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Propagation of Cosmic Rays Using the Time Parameter

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Abstract

A new code for calculation of the propagation of cosmic rays has been developed. The code uses time as the integration variable, instead of the g/cm^2, that has been used historically. The computational technique and parameters used in the code are described. Two results from application of the code are presented: (1) a numerically exact solution for the ^{10}Be/^{9}Be ratio over a wide range of energies in the Leaky Box model; (2) the consequences for the ^{10}Be/^{9}Be ratio when cosmic rays are observed from inside a cavity ("bubble") of low density.

1 Introduction:

In a previous paper (Stephens and Streitmatter, 1998; hereafter SSP1) we surveyed past work on the computation of cosmic ray transport and introduced the steady-state technique for obtaining exact numerical solutions of the large set of coupled differential equations which govern the propagation of cosmic radiation in the leaky box model. The variable of integration used in that work was the traditional "x" variable, loosely called "grammage", measuring g/cm^2 of interstellar material traversed. This variable has been used historically because it is easily adaptable for use in the weighted-slab method. Here, we present an evolution of the previous paper in which time is the independent variable.

The differential equation describing the propagation of cosmic rays in the Galaxy may be written using the general formalism given by Ginzburg and Syrovatskii (1964)

$$\frac{\partial N_i(E,t)}{\partial t} = \frac{\partial}{\partial E}\left\{\frac{dE}{dt} N_i(E,t)\right\} - N_i(E,t)\left\{\sigma_i(E) n \mathrm{v}_i + \frac{1.0}{\tau_i^{es}(E)} + \frac{1.0}{\tau_i^d \gamma_i}\right\} + Q_i(E,t) \qquad (1)$$

Here, for the i-th component of the cosmic radiation , $N_i(E,t)$ is the number density as a function of kinetic energy per nucleon, E, and time, t; σ_i is the interaction cross section; n is the mean number density of atomic hydrogen in the interstellar medium; v_i is the velocity; $\tau^{es}(E)$ is "the escape time" from the Galaxy; τ^d is the decay lifetime at rest for radioactive nuclei; γ_i is the Lorentz factor corresponding to kinetic energy E. The source term for the i-th component, Q_i, is comprised of three parts: the first term is

$$Q_i(E,t) = \sum_{j>i} N_j(E,t)\sigma_{ij}(E)\mathrm{v}_j n + \sum_{l \neq i} \frac{N_l(E,t)}{\tau_l^d \gamma_l} + q_i(E)$$

the contribution for fragmentation of heavier (j-th) nuclei, with partial cross section σ_{ij}; the second is the contribution from radioactive decay; the third term $q_i(E)$ is the primary source term, which has dimensions of number density / time / (energy / nucleon). The source spectra on the individual isotopes are taken as power laws in rigidity, with an index of -2.25, with the individual element abundances following those of Silberberg and Tsao (1990), and isotopes distributed in accordance with natural abundances. In SSP1, element abundances were adjusted slightly from the Silberberg and Tsao values to better fit the observed data which we had selected for comparison. The source abundances used here are those so determined.

2 The Computation:

The ensemble of propagating hadronic cosmic rays is represented by the set of coupled differential equations, Eqs. 1 above, one for each component (isotope). It will be noted that by characterizing particle loss by use of an escape time τ^{es}, and having no spatial dependence of any variable, we are implicitly adopting a leaky box model. The escape time τ_ι^{es} is related to the escape length $X_i(E)$ measured in g/cm^2 by $X_i(E) = \langle m \rangle\, n\, v_i\, \tau_i^{es}(E)$, where the composition of the interstellar medium is incorporated into <m>, the equivalent atomic mass. The enhancement in ionization loss due to partial ionization of the interstellar medium (ISM) has been noted by Ferrando and Soutoul (1989). Here, ionization has been calculated for an ISM consisting of 80% neutral hydrogen, 10% ionized hydrogen and 10% helium by number.

We use the cross sections for fragmentation of Webber, Kish and Shrier, 1990, and the total interaction cross sections of Silberberg and Tsao, 1990; and Kox et *al.*, 1987.

In the computations described here, the energy-dependence of the escape time (i.e. rigidity-dependence for any specific isotope) is incorporated as

$$\tau_i^{es}(E) = T_o \ \ (R/R_o)^{-a} \ \ \text{for} \ \ R > R_o$$

$$\tau_i^{es}(E) = T_o \ \ \text{for} \ \ R \le R_o$$

where R_o is the rigidity at which energy-dependent escape begins, α is the spectral index of the rigidity dependence of escape. For a given interstellar hydrogen number density, n, the value of T_o is a constant. The triad of constants (T_o, R_o, and α) is chosen to best fit the observed primary and secondary spectra. In the present work, we are not interested in fine optimization of such fits, but in understanding the effect of different interstellar densities upon the spectra of radioactive secondary nuclei. Noting $T_o = X_o / n \langle m \rangle c$, we follow SPP1 in using the optimized values: X_o = 15.9 g.cm^{-2}, R_o = 3.5 GV/c, and α = 0.55. These values are used in all the computations reported here.

We introduce the "steady state" technique of solving the set of coupled equations (1) representing the transport of hadrons in the cosmic radiation by simultaneous integration forward until all $\partial N_i/\partial t$ reach zero. This is by definition the equilibrium solution of the set of equations. We obtain thus the energy spectra of all isotopes used. In practice, we integrate 71 equations representing isotopes from ^{6}Li to ^{57}Fe. As a mathematical solution of the equations, the spectra are numerically exact, with no approximations, weighted slabs, etc. This is particularly important in comparing computed results with data. As an astrophysics problem, all the approximations are in the formulations -- use of the leaky box to represent the physical situation and the various input-parameters described above -- and any differences between the data and computations must be ascribed to these approximations.

3 Clock Isotopes; The Ratio ^{10}Be / ^{9}Be:

It is well understood that the secondary cosmic rays which are radioactive "clock isotopes" are of special interest. In the context of any particular model, the experimental data on stable secondary and primary nuclei in the cosmic rays allow one to obtain the magnitude and energy-dependence of the amount of material traversed by cosmic rays. The simple leaky box with uniform matter (ISM gas) distribution has to date been found adequate to explain these data (See, e.g., SSP1 and references therein). However, only data on the radioactive clocks can, within the context of a leaky box model, determine the density of the ISM or, within the context of models with non-homogeneous matter distributions, determine parameters such as the effective dimension of the Galactic halo (Prishchep and Ptuskin, 1975) or the local cosmic ray diffusion coefficient (Ptuskin and Soutoul, 1998). Further, data on multiple clock isotopes and on clock isotopes at energies where relativistic time dilation is important can be used to distinguish between models.

Data on clock isotopes in the ~100 MeV/nucleon range from four satellites are now available. The ACE data with definitive event statistics in this energy range should be reported at this conference. The ISOMAX spectrometer (Streitmatter *et al.*, 1993) made its first flight in 1998, measuring light isotopes including beryllium to ~ 1.7 GeV/nucleon. (See the papers of DeNolfo, Geier, Hams, and Mitchell, this conference.) In the flight planned for the year 2000, ISOMAX will measure beryllium isotopes to ~ 3.5 GeV/nucleon, and is capable of measurements beyond that energy.

Clearly, it is important to have the best possible calculations on clock isotopes for the various models. In Figure 1, we present results for the isotope ratio ^{10}Be / ^{9}Be as a function of energy in the steady state equilibrium solution to the leaky box. To allow comparison of the computed interstellar spectra and ratios with observations at earth, the calculated interstellar ^{10}Be and ^{9}Be spectra have been modulated using the code of Perko (1987). Four pairs of curves are shown. From bottom-to-top, these are for interstellar hydrogen densities of 0.10, 0.25, 0.50, and 1.0 cm^{-2}. In terms of the customary solar modulation parameter ϕ, the dashed curves correspond to ϕ = 500 MV, while the solid curves correspond to ϕ = 1000 MV. One notes: 1) as expected for two isotopes whose A/Z value differs by only 10%, solar modulation has a modest effect; 2) in general agreement with previous authors, the current low-energy data is fit by a leaky box with n = 0.25; 3) the higher-energy point of Buffington places no serious constraint on the model.

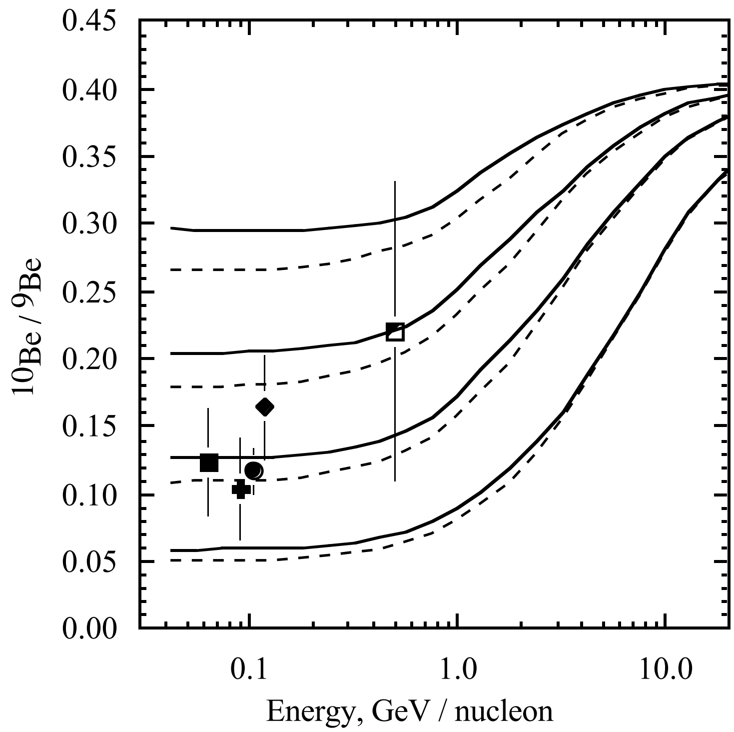

Figure 1. ^{10}Be / ^{9}Be Vs Energy. Computed curves are described in the main text. Data are: ■ Garcia-Munoz et al. 1981; ✚ Lukasiak et al. 1994; ● Connell 1987; ◆ Wiedenbeck and Greiner 1980; □ Buffington et al., 1978.

4 Non-homogeneous Models; A Low Density Bubble:

A density of 0.25 is less than the nominal density in the disk of 1/cm^{-2}. This is usually loosely interpreted as meaning that the cosmic rays observed have spent a large fraction of their life in the Galactic halo. However, as soon as one begins to deal with non-homogeneous models, the issue of the configuration of matter in the Galaxy and local ISM must be dealt with. After the leaky box, the next simplest model is the disk-halo model (Prishchep and Ptuskin, 1975; Berezinskii, et al., 1990). Even this is not fully satisfactory, since it is well known that the Solar System is located in a cavity, the Local Bubble (LB), of low density some hundreds of parsecs in extent. See, e.g., the review of Cox and Reynolds (1987); and Streitmatter *et al.*, 1985. Ptuskin and Soutoul (1998) have modeled the distribution of matter within several hundred parsecs of the Solar System and indicated how future data on clock isotopes can be used to determine the local diffusion coefficient.

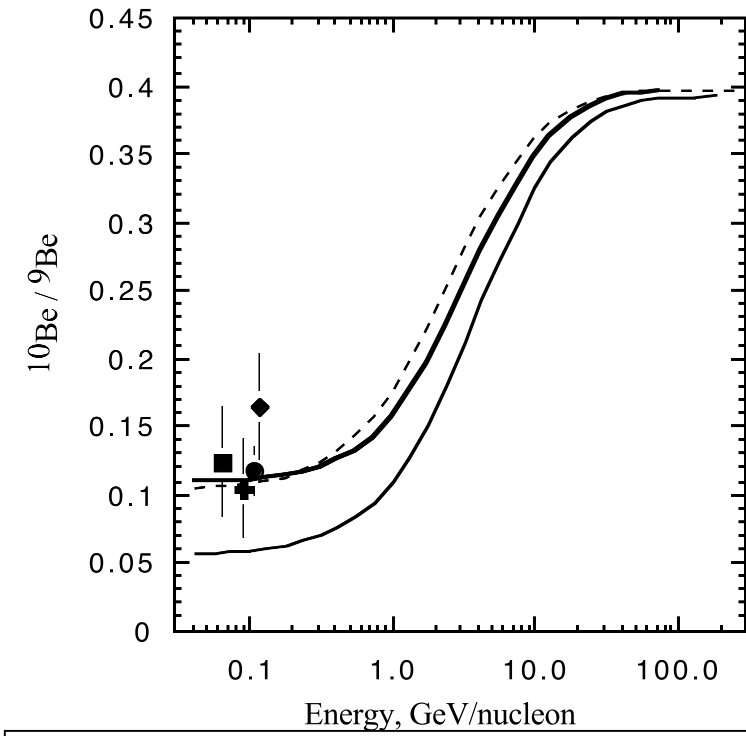

Figure 2. Bubble model predictions. Symbols same as Fig. 1.

As a simplest model, we assume the Solar System is located in a bubble whose interior matter density is zero, and that the cosmic rays which enter this bubble diffuse for a period of time encountering no interstellar matter before reaching the Solar System. We further assume that the cosmic rays entering the bubble have equilibrium spectra characterized by the input parameters and computation described in sections 2 and 3 above, with the only free parameter being the interstellar density of the matter exterior to the bubble. Within the bubble of zero density, the only change in the composition of the cosmic radiation is then the "aging" of the radioactive isotopes. The other free parameter of the model is the "aging distribution" of that part of the equilibrium spectra which reach the Earth. Our computer code can be adapted to accommodate any arbitrary choice of matter density as a function of time, allowing for a bubble with finite interior density and a distribution of ages due to the diffusion process within the bubble. Here, for illustration, we again make a simple choice: the cosmic rays reaching the Earth have all aged two million years since entering the local bubble. In Figure 2, the heavy solid curve is taken from Figure 1`, with n=0.25 and ϕ = 500 MV. The solid curve is for the simple bubble model just described. The dashed curve is for the same bubble model with an exterior density n = 0.6. In this model, cosmic rays reaching the Solar System have traveled mostly in the Galactic disk.

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