Propagation through matter : a model-based estimation of cosmic muon-intensities at various depths under water

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Abstract

Very recently Bugaev et al studied this aspect of the problem (alongwith some others as well) on the assumption of the validity of the Feyrman scaling even in the fragmentation region which is somewhat questionable. Besides, the chosen primary spectrum was of NSU-type which is also a bit oldish. We would concentrate here on the depth versus muon-intensity relations under water by incorporating some or all of the following few more uptodate views and points : (i) Modest violation of the Feynman scaling in the fragmentation region, (ii) Nature of the primary spectrum as proposed by JACEE collaboration, (iii) Contribution to the inelastic cress section as indicated by Block et al and some other very current considerations. Finally, we would compare our analytically calculated values with the experimental results for underwater muon-intensities.

1 Introduction

Cosmic muon and neutrino intensities provide important testing ground for verifying the models for production of secondary particles at high and ultrahigh energies and tools for checking up of some very basic observables related to cosmic ray physics phenomena, e.g., primary spectra, cosmic ray compositions etc. which always suffer from some degree of uncertainties [1-3]. Both underground and underwater muon intensities are of interest in this regard. Furthermore, muon and neutrino production-aspects contain additional information on production of charm particles which constitute a significant corner for developments in particle physics phenomenology.

In the past we already reported a model-based estimate of the same for under ground muon indentities [4]. The objective of the present work is to apply the same model for production of the major secondaries and to obtain the calculated values of the underwater muon intensities for various depths which would be presented here finally in a tabular form against a particular set of experimental values as obtained from Bugaev et al [1].

2 Muon Propagation through homogeneous medium : The Basics

The equation describing the high energy muon propagation through a homogeneous medium may be written in the form :

$$\partial\partial h\,[D_\mu(E,h)] - \frac{\partial}{\partial E}[.\beta(E)D_\mu(E,h)]$$

$$= \sum_{k=p,b,n} \int_0^1 \Big[(1-v)^{-1}\phi_k(v,E_0)D_\mu(E_u,h) \qquad (1)$$

$$-\phi_k(.u,E_v)D_\mu(E,h)]\,dv$$

with the boundary condition $D_\mu(E,0) = D_0(E)$ where $D_0(E)$ is the ground level muon spectrum and β is the rate of the muon energy loss which would be treated as continnous and $E_v \approx \frac{E}{1-v}$. The right hand site of the above equation describes the discrete muon energy loss resulting from direct e^+e^- pair production with $v > v_0$, bremsstrahlung and inelastic nuclear scattering ($k = n$). The corresponding macroscopic crossections $\phi_k(v,E)$ are defined by

$$\phi_k(v,E) \;=\; N_0\frac{d\sigma_k(v,E)}{dv} \;=\; N_0E\frac{d\sigma_k(E,E^1)}{dE^1}\; E^1 = (1-v)E \qquad (2)$$

Theoretically, the depth-intensity relation for muon given by

$$I_\mu(h) \;=\; \int_{Eth}^{\infty} D_\mu(E,h)dE \qquad (3)$$

with the letters having their usual contextual significance which could be obtained from Bugaev et al [1].

3 Model-based Inputs for calcutations

(A) Particle Production Model : The indusive crosssections for pion, kaon etc., used here are obtained on the basis of a model which was developed byacharyya [5] on the basis of the violation of the Feynman scaling pattern and a power law increase of multiplicity. This apart, the Kaon-poin ratio, according to this model has a specific type of energy-dependence and they are as follows.

$$K/\pi \;=\; 4.5\times 10^{-2}S^{0.05} \qquad (4a)$$

$$\left(E\frac{d3\sigma}{dp^3}\right)_c = cost \ \ exp\left[-c_1 pT^2\cdot\right] \ \ exp\left[-C_2\times\right] \tag{4b}$$

$$\text{and} \qquad < n >_c \sim cost \cdot S^\alpha \quad \text{with} \quad \alpha = 0.20 \tag{4c}$$

The constants and parameters are to be obtained from Bhattacharyya [5] and the references therein. The parameters C_1 and C_2, in their original derivations have $< n >$-dependence which introduce energy-dependence in the inclusive crosssections and spoil thus the rigid assumption of Feynman scaling factor ; in fact this is responsible for the introduction of the modest violation of the Feynman scaling.

(B) The primary cosmic ray flux can be represented by the following expression for primary nucleons at $x = 0$ [6].

$$\phi_p(E, x=0)\left[Cm^{-2}S^{-1}Sr^{-1}GeV^{-1}\right]$$

$$\left.\begin{aligned} &= 1.7\left(\frac{E}{GeV}\right)^{-2.7} \quad \text{for } E \leq E_0 \\ \text{and} \quad &= 174\left(\frac{E}{GeV}\right)^{-3} \quad \text{for } E \geq E_0 \end{aligned}\right\} \tag{5}$$

(C) The input for the total crosssection is obtained from Block et al [7] and is given by

$$\sigma_{log} \sim 12.56 \times 10^{-3} \ log^2 \frac{S}{S_0} \text{ [in mbs]} \tag{6}$$

(D) The prompt muon component is obtained in a straightforward manner from Pasquali et al [8] in the from $p \rightarrow$ chan $\rightarrow$ j $\rightarrow$ lepton and by

$$Z_{pj}(\alpha) \rightarrow Z_{p\ kc}(\alpha) Z_{kc\ j}(\beta_{kc}) \tag{7}$$

4 Calcutated Results and Comparison with a Particular Set of Data

In the following table (Table-1) we pesent both the experimental data of a particulat set (BAIKAL NT-361995] and the present model-basis calcutated values for various depths of water in meter.

TABLE - 1 Muon-Intensity versus depth in meter of water

Depth in meter(s) of water	Intensity in ($cm^{-2}s^{-}sr^{-1}$)	
	Experimental value(s)	Calculated value(s)
1000	8×10^{-7}	2.4×10^{-7}
2000	1×10^{-7}	6.5×10^{-8}
3000	2×10^{-8}	9×10^{-9}
4000	8×10^{-9}	2×10^{-9}
6000	1×10^{-9}	5×10^{-10}

5 Discussion and Conclusions

The used model for production of particles gives values for the underwater muon - intensities uniformly roughly one-third to $\frac{1}{4}$ of the experimental values, though the qualitative nature is well-reproduced. Before aarriving at any definite conclusion, we would try to recheck our model-based estimates with a view to searching for this factor-disparity. This is, of course, with the assumption that all other input factors which have gone into the calcutations are and should be correct and indisputable And obviously all this also is a source of great uncertainty for the final results.

References

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