois lâché, le cylindre est libre de tourner autour de son axe. Déterminez la tension de la corde, et les accélérations du cylindre et de la masse.

$F_T = ?$ $\quad a = ?$

$$\sum F_m = m \cdot a \Rightarrow F_W - F_T = m \cdot a = m \cdot R \cdot a_{ang} \quad (1)$$

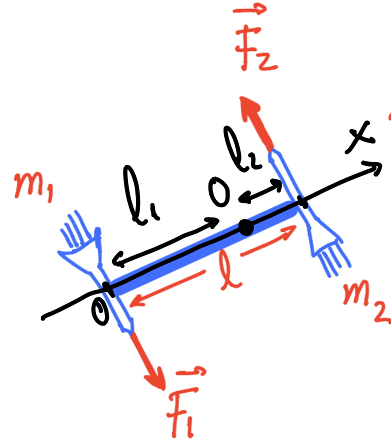
$$\sum \tau_o = I \cdot a_{ang} \Rightarrow F_T \cdot R = I \cdot a_{ang} \Rightarrow F_T = \frac{I \cdot a_{ang}}{R} \quad (2) \quad (+)\downarrow$$

$$(1) \xrightarrow{(2)} mg - I \cdot \frac{a_{ang}}{R} = m R\, a_{ang} \Rightarrow a_{ang} = \frac{mg}{I/R + mR}$$

pour cylindre $\quad I = \frac{1}{2} M_c R^2$

$$a_{ang} = \frac{mg}{mR + \frac{1}{2} M_c R} = \dots = 89.2 \text{ rad/s}^2 \Rightarrow a = R \cdot a_{ang} = 8.92 \text{ m/s}^2 < g$$

EXEMPLE



$m_1 = 100{,}000\ kg$

$m_2 = 200{,}000\ kg$

$l = 90\,m$

$F_1 = F_2 = 50{,}000\ N$

ω at $\Delta t = 30\,s$?

$\Downarrow$

a_{ang}?

$\boxed{W = a_{ang} \cdot \Delta t}$

$$\Sigma\tau = a_{ang} \cdot I$$

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 60\,m$$

$$\Sigma\tau = F_1 \cdot l_1 + F_2 \cdot l_2 = \cdots = 4.500.000\ Nm$$

$$I = \sum_i m_i r^2 = m_1 l_1^2 + m_2 l_2^2 = \cdots = 540.000.000\ kg/m^2$$

$l_1 = 60\,m$

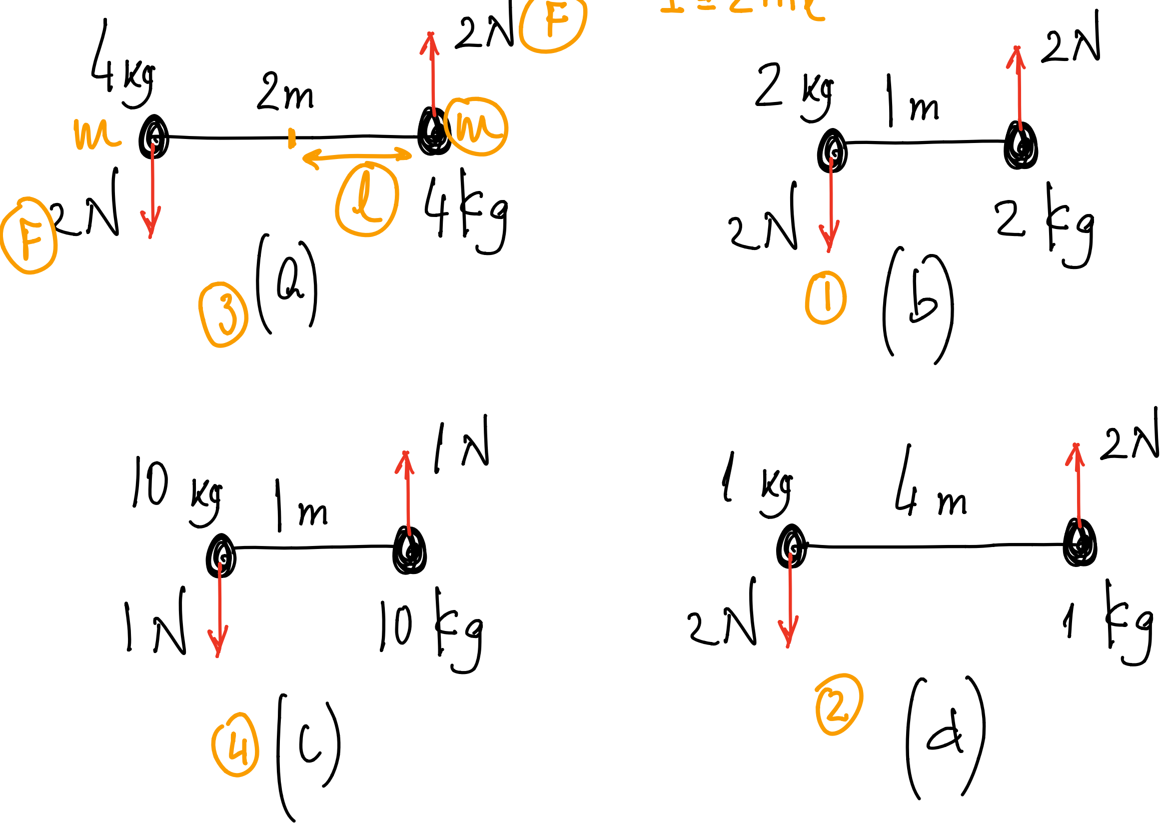
$l_2 = 30\,m$

$$a_{ang} = \frac{\Sigma\tau}{I} = 0.00833\ rad/s^2$$

$$W = a_{ang} \cdot \Delta t \Rightarrow \omega = 0.25\ rad/s$$

EXEMPLE

$$\left.\begin{aligned} a_{ang} &= \frac{\tau}{I} \\ \tau &= 2F\ell \\ I &= 2m\ell^2 \end{aligned}\right\} \Rightarrow a_{ang} = \frac{F}{m\ell}$$



Où a_{ang} max ?

MOMENT CINÉTIQUE

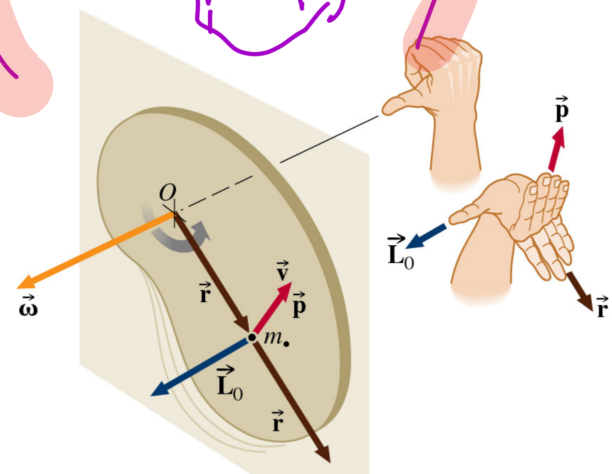
Eq. ROT $\quad \vec{F} : \vec{\tau}$

Eq. ROT $\quad \vec{p} : \vec{L}$

$$\left.\begin{array}{l} \vec{\tau} = \vec{r} \times \vec{F} \\ \vec{L} = \vec{r} \times \vec{p} \\ \vec{p} = m\vec{v} \end{array}\right\} \Rightarrow \boxed{\vec{L} = m\,\vec{r} \times \vec{v}} \;\Big|\; \rightsquigarrow L = \overbrace{\sum m_i r_i^2}^{I}\,\omega$$

$v = r.\omega$

$$\boxed{\vec{L} = I_0 \cdot \vec{\omega}} \Rightarrow \text{Axe de Rotation.}$$



CONSERVATION DU MOMENT CINÉTIQUE

$$\left.\begin{array}{l} \tau = I \cdot a_{ang} = I \cdot \dfrac{\Delta\omega}{\Delta t} \\ L = I \cdot \omega \Rightarrow \Delta L = I\,\Delta\omega \end{array}\right\} \Rightarrow \tau = \frac{\Delta L}{\Delta t}$$

$$\Delta t \to 0 \Rightarrow \tau = \frac{dL}{dt}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \left(\text{eq. } \vec{F} = \frac{d\vec{P}}{dt} \right)$$

Si $\vec{\tau} = 0 \Leftrightarrow \vec{L}$ conservé $\quad (d\vec{L} = 0)$

$(\vec{F} = 0 \Leftrightarrow \vec{p}$ conservé$)$

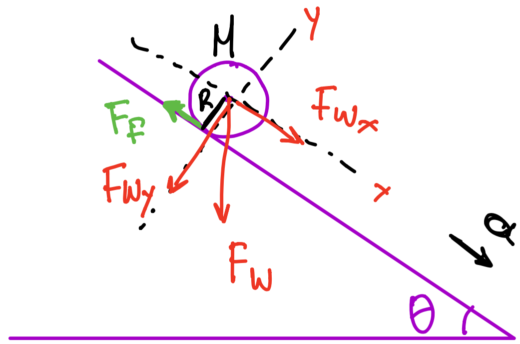
RECAP

Symétrie entre les lois de translation et celles de la rotation

$$\vec{F} = m\vec{a} \qquad \vec{\tau} = I \cdot \vec{a}_{ang}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I \cdot \vec{\omega}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$



$$a = R \cdot a_{ang}.$$

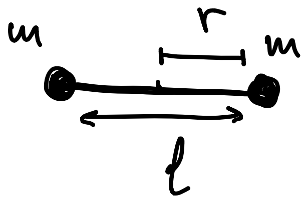
$$\Sigma F_x = F_w \sin\theta - F_f = M \cdot a \Rightarrow F_f = F_w \sin\theta - Ma \quad (1)$$

$$\left.\begin{aligned} \Sigma \tau &= I \cdot a_{ang} = F_f \cdot R \\ I &= \tfrac{1}{2} M R^2 \end{aligned}\right\} \Rightarrow \Sigma_\tau = \frac{1}{2} M R^2 a_{ang} = F_f \cdot R \quad (2)$$

$$\left.\begin{aligned} (1) &\rightarrow F_f = F_w \sin\theta - M R a_{ang} \\ (2) &\rightarrow \tfrac{1}{2} M R^2 a_{ang} = F_f \cdot R \end{aligned}\right\} \Rightarrow a_{ang} = \frac{2}{3} \frac{g \sin\theta}{R}$$

$$\Rightarrow \boxed{a = \frac{2}{3} g \sin\theta}$$

EXEMPLE



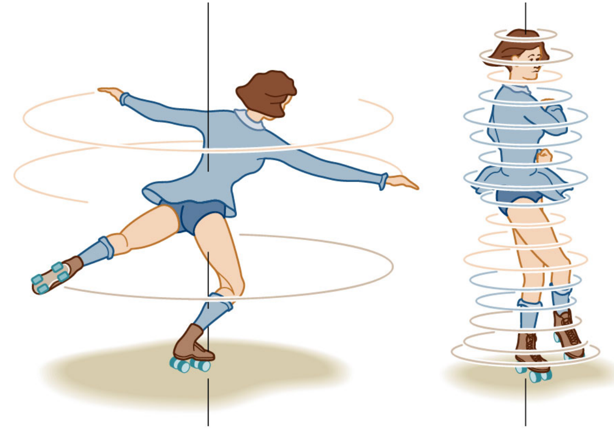
$r = \ell/2$

$\ell_i = 50\,cm$

$\omega_i = 2\,rev/s$

$\ell_f = 160\,cm$

$\omega_f = ?$



(a) (b)

$$L_i = I_i \cdot \omega_i = 2 m r_i^2 \cdot \omega_i$$

$$L_f = I_f \cdot \omega_f = 2 m r_f^2 \cdot \omega_f$$

$$L_i = L_f \Rightarrow r_i^2 \omega_i = r_f^2 \cdot \omega_f \Rightarrow \omega_f = \frac{r_i^2 \omega_i}{r_f^2}$$

$$\Rightarrow \omega_f = 0.2\,rev/s$$

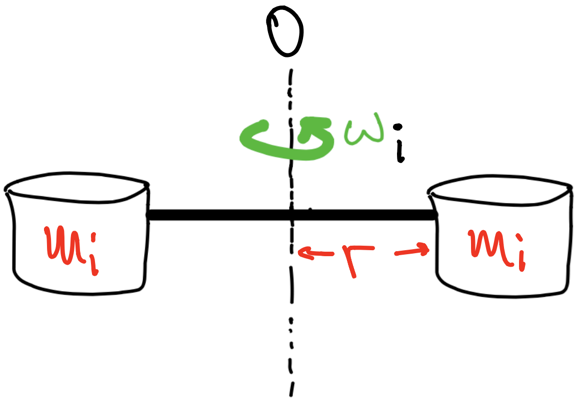
QUESTION

a) $\omega_f > \omega_i$

b) $\omega_i > \omega_f$

c) $\omega_i = \omega_f$

d) Rien de tout ça



$$L_i = 2 m_i r^2 \omega_i$$

$$L_f = 2 m_f r^2 \omega_f$$

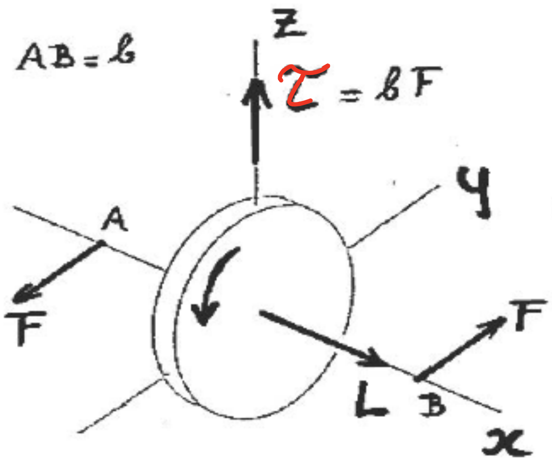
$$L_i = L_f \Rightarrow m_i \omega_i = m_f . \omega_f$$

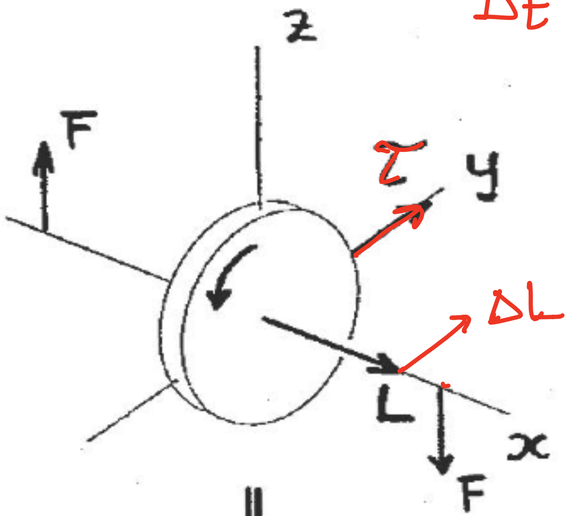
$$\Rightarrow \frac{m_i}{m_f} = \frac{\omega_f}{\omega_i} < 1$$

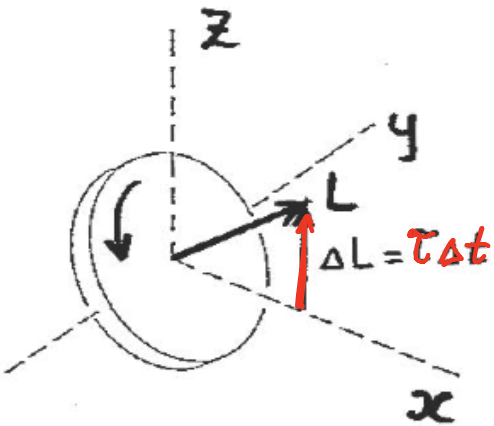
The most non-intuitive subject of 8.01

(perhaps of all physics)

$\tau = \frac{\Delta L}{\Delta t}$







LE GYROSCOPE