Universal multifractal parameters of fluctuations in E.A.S. produced by different primary cosmic rays

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Abstract

As it was pointed out in some recent papers, the fluctuations in secondary particle distributions of extensive air showers at observation level, exhibit a complex structure fitting in the frame of the universal multifractal theory by means of a set of multifractal parameters. In the present work, the multifractal parameters are calculated for samples of different cosmic primaries generated by the montecarlo CORSIKA. The performed analysis reveals the differences in the structure of the fluctuations of the different primary cosmic rays considered and could serve as basis and support to both chemical composition or separation methods applied to the cosmic radiation.

1 Introduction:

The study of fluctuations in the secondary charged particles distribution at earth observation level is a very interesting matter since it is found they have a complex structure when global fluctuations and trends representing mean behaviour are removed (E.Faleiro *et al*, 1997). Instead of founding a white noise behaviour that would represent the always present statistical fluctuations, it is found a correlation structure that can be interpreted as the combination of a white noise and a flicker $1/f$ noise which have statistical selfsimilarity properties (E. Faleiro & J.M.G.Gomez, 1998). Such a properties can be quantified by means of the universal multifractal formalism. In this context, statistical selfsimilarity of local particle distributions fluctuations, this is the $1/f$ noise, can be characterised using scaling exponents that depend of the fluctuations strength. The universal multifractal formalism quantify the multiple scaling properties by means of two parameters, the Levy's index α, and the mean codimension C_1, giving rise of fluctuations strength distribution on the physical support, this is the secondary particles detection area. A detailed analysis shows that for both primary protons and photons ranging from 10 to 50 TeV, the Levy's index of the secondary distribution fluctuations is near 2, which means in the universal multifractal language that there is no violent fluctuations. Related with C_1, it is found it ranges from 10^{-3} to 10^{-4} which express an extremally low intermittency or, equivalently, a great homogeneity (E. Faleiro & J.M.G.Gomez, 1999, E.Faleiro & J.L.Contreras, 1998). Since the primary cosmic rays that reach earth atmosphere can be of different type, it is interesting to study the universal multifractal parameters for each primary type in order to distinguish them. In the present work we will use helium, oxigen and iron nuclei as a primary particles with energy ranging from 10 to 70 TeV. Their extensive air shower will be simulated with the help of the montecarlo CORSIKA 4.50. Finally we will add the results obtained for protons and photons collected in references.

2 The correlation model of the fluctuations:

With de scope to eliminate the global fluctuations the generated events are clasified by the number of secondary charged particles N_s that reach a ring of radius $r_{min} = 50m$ and $r_{max} = 100m$ centered at the event core (E. Faleiro & J.M.G.Gomez, 1998). This ring is divided into 256 equal sectors in polar angle. In order to minimize global fluctuations we have considered intervalls ΔN_s and we assume that each intervall contains events without global fluctuations and whose intrinsic fluctuations have the same nature. In order to simplify we have considered only 3 of such intervalls whose amplitude is equivalent to 2500 secondary particles like

e^+ and e^-. Table 1 shows the simulated event distribution according to the intervalls clasification. Statistical analysis of fluctuations normally considers each event as the concrete realization of a stochastic process whose statistical properties are being determined. The stochastic process could be defined by the set of all primaries of the same type and energy, or even by the totality of events generated. Each process is then characterised by its statistical parameters, which may be studied in the coordinate or in the frequency domain. The autocorrelation

Primary	N_p	ΔN_s	A	B	C	H	α	C_1
proton	706	0-2500	0.62	1.14	0.0209	0.133	1.405	3.07E^{-2}
	572	2500-5000	1.61	1.12	0.0569	0.130	1.964	1.13E^{-2}
	450	5000-7500	2.51	1.13	0.0970	0.130	1.997	6.30E^{-3}
γ	514	0-2500	0.054	1.0	0.0235	0.129	1.863	2.89E^{-3}
	359	2500-5000	0.119	1.0	0.0532	0.128	1.976	1.19E^{-3}
	201	5000-7500	0.21	1.0	0.0929	0.130	1.996	7.18E^{-4}
Helium	176	0-2500	0.96	1.21	0.0292	0.177	1.824	2.40E^{-2}
	443	2500-5000	2.00	1.08	0.0557	0.151	1.951	1.51E^{-2}
	355	5000-7500	2.72	1.11	0.0957	0.156	1.996	7.56E^{-3}
Iron	1829	0-2500	0.80	1.00	0.0173	0.132	1.699	5.09E^{-2}
	294	2500-5000	2.56	1.06	0.0526	0.146	1.916	2.23E^{-2}
	38	5000-7500	5.2	1.13	0.0780	0.170	1.969	1.85E^{-2}
Oxigen	806	0-2500	0.87	1.04	0.0255	0.138	1.784	3.11E^{-2}
	432	2500-5000	2.15	1.09	0.0518	0.152	1.932	1.87E^{-2}
	194	5000-7500	3.6	1.13	0.0965	0.163	1.984	1.04E^{-2}

Table 1: Main features of the Monte Carlo simulation of several type primary samples ranging from 10 to 70 TeV. Here N_p is the number of primary particles whose number of secondary particles arriving to the ring is within the bin ΔN_s; A, B and C are the parameters of the power spectrum density proposed model; H is the first-order structure function exponent, and α and C_1 are the universal multifractal parameters

information about a process is contained in the autocorrelation function $R(t, t+\tau) = < \epsilon(t)\epsilon(t+\tau) >$, where $\epsilon(t)$ is the content of sector t and the simbol $< . >$ indicates ensemble averaging (W.Feller, 1977). For a special class of processes, the so called *wide sense stationary processes* (WSS), it simplifies to $R(\tau)$. In this case it is related to the mean power spectrum density (PSD) denoted $P(k)$, by a Fourier transformation (P.Z.Peebles, 1987)

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(k)e^{ik\tau}dk \quad , \tag{1}$$

where τ is a variable that here represents the separation between sectors.

When the natural logarithm of $P(k)$ for our events is represented versus $\log k$ (E. Faleiro & J.M.G.Gomez, 1999), this plots show two important facts. In the first place, instead of finding a constant $P(k)$, as would be the case for a signal whose fluctuations were uncorrelated, a PSD showing a complex structure is found. As a second remark, the shape of $P(k)$ suggest a simple model of functional form

$$P(k) \sim \frac{A}{k^B} + C \quad . \tag{2}$$

The constant term C represents a white noise, the need for it is clearly seen in the constant behaviour at high k, while the first term stems for the nearly linear behaviour in $1/k$ seen at low k.

Table 1 also contains the best fit parameters A, B and C obtained for our model power spectrum. The fit errors of A, B and C affect the last digit of the values given in the table. We see that the model power spectrum (2) fits well the data. Thus it can be concluded that the fluctuations of the particle density are composed of a white noise plus a scaling noise $1/f$ which gives rise to its complex structure. This kind of noise is usually called a *flicker noise* in the literature (M.B. Weissman, 1988). The value found for the parameter B, nearly one, shows that the scaling component of the signal can *a priori* be assumed to be *conservative* within the multiplicative cascade model that we should now propose (Schertzer D. and Lovejoy S., 1991). However, in this work we will analize the first order structure function of this signal component as a test of the conservative assumption. We have then filtered the white noise component of the signal using a real non random filter with transfer function

$$h(k) = \sqrt{1 - \frac{C}{\frac{A}{k^B} + C}} \ . \quad (3)$$

The filter is a variant of the Wiener filter, which is known to be adecuate for elimination of the white noise in the signal provided that it is uncorrelated to the $1/f$ component. In the folowing sections we will analyze this scaling component of the signal that results after filtering.

3 Multifractal analysis of the resulting $1/f$ noise:

Although the values for the B parameter of the proposed model are near to one, and therefore stand for conservativity, the study of structure functions of the scaling component give rise to scaling power laws of the type

$$< |\epsilon(t+\tau) - \epsilon(t)|^q > \sim \tau^{\zeta(q)}. \quad (4)$$

We note that a scaling law is fulfilled by the first order structure function with exponent $\zeta(1) = H$, which is a measure of the non conservativity in the context of the multiplicative cascade model (Schertzer D. and Lovejoy S., 1993). The calculated values for H for all samples are also displayed in table 1. It is found that in all cases the conservative aproximation (H=0) may be adopted. Finer results can be found if we consider that the small values of H arise of an H-order fractional integration of the conservative associated signal. To obtain this conservative signal we have to H-order fractionally derivate the non conservative one. This is equivalent to filtering the signal with a non random filter whose transfer function is $h(k) = k^H$. Considering the resulting $1/f$ noise as conservative, we study the scaling properties of its statistical moments $< \epsilon_\lambda^q >$, where ϵ_λ is the content of a generic sector at scale ratio λ and $q > 0$ is a real mumber. The scaling law for statistical moments in a multifractal scheme states that

$$< \epsilon_\lambda^q > \sim \lambda^{K(q)} \quad (5)$$

where the $K(q)$ function characterises the scaling behaviour and also the statistics of the data and it is called *moment scaling function*. The non linearity of $K(q)$ involves a multiscaling description corresponding to a nontrivial power law.

Let us now see whether this multifractal fits into the universal multifractal scheme . In this scheme the theoretical value of $K(q)$ is given by

$$K(q) = \frac{C_1}{\alpha - 1}(q^\alpha - q) \ , \ \alpha \neq 1 \quad (6)$$

with $0 \leq \alpha \leq 2$, called Levy index, and $0 \leq C_1 \leq 1$ for unidimensional data, called mean fractal inhomogeneity measure. Eq.(6) arises considering the empirical signal as an outcome of a multiplicative stochastic process governed by Levy stable probability distributions of index α. To determine the indices α and C_1 we applied a direct analysis technique called DTM (Double Trace Moment technique) (Ratti S. P., 1994, D. Lavallee, 1991) to the data. Figures representing the empirical function $K(q)$ and their theoretical counterpart derived from universal multifractal scheme can be found in references (E. Faleiro & J.M.G.Gomez, 1998, E. Faleiro & J.M.G.Gomez, 1999). The agreement between the empirical and theoretical $K(q)$ is very good. In all the other bins ΔN_s the agreement is also good at least up to $q = 4$.

The parameters of the DTM analysis are given in table 1. We can see that all primary types show an extremely low intermittency (small values of C_1), corresponding to a great homogeneity. On the other hand, the value of the α parameter, near $\alpha = 2$, shows a typical log-normal behaviour. We can appreciate that charged cosmic primaries have parameters that behave quite closely. On the oposite primary photon parameters are clearly different.

References

E. Faleiro *et al.*, *Power spectrum analysis and multifractal study of fluctuations of secondary particles in Monte Carlo simulation of EAS*. Proceedings 25th International Cosmic Ray Conference, Durban, (1997) 261-264.

E. Faleiro and J.M.G. Gómez, *Multifractal characterisation of fluctuations in simulated extensive air showers*, Europhys. Lett. **45** (4), pp. 437-443 (1999).

E.Faleiro, J.M.G. Gómez, A study of fluctuations in simulated extensive air showers in Fractals and Beyond, Ed. M.M.Novak (World Scientific, 1998), pp 279-288.

E. Faleiro and J.L. Contreras, *A study of fluctuations in extensive air showers simulated by Monte Carlo methods*, J. Phys. G, **24** (1998) 1795-1804.

Peebles P.Z. Probability, Random Variables and Random Signal Principles (2nd ed.), McGraw-Hill (1987).

M.B. Weissman Rev. Mod. Phys. **60** (1988) 537.

Schertzer D. and Lovejoy S. in Scaling, Fractals and Nonlinear Variability in Geophysics. edited by Schertzer D. and Lovejoy S. (Kluwer, Holland) (1991) 41.

Schertzer D. and Lovejoy S. in Nonlinear Variability in Geophysics 3: Scaling and multifractal processes. (Lecture Notes NVAG3, EGS) (1993).

W. Feller, *An introduction to probability theory and its applications*, Vol II, 2nd ed., Wiley 1977.

Ratti S. P. et al. Z. Phys. C **61** (1994) 229.

D. Lavallee, Ph.D. Thesis, McGill University, Montreal, Canada (1991).