Origin of the Speed and Spin of Pulsars

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Abstract

Statistical analysis of the properties of pulsars and their location with respect to the galactic plane or to the supernova remnants in which they were born indicate they had initial speeds of several hundred km s^{-1}. Similarly their evolution on the $(p, \dot{p})$ - plane indicates that they were born with spin periods of 0.1 - 0.2 s. We suggest that the speed and spins of pulsars at birth are causally connected. Accordingly these are attributed to the recoil momentum and torque impulse imparted to the neutron star by anisotropic and asymmetric emission of neutrinos during the collapse of the core in a supernova.

Statistical analysis of the properties of pulsars and their location with respect to the galactic plane or to the supernova remnants in which they were born indicate that they were born with typical speeds v_o of several hundred km s^{-1} [Cowsik 1998 and references therein, Spruit and Phinney 1998 and references therein]. Similarly, their evolution on the $(p, \dot{p})$ plane indicates that they are born with periods of duration typically $0.1 - 0.2$ s. Here we suggest that the speed and spin of pulsars at birth are both direct consequences of the anisotropic and nonaxisymmetric emission of neutrinos by the stellar core during its collapse, leading to the formation of the neutron star. The familiar example of the Crab pulsar spinning with period of 33 ms will be discussed near the end of this paper. The millisecond pulsars, which are recycled and spun up in accreting binary systems, are excluded in this analysis.

Considerable effort has gone into the understanding of the speeds of pulsars at birth: Pulsars born when one of the stars in a close binary or a multiple stellar system explodes as a supernova may be released by the "sling effect"with high speeds, as discussed in detail by Iben and Tutukov and others. The speed of pulsars may also be attributed to the recoil that the neutron star will suffer due to possible asymmetries in the explosion or in the emission of neutrinos. The suggestion that neutrino recoil effects are responsible for the proper motions of pulsars is gaining increasing observational support. Here one notes that during the process of its collapse into a neutron star the core of a presupernova star emits neutrinos with a total energy $S_\nu \approx 3 \times 10^{53}$ erg. The scalar momentum associated with these neutrinos $P_\nu = S_\nu / c \approx 10^{43} gcms^{-1}$. If the neutrino emission is perfectly isotropic, then the vector sum of the neutrino momenta $\vec{p}_\nu$ will vanish;

$$\sum \vec{p}_\nu = 0 \tag{1}$$

However, the neutrino emission is expected to be anisotropic either due to its convective-diffusive transport in the asymmetrically collapsing core or due to neutrino flavor oscillations in the material of the core threaded by magnetic fields. As per this suggestion, the speed of pulsars at birth is given by

$$v_o = \frac{|\sum \vec{p}_i|}{m_*} \equiv \frac{p_o}{m_*} \equiv \frac{\eta P_\nu}{m_*} \approx \left(\frac{\eta}{.01}\right) \times 360 \;\; kms^{-1}, \tag{2}$$

where $\vec{p}_i$ are individual momenta of the neutrinos, m_* is the mass of the neutron star and the asymmetry parameter $\eta = \left|\sum \vec{p}_i\right| / P_\nu$. A distribution in the value of η, spreading over a factor of 2 to 3 on either side of its mean value of $\sim$ 0.01, can reproduce the observed proper motions of pulsars.

As much attention has not been focussed on the origin of the spin of pulsars, but it has been assumed generally that angular momentum conservation during the formation of a neutron star may easily generate the requisite periods. However a neutron star which carries the same angular momentum per unit mass as a typical star will have a spin period of $\sim$ 0.1 - 0.5 ms ! This is several hundred times faster than the typical periods of pulsars at birth. If this mechanism were the correct explanation of the spin of pulsars at birth, which as already noted has a mean of $\sim$ 0.1 - 0.2 s, then we have to find the mechanisms by which precisely the right fraction of $\sim 10^{-3}$ of the angular momentum is left behind, irrespective of the widely varying angular momenta and other characteristics of their progenitors, observed during their evolution along the main sequence. The catalogue of the observed properties of 558 pulsars and a search for fast pulsars in the galactic plane show that the observed periods peak at $\sim$0.5 s and more than 90% of the pulsars lie within a factor of $\sim$ 3 on either side of this central value. When one accounts for the loss of energy and angular momentum through radiation by the pulsars it has been estimated that their periods at birth peak around 0.2 s with a similar dispersion. Irrespective of these small differences, we note below that O-B supergiants, which are the progenitors of most pulsars, are spinning so slowly that the neutron stars formed during the collapse of their core will actually be spinning far too slowly to be pulsars, unless there exist mechanisms other than the initial angular momentum which can spin them up to the requisite periods. Iben and Tutukov have also made this point and estimate the spin period of neutron stars born in suprnova explosions of single O-B supergiants to be longer than $\sim$ 200s, far too long to contribute significantly to the spin of pulsars. The following discussion shows why this is so:

For a realistic estimation of the contribution of stellar angular momenta to the spin of pulsars, we note that most of the neutron stars/pulsars are born in supernova explosions of O-B stars of mass greater than $\sim 10M_\odot$. Even though there are several stages of evolution before all the nuclear fuel in the interior of the star is used up and a core of mass $\sim 1.4M_\odot$ collapses into a neutron star triggering the supernova explosion, the first stages of evolution up to the formation of O-B supergiant are the most relevant here.

(i) On the main sequence hydrogen is burnt in the core to helium, with extensive convective mixing in the central regions of the star. This stage takes up typically 10^7 years and generates a core of He of ~ 4 $M_\odot$; hydrogen continues to burn in a shell surrounding the core. The convection enhances the radial transport of angular momentum and ensures that during this phase the whole star from the core to the surface rotates essentially with the same angular frequency.

(ii) After the hydrogen burning in the core is complete the star leaves the main sequence as the core of a few $M_\odot$ contracts, heating it to T$\sim 10^{8^o}$K and $\rho_c \sim 10^3 g\ cm^{-3}$ and causing helium nuclei to fuse initially to ^{12}C and subsequently to ^{16}O. Concommittantly the outer regions of the star expand up to $\sim 5 \times 10^{13} cm$ ($\sim 700\, R_\odot$) to form a red supergiant. This phase lasts about 1-2 million years or $\sim 10^4 - 10^5$ sound travel times from the core to the surface. Now the important question is, does the core still corotate with the rest of the star?

Even though the details of the mechanism through which the slow rotation of the outer regions of the star decelerates the core are not well understood, it is generally accepted that transport of angular momentum does indeed take place at non-negligible rates. Tassoul, in his book on stellar models, structure and evolution concludes that stellar cores are not perfectly isolated from their envelopes. The key point is that when there is convection and/or when magnetic fields are present threading the various shells then there would be adequate transfer of angular momentum and the core would be coupled to the rest of the star. In intermediate mass stars during the He burning phase, and indeed even subsequently, extensive convective transport does obtain down to the core, coupling it to the rest of the star. We may now estimate an upper bound on the rotational period of the supergiant, by making the assumption that it carries the same angular momentum (J $\sim 10^{51} - 10^{52} g\ cm^2 s^{-1}$) as the bulk of the stars observed on the main sequence:

$$T_{sg} = \frac{2\pi I}{J} \approx 10^{10}\ s \tag{3}$$

Note here that winds emitted during early phases of evolution of the O, B stars, especially if they are asymmatric or magnetically coupled to the star, carry away a disproportionately high fraction of the angular momentum and will make the supergiant of spin even more slowly than that estimated here.

Instead of following the further stages of evolution of this, let us for the purposes of estimating the contribution of stellar angular momenta to the spin of the pulsars assume that $\sim\ 1.5M_{\odot}$ of this He-core with $\rho_c\ \sim\ 10^3\ gcm^{-3}$ collapses, conserving angular momentum to form a neutron star of central density $\rho_n \sim 10^{15} g\ cm^{-3}$. The rotational period of this neutron star would be

$$T_n\ \sim\ T_{sg}\,(\rho_c/\rho_n)^{2/3} \approx\ 10^2\,s \tag{4}$$

Thus it is seen that the expected periods are too long by several orders of magnitude to account for the observed periods of pulsars. Iben and Tutukov also estimate $T_n \sim 300$ s for neutron stars born in the supernova explosions of single O-B supergiants. Increasing the specific angular momentum within reasonable limits, or processes taking place in the later phases of the evolution of the star, do not help to generate adequately fast rotations. On the other hand, white dwarfs (which are formed after sloughing of a few solar masses of their envelopes in stellar winds) have periods ranging from days to roughly a year, consistent with corresponding estimates made with equations similar to eq. 3,4. With such long periods implied by eq.4 for neutron stars at birth we are forced to search for alternative mechanisms that can spin up the pulsars to typical periods of 0.1-0.2s with which they are born.

In this paper, we propose that the same asymmetric neutrino emission that generates the proper motion also provides just the right amount of angular momentum to yield the spin period of pulsars at birth. Qualitatively, the basic idea is that when the asymmetric neutrino emission imparts momentum to the neutron star it would, with a high probability, also impart angular momentum that will make the neutron star spin. If this asymmetry in the neutrino emission is characterized by an effective impact parameter b, with respect to the centre of the neutron star, then the neutron star would be subjected to a torque impulse $m_* v_o b$. This will cause the neutron star of moment of inertia $I \approx 4 \times 10^{44}$, to spin with an angular frequency $\omega_o = m_* v b / I$ or, in other words, the spin period t_o at birth will be

$$t_o = \frac{2\pi}{\omega_o} \approx 2\pi \left\{ \frac{m_* v b}{I} \right\}^{-1} \approx \frac{1}{\eta} \left(\frac{r_*}{b} \right)\ 2.5 \times 10^{-4}\ \ s \tag{5}$$

If $\eta \approx 0.01\ and\, b \approx 0.1\ r_*$, the spin period of pulsars at birth can be reproduced. Note that this is the same value of η which gives the correct translational speed for the pulsar. To see this result in some detail, we write eq.5 more precisely:

$$t_o = \frac{2\pi}{\omega_o} = 2\pi I \left\{ \left| \sum \vec{p}_i \times \vec{b}_i \right| \right\}^{-1} \tag{6}$$

where $\vec{b}_i$ is the impact parameter of the neutrino with momentum $\vec{p}_i$. The equations 5, 6 imply a clear interrelationship between the period and the speed of the pulsars, both arising from a common cause, namely, asymmetric neutrino emission.

Before assessing the possible distribution of initial pulsar periods implied by the hypothesis of spin-up by neutrino recoil torques, we wish to note, following Burrows and Hayes for example, that asymmetries in neutrino emission are quite generic and will occur whether neutrino oscillations play a role or not. According to this picture, the collapsing core receives a series of small impulses as the neutrinos spurt out of a matrix of advection cells. To see this, note that the neutrino mean free path in the collapsing core is a small fraction $\sim$ 0.01 - 0.1 of the typical dimensions of the neutron star so that the neutrinos diffuse outwards as they are being convected inwards by the collapsing flow. Thus the neutrinos exert pressure $\wp$ on the infalling material and the gradient of this pressure exerts a force comparable to the gravitational force responsible for the infall. This makes the system suffer Rayleigh-Taylor-like instabilities whose dispersion relation may be written as

$$\omega^2 = \frac{k}{\rho} \frac{\partial \wp}{\partial r}, \tag{7}$$

which indicates that disturbances with the largest k have the highest growth rates and are consequently the most important. The largest k or the smallest $\lambda = 2\pi/k$ that is relevant in the present context is the neutrino mean free path. Thus the picture that emerges is that the collapsing core has isodensity and isovelocity contours that undulate with typical wavelengths of the order of the neutrino mean free path. These undulations grow rapidly releasing minijets of neutrinos which tend to stabilise the further collapse of the material under gravity. It is the statistical sum of the momenta associated with these minijets which yields the net momentum $m_* v_o$ to the pulsar.

The observations of pulsars, as mentioned earlier, show that their periods are broadly peaked around a central value, and imply a similar distribution of periods at their birth. To see, qualitatively, that minijets of neutrinos can lead to such a distribution, note that each of these minijets will have impact parameters $\vec{b}$ distributed randomly in the three directions according to some distribution dictated by the instabilities in the neutrino transport. By the central-limit theorem, the sum of these will lead to Gaussian distributions of impact parameters along each of the three axes with a clear maximum of r_*. The scalar impact parameter will therefore be distributed as

$$P(b)db \sim b^2 exp(-\frac{b^2}{b_c^2})db \tag{8}$$

As b approaches r_* the distribution will become steeper than Gaussian and will be cut off sharply. Thus we may expect that the initial periods may be peaked at the value obtained by inserting into eq.4 the value of b at which P(b) has a maximum, or more explicitly

$$P(t)\, dt \;\sim\; t^{-4}\, exp\left(-t_c^2/t^2\right)\; dt \tag{9}$$

With $t_c \sim$ 0.25s, or $b_c \sim 0.1 r_*$ the observed distribution of t_o can be reproduced, as noted above.

A possible observational confirmation of this idea might come from the search for the correlations between v_o and t_o, since high v_o implies small t_o statistically. Also three dimensional simulations of the collapse should lead to a deeper understanding of this problem since simulations in lower dimensions tend to average anisotropies out and will not be able to generate angular momentum at all. It is prudent to note here that the correlations mentioned are not simply one to one but indeed we may have occasionally high v_o and high t_o or low v_o and a low t_o as shown by the examples described below. The pulsar inside the Crab-nebula has a period of $\sim$ 33 ms and yet with a speed of $\sim$ 150 kms^{-1}, it is not displaced substantially with respect to the centre. Could such a situation obtain if the phenomenon of neutrino recoil was responsible for its spin-up? Let two subsets of mini-jets generate recoils $\vec{p}_1$ and $\vec{p}_2$ with impact parameters $\vec{b}_1$ and $\vec{b}_2$; if fortuitously $\vec{p}_1 \approx -\vec{p}_2$ and $\vec{b}_1 \approx -\vec{b}_2$ then the torques due to the two subsets will add and momentum impulses would cancel each other, giving rise to a situation like the crab-nebula and its pulsar. Similarly, a slowly rotating run away pulsar without any nebula associated with it can be generated when there is a large $\left|\vec{p}\right|$ and a small $\left|\vec{b}\right|$.

The key ideas presented in this paper were discussed by us in 1996 soon after the seminar by Blaauw [1996], and an abridged version of the paper was published in 1998 [Cowsik 1998]. Recently our attention was drawn to the excellent paper by Spruit and Phinney [Spruit and Phinney 1998], who have discussed this problem more completely. The strict correlation that is expected between speed and spin when a single impulse is responsible for both (as in the case of the sling effect) appears to be ruled out from their analysis of the observed data and their simulations of pulsars on the v_t plane. Also detailed polarization and other features of pulsars are being studied to answer carefully the questions raised in this.

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