Fractal Study of Extensive Air Shower Time Series

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Abstract

Chaotic behaviors in the arrival time of cosmic rays were studied, using the air showers with the mean energy 1.1×10^{15}eV, collected by the air-shower array at Mitsuishi(34.8°N, 134.3°E), Japan, during the period from January 1989 to October 1998. We found 40 chaotic events with the fractal dimensions of $1.2 \sim 5.0$, using the Takens' method, applied for the arrival time sequences of air showers with 256(or 128) series. The right ascension distribution of the chaotic events has a peak at 9 hour with 3.9 σ. No difference was found in the size and the zenith angle distributions of the air showers in the chaotic events from the normal air showers.

1 Introduction

Fractal dimension analysis has received much attention in the last several years, to study the origins and the propagation mechanisms of cosmic rays. We applied the Takens' method to the arrival time series of the air showers, collected by the Mitsuishi Air Shower Array (34.8°N, 134.3°E). In 1995 we reported the chaotic features, found in the 128 and 256 time series, with the fractal dimensions of $2.5 \sim 4.1$, using the data collected from January 1989 to December 1994(Katayose *et al.*, 1995). In this paper, we present the results of the analysis using the data from January 1989 to October 1998. In addition, we studied the time development of each chaotic event, by shifting the boundaries of the time series.

2 Data Analysis and Results

The used number of air showers was 4,272,333, collected by the Mitsuishi Air Shower Array, with the averaged trigger rate of approximately 1 event/min before Nov. 1997 and 2 events/min after Nov. 1997. The selection of air showers was done using the air-shower analysis method, mentioned previously(Katayose *et al.*,1998). We required the existence of muon(s) with energy $\geq$ 6GeV, accompanied with the air showers. The mean energy of the air showers was 1.1×10^{15}eV. After selecting the air showers by the zenith angle, $\cos\theta \geq 0.7$, the Takens' method(Takens,1981) was applied to the arrival time sequences. Time intervals are defined as(Katayose *et al.*, 1995),

$$x(i) = t_i - t_{i-1} \tag{1}$$

where $x(i)$ denotes the i-th time interval and t_i is the arrival time of the i-th air shower. Those time interval series with n data points are embedded in the m-dimensional pseudo-phase space by constructing a vector $V_m(i)$ as,

$$V_m(i) = [x(i), x(i+1\tau), x(i+2\tau), ..., x(i+(m-1)\tau)], \tag{2}$$

where m is the embedding dimension and τ is the delay time. The correlation dimension, D_m, is defined by,

$$D_m = \frac{d \log C_m(r)}{d \log r} \tag{3}$$

where C_m is the number of vector point pairs, $V_m(i)$ and $V_m(j)$, of which the mutual distance in the m-dimensional phase space is less than r. This definition of D_m was introduced by Takens and the range of its applicability for noisy data was studied by Ohara *et al*(Ohara *et al.*,1992) using the synthesized time series. In our analysis, air showers were divided into the buckets with 256(or 128) time intervals, which corresponds to a period of the order of 4(or 2) hours. We applied the Takens' method with $\tau = 1$, for the 256(or 128) time series of air showers in the each bucket: n=256(or 128). If the arrival time is random, the correlation dimension should have no flat region in the D_m-log(r) plot. Existence of a flat region in the D_m-log(r) plot indicates the existence of fractal features. We use this characteristics to select the events. Fig. 1 shows a typical event with n=256, whose period was approximately 6 hours: observed in 2/26/20h $\sim$ 2/27/02h in 1989. Fig. 2 shows the fractal dimension as a function of the embedding dimension for this event. As seen in Fig. 2, D_m of the flat region increases with the embedding dimension m, then showing a plateau at 3.85 for $m \geq 8$. The existence of the plateau is an indication that the air showers in this fractal event have a chaotic feature. We have found 40 events with the chaotic feature, listed in Table 1. In Table 1, the starting time of the chaotic event is coded in the format: year/month/day/hour. The events, 1, 2, 17, 23, 26 have chaotic features in two consecutive buckets.

To study the time development of the chaotic features of these events, we changed the bucket boundaries by shifting the starting time of the bucket(n=128) by ten minutes, and measured the fractal dimensions. Fig. 3 shows the time variation of the fractal dimension for a typical chaotic event. As seen in this example, a characteristic pattern was observed in the time development of the fractal dimension. The fractal dimension appears at the start of the event with high value. Then the dimension decreases and stays at a minimum during the event. And it increases again and disappears at the end. The event in Fig. 3 has a minimum D_m = 2.0.

We found that the incident directions of air showers in the chaotic events are not uniform. The right ascension distribution of the all air showers in the all chaotic events is shown in Fig.4. The distribution has a peak at 9 hour. The observational period(acceptance) in right ascension is approximately uniform; its variation is smaller than 1 %. The direction of the chaotic event was measured by the averaged right ascensions of the air showers in the chaotic event with the accuracy of 1 hour. Fig. 5 shows the directions of the chaotic events as a function of right ascension. The distribution has a peak at 9 hour with 3.9 σ. Fig. 6 shows the directions of the chaotic events plotted in the galactic coordinate. The events are clustered in the galactic latitude $15^\circ \sim 80^\circ$. The observable region was $-10^\circ <$ declination $< 80^\circ$. The large closed loop, shown in Fig. 6, shows the trajectory of the vertical direction at Mitsuishi. There was no difference in the size and the zenith angle distributions between the air showers in the chaotic events and the normal air showers. Also, no difference was found in the distribution of the number of accompanying muons.

3 Summary

Fractal analysis has been done to the air showers, collected by the Mitsuishi Air Shower Array, during the period of ten years; from January 1989 to October 1998. The average event rate was 1 $\sim$ 2 event/min. The mean size of the air showers was $N_e \simeq 1.1\times10^5$. The air showers were required to have accompanying muon(s) with $E_\mu \geq$6 GeV. We found 40 chaotic events with the fractal dimensions of 1.2 $\sim$ 5.0, using the Takens' method, applied to the arrival time of the air showers in 256 series, which corresponds to approximately 4 hours. The arrival directions of the chaotic events in the right ascension has a peak at 9 hour with 3.9σ. In the galactic coordinate, the chaotic events are clustered in the galactic latitude $15° \sim 80°$. The size and the zenith angle of the air showers in the chaotic events have no significant difference from the normal air showers.

References

Y.Katayose et al., 1995, Proc. of the 24th ICRC(Roma,1995), **1** 301.
Y.Katayose et al., 1998, IL Nuovo Cimento, **21** 299.
F.Takens, Lecture Note in Math., 1981, **898**(Springer Verlag), 366.
S.Ohara et al., 1992, Science and Technology, Kinki University, **5** 59.

Table 1: Chaotic events; DATE/TIME shows the starting time of the chaotic event in year/month/day/hour. The events 1, 2, 17, 23 and 26 have chaotic features in two consecutive buckets.

No.	DATE/TIME (U.T.)	$D_{\rm m}$		No.	DATE/TIME (U.T)	$D_{\rm m}$
1	1989/2/26/20	3.85		20	1992/8/19/16	3.60
	1989/2/26/22	3.55		21	1992/9/12/8	2.85
2	1989/6/16/22	3.55		22	1993/6/17/14	2.85
	1989/6/17/0	3.65		23	1994/5/24/12	3.20
3	1989/8/13/4	4.30			1994/5/24/14	3.95
4	1989/9/14/12	4.30		24	1994/5/27/0	3.85
5	1989/9/16/8	3.20		25	1994/10/8/22	3.60
6	1989/9/24/22	4.70		26	1994/10/26/6	1.45
7	1989/10/8/14	4.05			1994/10/26/8	1.20
8	1989/10/17/8	4.20		27	1994/11/14/16	4.40
9	1989/12/2/16	3.45		28	1995/2/18/14	3.50
10	1990/2/10/10	4.00		29	1995/2/27/22	5.00
11	1990/4/25/2	2.45		30	1995/3/13/12	3.80
12	1990/5/2/20	2.75		31	1995/8/3/10	3.95
13	1990/5/15/18	3.75		32	1995/8/16/10	4.55
14	1990/5/30/2	3.50		33	1995/8/20/16	3.90
15	1991/5/12/22	3.65		34	1995/10/12/4	4.35
16	1991/6/5/12	3.95		35	1998/5/2/2	3.60
17	1992/2/17/18	3.80		36	1998/5/15/0	4.40
	1992/2/17/20	4.15		37	1998/6/16/6	3.20
18	1992/7/30/0	4.30		38	1998/8/1/12	4.24
19	1992/8/14/8	4.00		39	1998/8/3/4	4.64
				40	1998/10/5/8	3.60

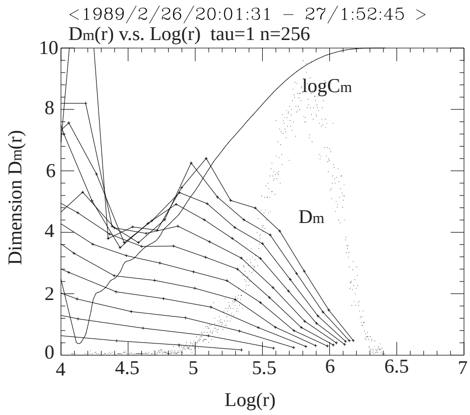

Figure 1: Correlation between Dm and log(r); Feb. 26th 1989.

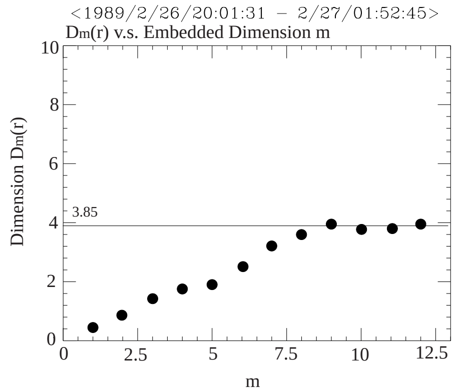

Figure 2: Embedding dimension vs Fractal dimension; Feb. 26th 1898.

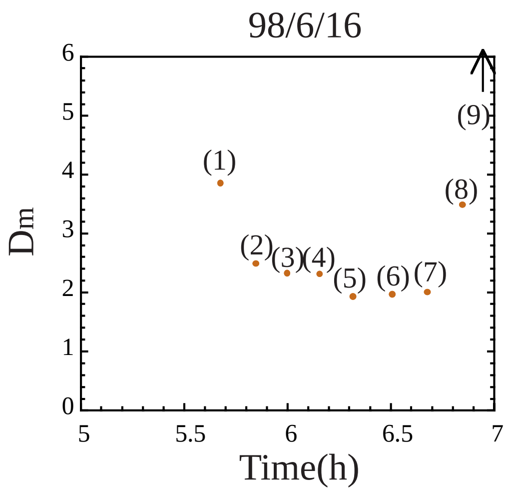

Figure 3: Variation of dimension with time; Jun. 16th 1998.

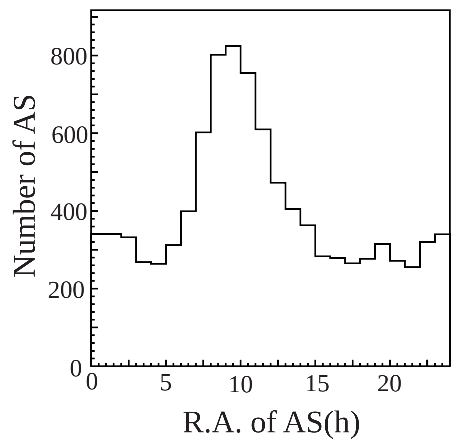

Figure 4: Right ascension distribution of the all air showers in the all chaotic events.

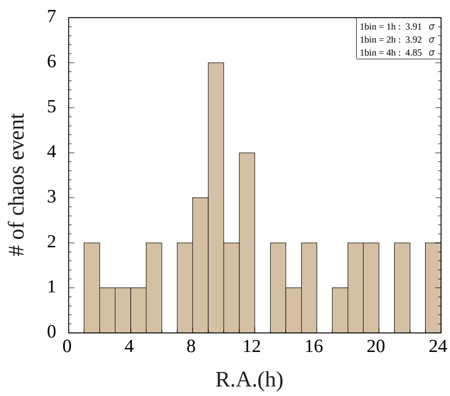

Figure 5: Right ascensions distribution of the chaotic events.

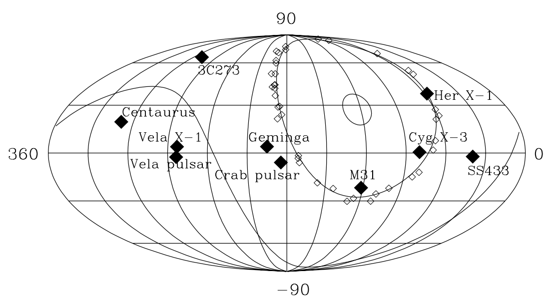

Figure 6: Arrival directions of the chaotic events(open diamonds) in the galactic coordinate.