A Simulation Study of Linsley's Approach to infer Elongation Rate and Fluctuations of the EAS Maximum Depth from Muon Arrival Time Distributions

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Abstract

An indirect approach to deduce the elongation rate D_e and the fluctuations of the atmospheric depth X_m of the EAS maximum from muon arrival time distributions has been scrutinized on basis of Monte Carlo simulations of the EAS development and of the longitudinal profile of various EAS parameters. Special attention is made to the behaviour of a scaling parameter relating the variations at the height of the shower maximum to the arrival time of muons at observation level.

1 Introduction:

The early development of extensive air showers induced by high-energy cosmic particles, is critically influenced by the basic parameters of the particle-air interaction, i.e. by the mean free path, the interaction inelasticity and the multiplicity of secondary-particle production. A shorter initial free path, high inelasticity or high multiplicity can be also associated to a primary particle with high atomic number. The early stages, difficult to be directly observed, influence the position of the atmospheric depth of the maximum EAS development. Thus experimental studies which are able to observe the EAS maximum, like the observation of the fluorescence light of the atmospheric nitrogen do provide important information on the nature of the primary and of its interaction properties.

The average depth of the maximum X_m of the EAS development depends on the energy E_0 and the mass of the primary particle, and its dependence from the energy is traditionally expressed by the so-called elongation rate D_e defined as change in the average depth of the maximum per decade of E_0:

$$D_e = dX_m/dlog_{10}E_0$$

Invoking the superposition model approximation i.e. assuming that a heavy primary (A) has the same shower elongation rate like a proton, but scaled with energies E_0/A

$$X_m = X_{init} + D_e log_{10}(E_0/A)$$

or for a mixed composition, characterized by $< log_{10}A >$

$$< X_m >= X_{init} + D_e(log_{10}E_0 - < log_{10}A >)$$

As long as D_e is only weekly dependent from the energy, X_m shows practically a linear dependence from $log_{10}E_0$, and any change in this dependence is indicative for a change either of D_e or of the composition ($< log_{10}A >$).

In 1977 (Linsley 1977, Linsley and Watson 1981) an indirect approach studying D_e has been suggested. This approach can be applied to shower parameters which do not depend explicitely on the energy of the primary

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particle, but do depend on the depth of observation X and on the depth X_m of shower maximum. EAS quantities like arrival times and their dispersions characterizing the time structure of the muon shower disc and mapping the longitudinal EAS development, are of this type. Actually the basic idea arises from the simple fact that the muon arrival time distributions reflect largely the time-flight of the muon travelling through the atmosphere i.e. being dependent on distance (path length) of the observation level X from X_m. Hence adequately defined arrival time parameters T do implicitely depend on the primary energy E_0 and the angle-of-EAS incidence Θ.

Prompted by recent experimental investigations of these dependences by the KASCADE collaboration (Brancus 1998, Antoni 1999) we did scrutinize Linsley's approach on basis of Monte Carlo simulations of the EAS longitudinal development of the muon component. The interest in such studies arises from the question, how various influences of the muon propagation (Coulomb scattering e.g.) may obscure the information expected from the observation of muon arrival time distributions and if the basic assumptions involved are correct. From the calculated longitudinal profiles of the shower size N_e and the muon number N_μ the corresponding maxima $X_m^{(e,\mu)}$ and their fluctuations $\sigma(X_m^{(e,\mu)})$ are determined. These results are subsequently compared with the application of a procedure, essentially based on the relation between the variation of EAS time dispersion (characterised by various adequate moments T of the muon arrival time distributions like the mean or the median values e.g.) with the primary energy and the variation with the zenith angle of incidence.

2 Basic relations:

The distribution of the EAS muon arrival times, measured at a certain observation level relatively to the arrival time of the shower core reflect the pathlength distribution of the muon travel from locus of production (near the axis) to the observation locus. The basic a-priori assumption is that we can associate the mean value or median T of the time distribution to the height of the EAS maximum X_m, and that we can express $T = f(X, X_m)$. Here X is slant depth at the observation level X_v (the vertical atmospheric thickness X_v)

$$\partial T/\partial log_{10} E_0|_X = D_e \partial T/\partial X_m|_X$$

The change of T with the energy E_0 at a given $X = X_v/cos\Theta$ is proportional to the variation of T with X_m for a given energy. However, at observation level we do not observe $\partial T/\partial X_m$, which could be related, if specifying the function $f(X, X_m)$ and

$$F = -(\partial T/\partial X_m)_X/(\partial T/\partial X)_{X_m}$$

respectively. Thus

$$\partial T/\partial log_{10} E_0|_X = -F \cdot D_e \cdot 1/X_v \cdot \partial T/\partial sec\theta|_{E_0}$$

In order to derive from the energy variation of the arrival time quantities information about elongation rate, some knowledge is required about F, in addition to the variations with the depth of observation and the zenith-angle dependence, respectively.
In a similar way the fluctuations $\sigma(X_m)$ of X_m, may be related to the fluctuations $\sigma(T)$ of T

$$\sigma(T) = -\sigma(X_m) \cdot F_\sigma \cdot 1/X_v \cdot \partial T/\partial sec\theta|_{E_0}$$

with F_σ being the corresponding scaling factor for the fluctuation of F. In previous applications (Walker and Watson 1981 and Blake 1990) of this concept on basis of data measured with the Haverah Park water Cerenkov detectors simple assumptions have been made for two extreme forms of $f(X, X_m) = f(X - X_m)$ or $f(X, X_m) = f(X/X_m)$ leading to $F = 1$ or $F = X/X_m$, respectively. At a closer look these assumptions appear to be not very convincing, since the arrival times are related directly only to the travel distances of the muons rather than to the differences in the traversed grammage of the atmosphere. That fact will complicate the dependence from X and X_m. We try to scrutinize this aspect on basis of detailed EAS Monte Carlo simulations.

3 Monte Carlo simulations:

The simulations of the air shower development have been performed by use of the Monte Carlo air shower simulation program CORSIKA (Heck 1998) (vers.5.621). The actual set of simulations calculations (based on the QGSJET model) comprise the EAS development of proton and iron induced EAS for three different primary energies (10^{15} eV, $3.16 \cdot 10^{15}$ eV, 10^{16} eV) and three different angles-of-incidence ($\Theta = 15^\circ, 25^\circ, 35^\circ$) with a set of 1000 simulated EAS for each case. The EAS quantities of interest are evaluated at six different observation levels, in the case of the EAS muons generally with two different energy thresholds of 0.25 and 2.0 GeV. The longitudinal profiles of the electromagnetic and the muon component develop differently.

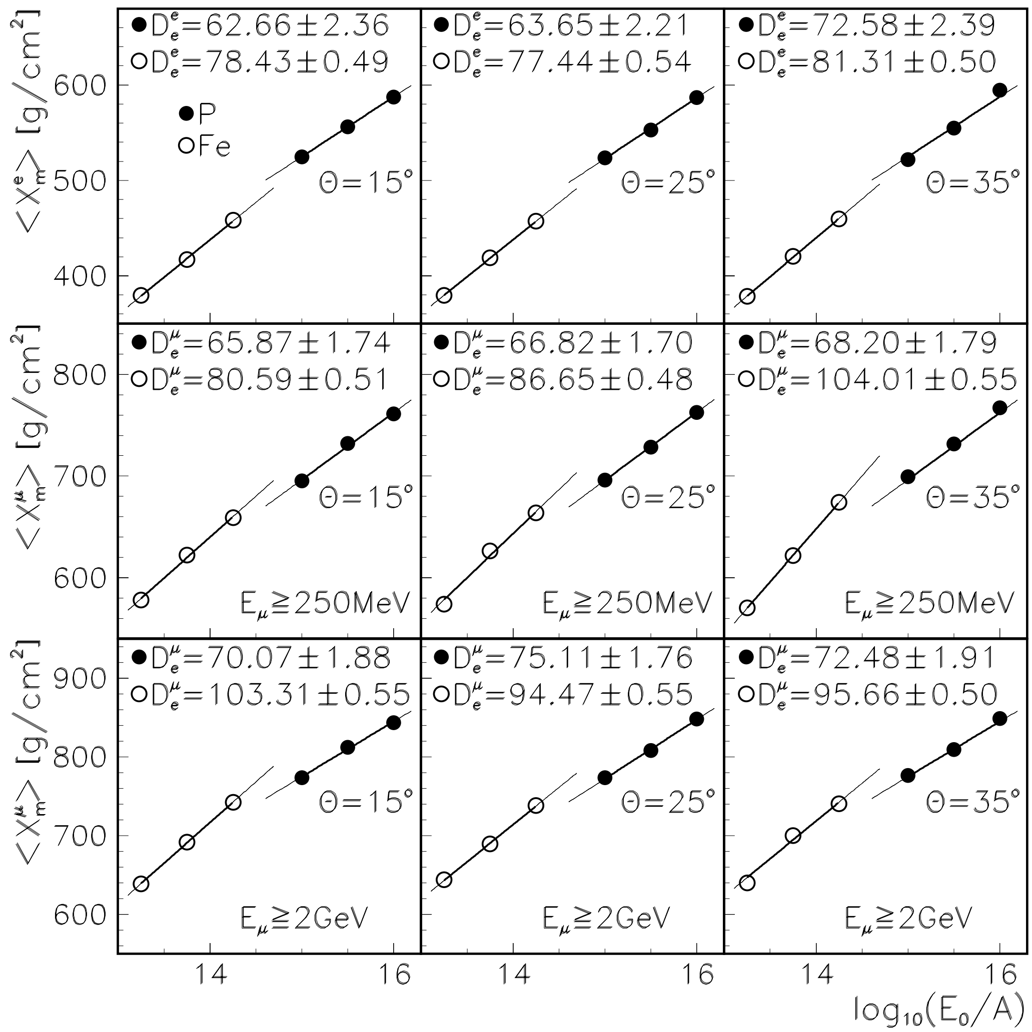

Figure 1: $D_e^{e,\mu}[g/cm^2/dec]$ for electromagnetic and muon components.

While the electromagnetic component exhibits a relatively well pronounced maximum of the shower size N_e, characterized in the Greisen parametrisation by the shower age $s = 1$, the maximum of the penetrating muon component (N_μ) appears to be shifted deeper and rather shallow, since the muon losses, after reaching a kind of plateau of N_μ, are relatively small, especially for higher energy muons. As the maximum depths are not necessarily identical, we discriminate between X_m^e and X_m^μ, D_e and D_μ, respectively. The fluctuations $\sigma(X_m)$ (standard deviation of the X_m distributions) prove to be practically energy independent, but they are different for different cases: $\delta(X_m^e)(p) \approx 80 g/cm^2$, $\delta(X_m^e)(Fe) \approx 17 g/cm^2$, $\delta(X_m^\mu)(p) \approx 100 g/cm^2$, $\delta(X_m^\mu)(Fe) \approx 30 g/cm^2$. There is a trend of slightly increasing values D_e^e (60-70 g/cm/dec for protons) with the zenith angle, which can be understood as effect of the different path lengths of the particles traversing

the same grammage layers at different zenith angles. Fig.1 illustrates the features of D_e. Though there are differencies for different primaries, globally the superposition model appears to be a good approximation.
In deriving the muon arrival time distributions from the EAS simulations we restrict our considerations to the cases of distributions of the mean ($T = \Delta\tau_{mean}$) and median ($T = \Delta\tau_{0.5}$) arrival time of EAS registrated relatively to the foremost muon in an interval of a distance from the shower core of $R_\mu = 90 - 110\,\mathrm{m}$ with a detection multiplicity $n{\geq}4$, for different energies E_0 and zenith angles Θ of incidence, and for different observation levels, especially for the level of the KASCADE experiment, Germany (Brancus 1998) on sea level and for the ANI installation on Mt.Aragats, Armenia (3250 m a.s.l.). The variations display a linear dependence, and $\epsilon_E = \partial T/\partial sec\theta|_{E_0}$ and $\epsilon_\theta = \partial T/\partial log_{10} E_0|_{sec\theta}$ can be determined.

4 The scaling factor F deduced from arrival time parameters:

Using the coefficients ϵ_E and ϵ_Θ characterising the variation of the mean or median distributions with the energy and the angle of EAS incidence and adopting the value of the elongation rate, as predidicted by the simulations, we infer for different observation levels and different primaries the values of the scaling parameter F, whose knowledge would be in turn a prerequisite to evaluate experimental data in terms of the elongation rate and fluctutions of the height of EAS maximum.
The results display a rather complex dependence of F from X, X_m, from the type of the primary (p or Fe: $F_{fe} > F_p$) and from the energy threshold of the detected muons, varying between ($\approx 0.9 - 2.0$). Within all uncertainties and fluctuations of the results, eventually arising from the fact that the longitudinal development $T = f(X, X_m)$ can be never fairly expressed by a single form f for each observation level X_v, there may be a tendency with

$$F(X_v^1)/F(X_v^2) \propto X_v^1/X_v^2 \cdot sec\theta \quad \text{and} \quad F(Fe)/F(p) \propto X_m(p)/X_m(Fe).$$

5 Summary:

The relation between the arrival time observables and the changes of the longitudinal EAS profiles implies a scaling factor F, which depends from the height of the shower maximum, the observation level and zenith angle of EAS incidence. On basis of Monte Carlo simulations of the EAS development the ingredients for a determination of F have been deduced, and the variation of F has been studied for a number of cases. In the present status of our understanding we have to conclude that the scaling factor has a rather complex behaviour.It is affected by the EAS fluctuations and, though there are some trends, the dependences of average values are not yet established. In conclusion, unfortunately the Linsley approach does not provide a way to relate muon arrival time observations directly to the elongation rate and fluctuations of X_m without invoking detailed Monte Carlo simulations.

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