DYNAMIQUE DE ROTATION

PGC-04

LE MOMENT DE FORCE

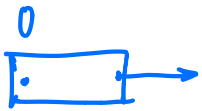
$$\tau_O = r_\perp \cdot F = r \cdot \sin\theta \cdot F =$$

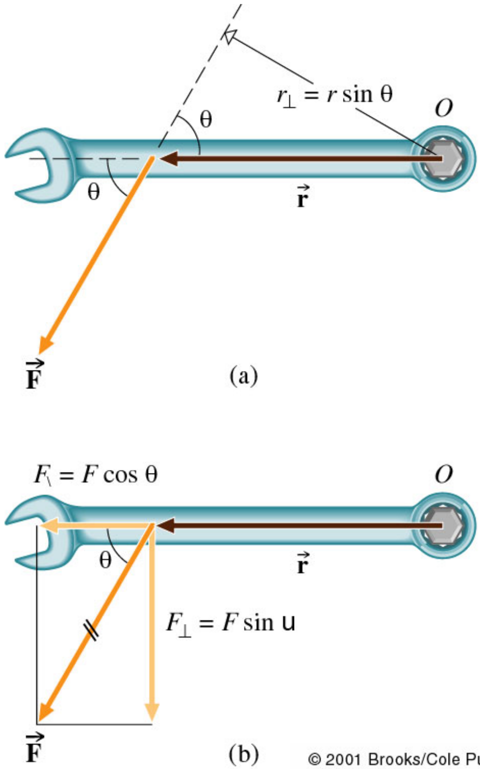
$$= r \cdot F \cdot \sin\theta =$$

$$= r \cdot F_\perp$$

bras de levier

$$[\tau] = [F] \cdot [r] = N \cdot m$$





DYNAMIQUE DE ROTATION - INTRO

$a \Leftrightarrow F$

$\vec{F} = m\vec{a}$

$a_{ang} \Leftrightarrow I$

$\vec{\tau} = I\,\vec{a}_{ang}$

$$I = \int_0^R r^2\,dm$$

m : inertie pour mouvement en translation

I : inertie pour mouvement en rotation.

Depend de la masse et <u>comment</u> elle est répartie autour du point de rotation.

MOMENT D'INERTIE

$$\left.\begin{array}{l} \tau_0 = r \cdot F \\ F = m_0 a_t \end{array}\right\} \Rightarrow \left.\begin{array}{l} \tau_0 = r \cdot m_0 \cdot a_t \\ a_t = r \cdot a_{ang} \end{array}\right\} \Rightarrow \tau_0 = \underbrace{m_0 r^2}\cdot a_{ang}$$

$$\tau_0 = I_0 \cdot a_{ang}$$

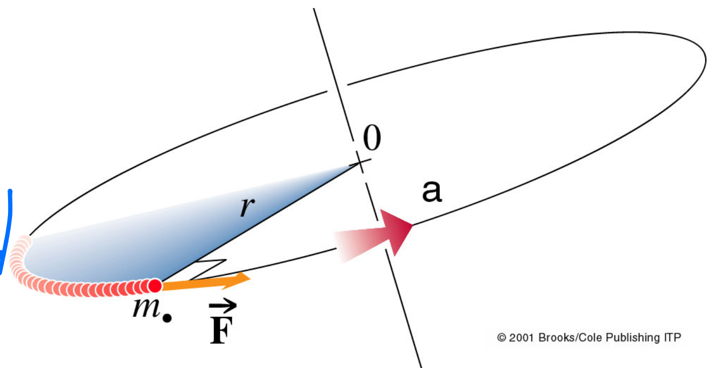
$$F = m\,a$$

$$\tau_0 = I_0\, a_{ang}$$

$$\tau_0^{TOT} = \sum_{i=1}^{N} \tau_0 = \underbrace{\sum_i \left(m_{0i} r_i^2\right)} a_{ang}$$

$$\boxed{I_0 = \sum_i m_{0i} r_i^2}$$

$$\left.\begin{array}{l} \boxed{I_0 = \int r^2 dm} \\ dm = \rho\, dV \end{array}\right\} \Rightarrow I_0 = \int r^2 \rho\, dV$$



MOMENT D’INERTIE DES CORPS SIMPLES

$\vec{\tau} = I \cdot \vec{a}_{ang}$

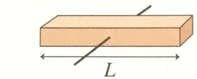
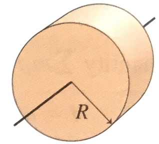
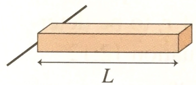
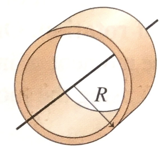
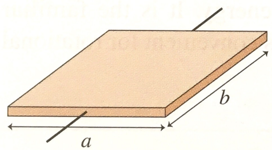
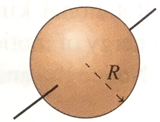
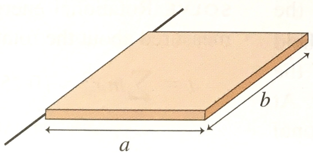
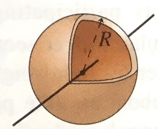
4. $I_1 = I_2$

$\tau_1 = \tau_2$

$a_{ang1} > a_{ang2}$

$\boxed{I = \sum m r^2}$

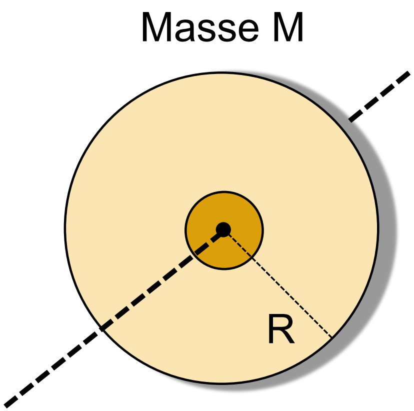
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		I_1 $\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		I_2 $\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

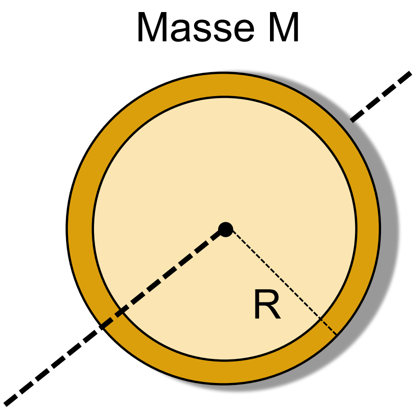
MOMENT D'INERTIE

$$I = \sum m r^2$$

$$I = \int r^2 \, dm$$



Objet 1



Objet 2

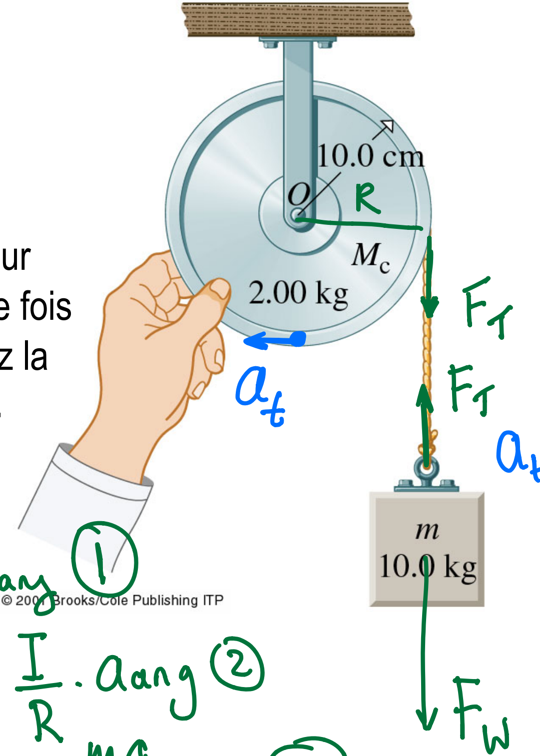
(a) $I_1 > I_2$

(b) $I_1 = I_2$

(c) $I_1 < I_2$

$$\Sigma F = m a_t$$
$$\Sigma \tau = I \cdot a_{ang}$$

EXEMPLE



Une masse $m = 10.0$ kg est suspendue à une corde enroulée autour d'un cylindre de rayon $R = 10.0$ cm et de masse $M_c = 2.00$ kg. Une fois lâché, le cylindre est libre de tourner autour de son axe. Déterminez la tension de la corde, et les accélérations du cylindre et de la masse.

$F_T = ?$ $\qquad a_t = ?$

$$\Sigma F = m a_t = m \cdot R \cdot a_{ang} \Rightarrow F_W - F_T = m R a_{ang} \quad (1)$$

$$\Sigma \tau_o = I \cdot a_{ang} \Rightarrow R \cdot F_T = I \cdot a_{ang} \Rightarrow F_T = \frac{I}{R} \cdot a_{ang} \quad (2)$$

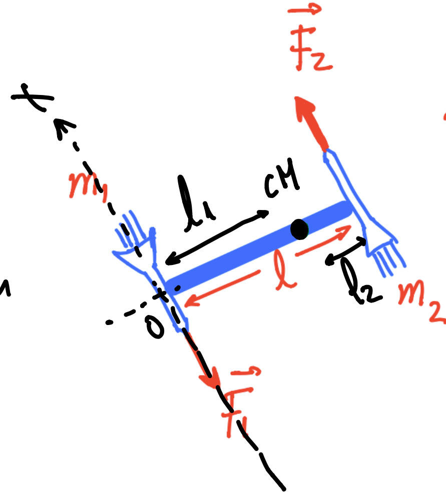
$$(1) \xrightarrow{(2)} F_W - \frac{I}{R} a_{ang} = m R a_{ang} \Rightarrow a_{ang} = \frac{mg}{mR + I/R} \quad (3)$$

$$I = \frac{1}{2} M_c R^2 \text{ (pour le cylindre)}$$

$$(3) \rightarrow a_{ang} = \frac{mg}{mR + \frac{1}{2} M_c R} = \dots = 89.2 \frac{rad}{s^2}, \quad a_t = 8.92 \frac{m}{s^2} < g$$

$\Sigma F = ma$
$\Sigma \tau = I a_{ang}$

EXEMPLE



$m_1 = 100{,}000$ kg
$m_2 = 200{,}000$ kg
$l = 90$m
$F_1 = F_2 = 50{,}000$ N
ω at $\Delta t = 30$s ?
$\Downarrow$
a_{ang} ?

$$x_{CM} = \cancel{\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}} = \frac{m_2 \cdot l}{m_1 + m_2} = 60\text{m}$$

$l_1 = 60$m $\quad l_2 = 30$m

$$\Sigma \tau = I \cdot a_{ang}$$

$$I = \sum_i m_i r_i^2 = m_1 \cdot l_1^2 + m_2 \cdot l_2^2 = 540.000.000 \text{ kg} \cdot \text{m}^2$$

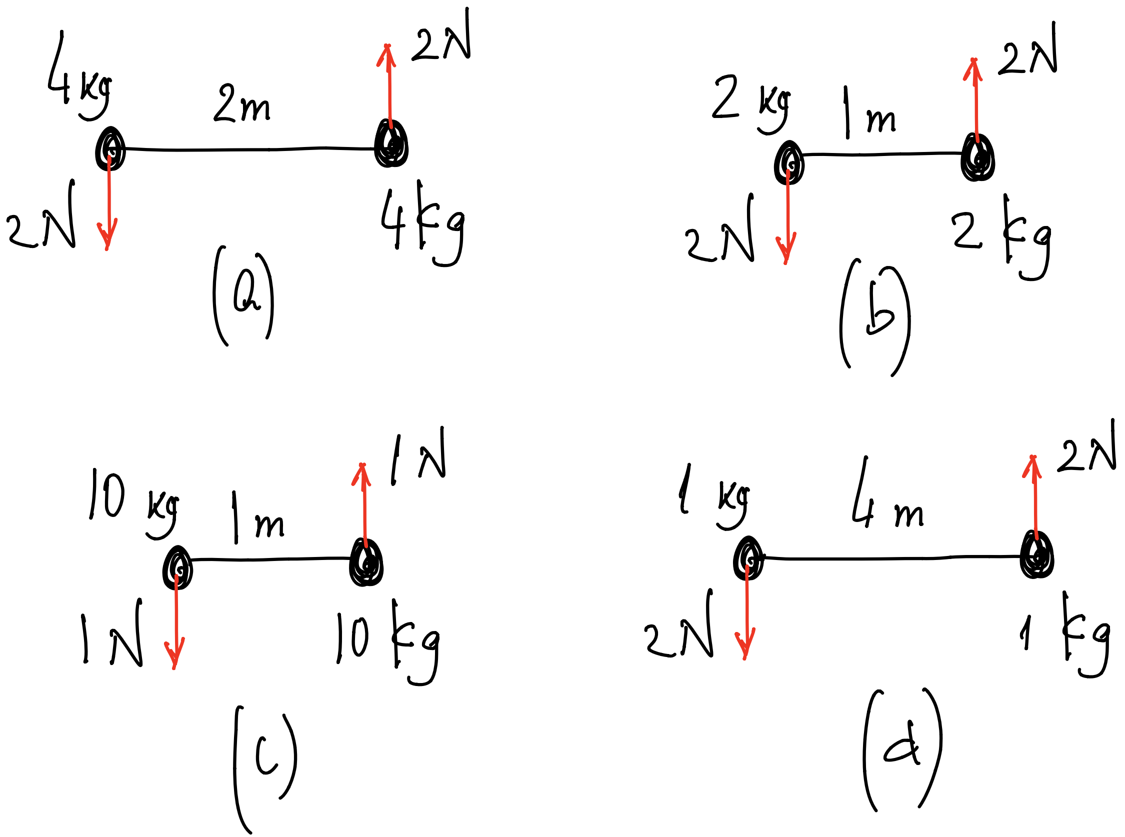
$$\tau = F_1 \cdot l_1 + F_2 \cdot l_2 = \cdots = 4.500{,}000 \text{ N} \cdot \text{m}$$

$$\tau = I \cdot a_{ang} \Rightarrow a_{ang} = \frac{\tau}{I} = 0.00833 \text{ rad/s}^2$$

$$\omega = a_{ang} \cdot \Delta t \Rightarrow \omega = 0.25 \text{ rad/s}$$

EXEMPLE

Où Q_{ang} max?



MOMENT CINÉTIQUE

Equiv. Rotation $\quad \vec{F} \Rightarrow \vec{\tau}$

-//- $\quad \vec{p} = m\vec{v} \Rightarrow \vec{L}$ moment cinetique

$$\left.\begin{array}{l} \vec{\tau} = \vec{r} \times \vec{F} \\ \vec{L} = \vec{r} \times \vec{p} \\ \vec{p} = m\vec{v} \end{array}\right\} \Rightarrow \vec{L} = m\,\vec{r} \times \vec{v}$$

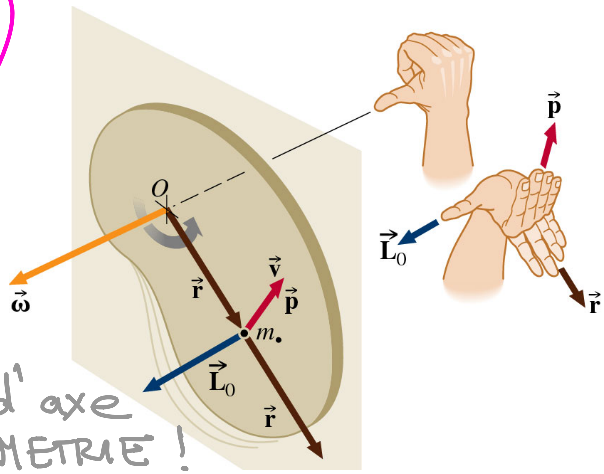
Considerons le cas simple:

$L = mrv \quad (v \perp r)$

Puisque $v = r\omega$:

$L = mr^2\omega \Rightarrow$

$$\boxed{L = I\omega}$$



Generalisable pour Rotation autour d'axe de SYMMETRIE !

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

CONSERVATION DU MOMENT CINÉTIQUE

$$\left.\begin{array}{l} \tau = I \cdot a_{ang} = I \cdot \dfrac{\Delta \omega}{\Delta t} \\ L = I\omega \Rightarrow \Delta L = I \Delta \omega \end{array}\right\} \Rightarrow \tau = \frac{\Delta L}{\Delta t}$$

$$\Delta t \to 0 \;:\; \tau = \frac{dL}{dt}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \text{eq.} \quad \vec{F} = \frac{d\vec{P}}{\Delta t}$$

Si $\vec{\tau} = 0 \Leftrightarrow \vec{L}$: conserve! $\Delta \vec{L} = 0$

(si $\vec{F} = 0 \Leftrightarrow \vec{p}, \vec{v}$ conserve!)

RECAP

Symétrie entre les lois de translation et celles de la rotation

$$\vec{F} = m\vec{a} \qquad \vec{\tau} = I \cdot a_{ang}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\vec{\omega}$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \qquad \vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}$$

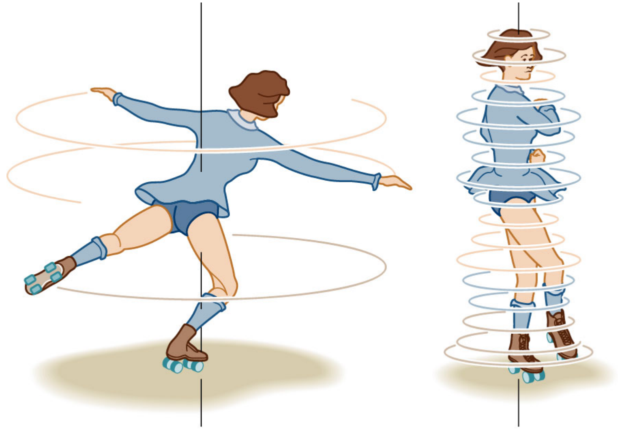
$$m \qquad I$$

EXEMPLE

(A) : $L_A = I_A \cdot \omega_A$

(B) : $L_B = I_B \cdot \omega_B$

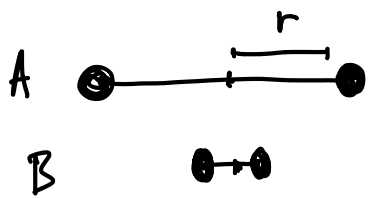
$$\left.\begin{aligned} I_A &> I_B \\ L_A &= L_B \end{aligned}\right\} \quad \omega_A < \omega_B$$



(a) (b)

A B

Personne sur chaise avec altères.

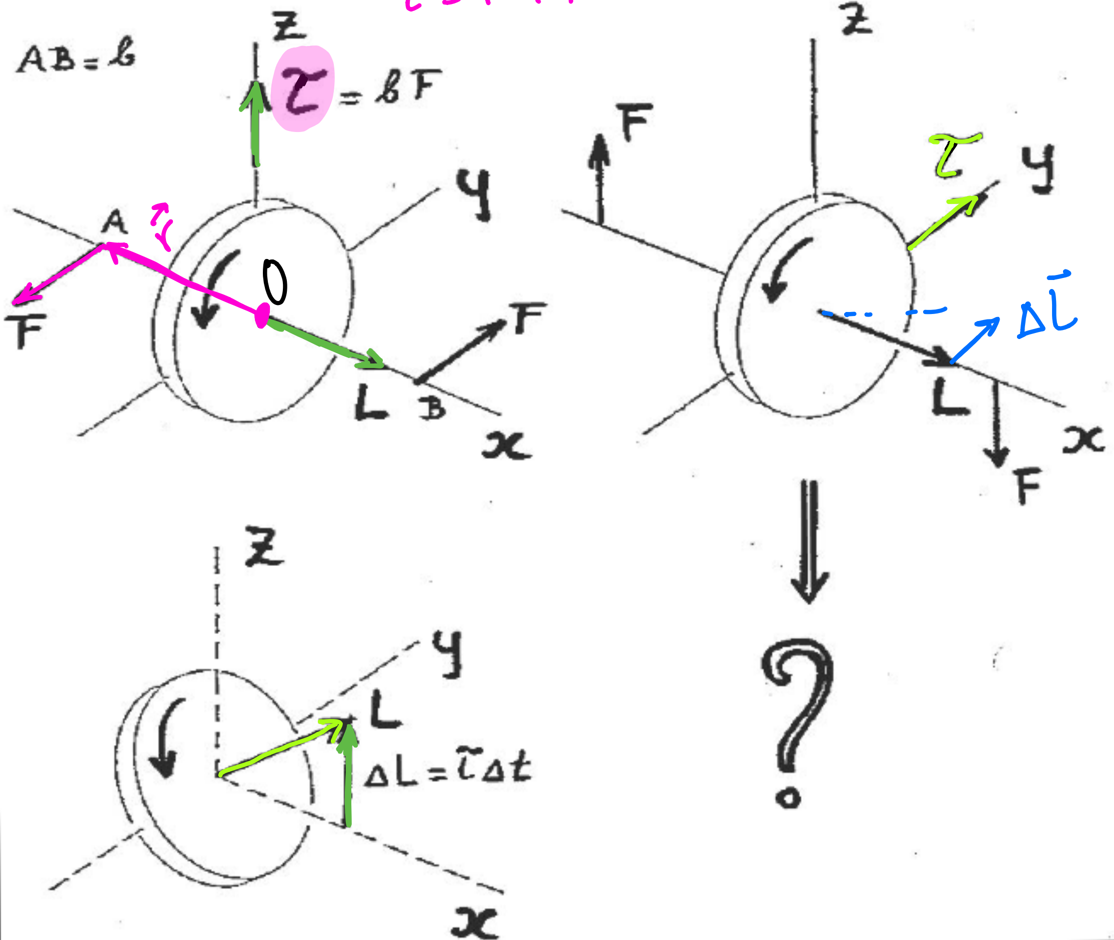


$I = 2mr^2$

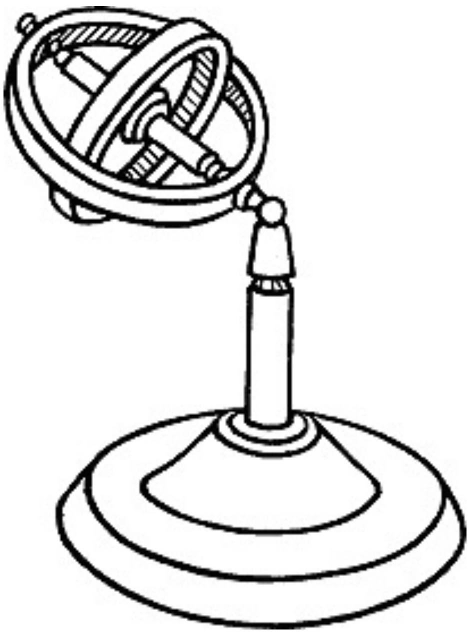
The most non-intuitive subject of 8.01

(perhaps of all physics)

$\tau = \vec{r} \times \vec{F}$



LE GYROSCOPE



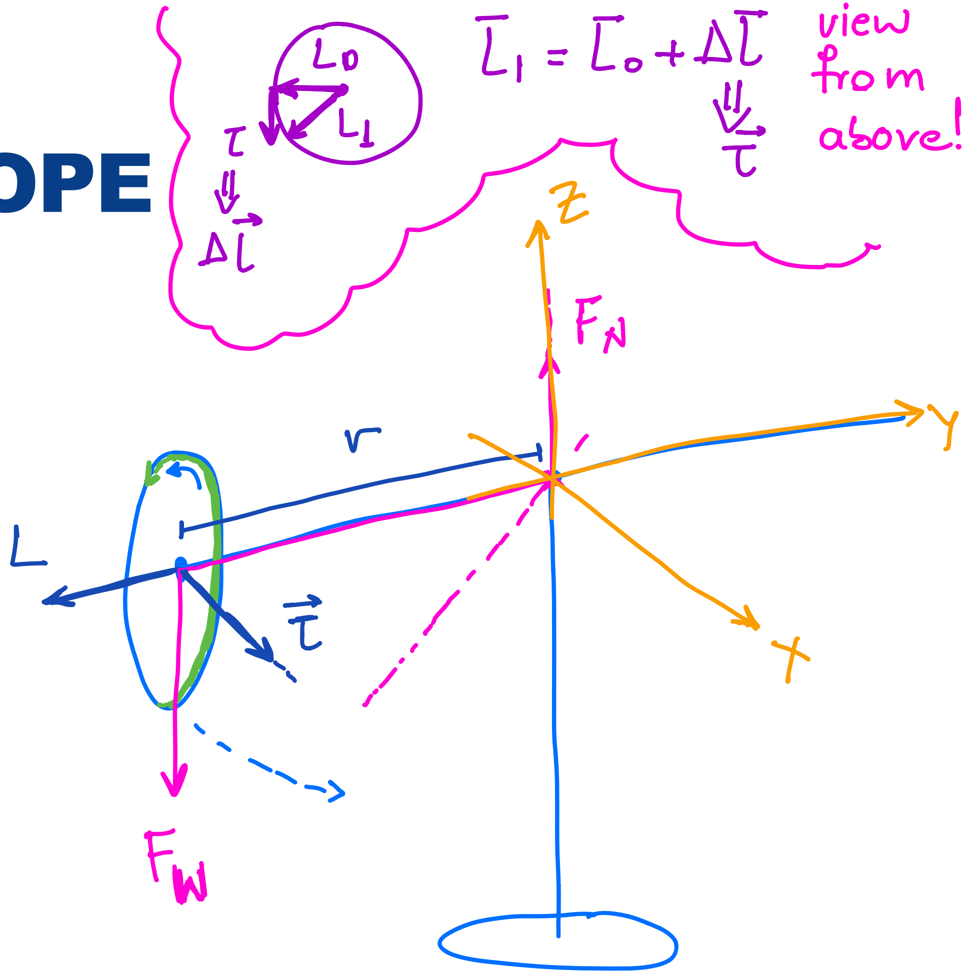
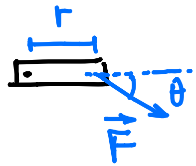


Figure from merriam-webster website

RESUMÉ

- **Moment de Force**



$$\tau_o = r \sin\theta\, F$$

$$\vec{\tau}_o = \vec{r} \times \vec{F}$$

- $\sum \vec{F} = 0 \Leftrightarrow \Delta\vec{v} = 0$

Equilibre Translationnel

$\sum \vec{\tau} = 0 \Leftrightarrow \Delta\omega = 0$

Equilibre Rotationnel

- $x_{CM} = \dfrac{\sum_i F_{w_i} x_i}{\sum_i F_{w_i}}$ CENTRE DE MASSE.

- Point d'application de la Force pesanteur avec même resultat mèchanique.
- Un corps soutenu par son centre de gravité ne subit aucun couple de rotation gravitationnel, quelque soit son orientation.

- $\sum \tau_o = I_o\, a_{ang}$ moment Force

$I_o = \sum_i m_i r_i^2$ moment d'inertie

$\vec{L}_o = I_o \vec{\omega}$ moment cinétique