The Modified Weighted Slab Technique: Description of the Technique and Models

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Abstract

In an atempt to understand the source and propagation of galactic cosmic rays (Webber, 1997) we have employed the modified weighted slab technique of Ptuskin, Jones, and Ormes along with recent values of the relevant cross sections to compute primary to secondary ratios including B/C and Sub-Fe/Fe for different galactic propagation models. The models that we have considered are the standard diffusion model (leaky box or thin disk model), the dynamical halo model, the turbulent diffusion model and a model with minimal reacceleration. The modified weighted slab technique will be briefly discussed and a more detailed description of the models will be given. The results of our calculation will be given in an accompanying paper.

1 Simple test models to fit the observed escape length

Ptuskin, Jones and Ormes (1996) showed how the weighted slab technique could be made exact for galactic propagation models in which energy gains and losses were proportional to the same mass density that determined nuclear fragmentation and time dependant processes, eg. radioactive decay, do not play a role. This modification allows for the fact that particles had different (usually higher) energies in the past and hence different propagation properties and that propagation is considered to be a function of rigidity rather than energy per nucleon which is the proper parameter for nuclear fragmentation calculations. Strictly, this technique is rigorous only for models in which the particle propagation parameters are proportional to a single function of energy for each particle species, and hence does not apply to galactic wind models or turbulent diffusion models. However, most of these models may be closely approximated by simplified homogeneous models in which the mean path length has an exponential distribution (equivalent to the leaky box) with a mean path length that is a particular function of rigidity. It is models of this type and approximation that we discuss in this study. Detailed results of this work are given in a companion paper (Ptuskin *et al.* 1999).

2 Galactic wind model.

We consider a one-dimensional galaxy model shown in Figure (1) (the coordinate z is perpendicular to the galactic plane). Cosmic ray sources and interstellar gas are concentrated in a thin disk at $z = 0$ (For details of this model see Jokipii, 1976; Jones, 1979). The distribution function $f(z, p)$ normalized as $N = 4\pi \int dpp^2 f$ (where N is the total cosmic ray number density) obeys the equation

$$-\frac{\partial}{\partial z} D \frac{\partial f}{\partial z} + u\frac{\partial f}{\partial z} - \frac{du}{dz}\frac{p}{3}\frac{\partial f}{\partial p} + \frac{\mu v \sigma}{m}\delta(z) f + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(\frac{dp}{dt}\right)_{ion} f\right] = q_0(p)\delta(z). \quad (1)$$

Here $D(p, z)$ is the particle diffusion coefficient; $u(z)$ is the wind velocity; μ is the surface mass density of the galactic disk (in g/cm^2); $v = \beta c$ is the particle velocity; σ is the total spallation cross section; m is the mean mass of interstellar atom; $(dp/dt)_{ion} = \mu\delta(z) b_0(p)/m < 0$ describes the ionization energy losses; $q_0(p)\delta(z)$ is the source term that may include the yield from the fragmentation of more heavy nuclei.

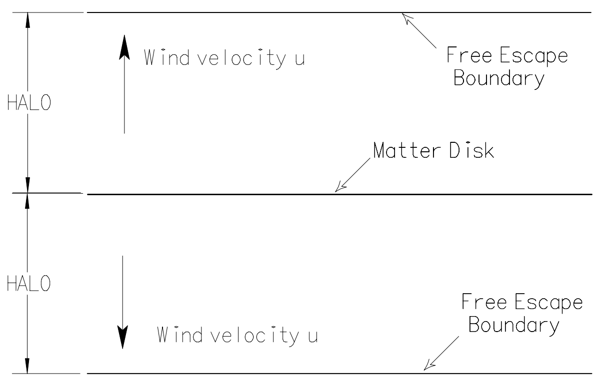

Figure 1: Simplified galactic model with a halo wind; galactic matter disk is of infinitessimal thickness.

We shall assume that diffusion does not depend on position, i.e. $D = D(p)$, and the wind velocity u is constant and directed outward the galactic plane. There is a cosmic ray halo boundary at $|z| = H$ where cosmic rays freely exit from the Galaxy.

Integrating equation (1) in the vicinity of galactic plane $\lim \int_{-\varepsilon}^{\varepsilon} dz(...)$ at $\varepsilon \longrightarrow 0$ one can find the boundary condition at $z = 0 + \varepsilon$:

$$-2D\frac{\partial f_0}{\partial z} - \frac{2up}{3}\frac{\partial f_0}{\partial p} + \frac{\mu v \sigma}{m} f_0 + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \frac{\mu b_0}{m} f_0\right] = q_0(p), \quad (2)$$

where $f_0(p) = f(z = 0, p)$ is the distribution function at the galactic midplane.

The solution of equation (1) at $z \neq 0$ under the boundary condition $f(|z| = H) = 0$ is:

$$f = f_0 \frac{1 - \exp\left(-u(H - |z|)/D\right)}{1 - \exp\left(-uH/D\right)}. \quad (3)$$

Calculating $-D\frac{\partial f_0}{\partial z}$ from equation (3) and substituting it in equation (2), one can get the following closed equation for $f_0(p)$:

$$\frac{2uf_0}{\mu v\left(\exp(uH/D) - 1\right)} - \frac{2u}{3\mu v} p \frac{\partial f_0}{\partial p} + \frac{\sigma}{m} f_0 + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \frac{b_0}{mv} f_0\right] = \frac{q_0}{\mu v}. \quad (4)$$

Now we introduce cosmic ray intensity as function of kinetic energy per nucleon $I(E_k)dE_k = vf_0(p)p^2dp$, so that $f_0(p) = I(E_k)A/p^2$ (A is the atomic number). Equation (4) gives the following equation for $I(E_k)$:

$$\frac{I}{X_w} + \frac{d}{dE_k}\left[\left(\left(\frac{dE_k}{dx}\right)_{ad} + \left(\frac{dE_k}{dx}\right)_{ion}\right) I\right] + \frac{\sigma}{m} I = \frac{q_0(p)p^2 A}{\mu v} \quad (5)$$

with the effective escape length

$$X_w = \frac{\mu v}{2u}\left(1 - \exp\left(-uH/D\right)\right), \quad (6)$$

and the adiabatic energy loss rate per g/cm^2

$$\left(\frac{dE_k}{dx}\right)_{ad} = -\frac{2u}{3\mu c}\sqrt{E_k(E_k + 2E_0)}. \quad (7)$$

At this point we see that we have arrived at the basic equations for the ”leaky box” model in the sense that the path length distribution will be an exponential with the mean path length given by X_w. We may therefore apply the modified weighted slab technique to solve the equations.

The scaling of diffusion is $D = \beta\kappa_0 R^a$ (here $\kappa_0 = const$) if cosmic ray scattering is produced by hydromagnetic turbulence with a power law spectrum on wave number, $W_k dk \propto k^{-2+a} dk$. The asymptotic scaling of X_w in this case is

$$X_w = \frac{\mu v}{2u} \propto v \text{ at small rigidities (when } uH/D \gg 1); \quad (8)$$

and

$$X_w = \frac{\mu v H}{2D} \propto R^{-a} \text{ at large rigidities (when } uH/D \ll 1). \tag{9}$$

Equation (6) may be presented in the following form usefull to fit the observations:

$$X_w = \beta X_0 \left(1 - \exp\left(-\frac{1}{\beta\left(R/R_0\right)^a}\right)\right). \tag{10}$$

Here $X_0 = (\mu c)/(2u)$, and $R_0 = (uH/\kappa_0)^{1/a}$. The adiabatic energy loss term, equation (7) in these notations reads as

$$\left(\frac{dE_k}{dx}\right)_{ad} = -\frac{1}{3X_0}\sqrt{E_k(E_k + 2E_0)}. \tag{11}$$

3 Turbulent diffusion

There is no regular convective (wind) transport in this model (See Figure 2) and equation (1) reads as

$$-\frac{\partial}{\partial z} D \frac{\partial f}{\partial z} + \frac{\mu v \sigma}{m}\delta(z) f + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \left(\frac{dp}{dt}\right)_{ion} f\right] = q_0(p)\delta(z). \tag{12}$$

The equation for $I(E_k)$ is the following

$$\frac{I}{X_{dif}} + \frac{d}{dE_k}\left[\left(\frac{dE_k}{dx}\right)_{ion} I\right] + \frac{\sigma}{m} I = \frac{q_0(p)p^2 A}{\mu v} \tag{13}$$

where

$$X_{dif} = \frac{\mu v H}{2D} \tag{14}$$

.

We assume now that cosmic ray diffusion is provided simultaneously by turbulent diffusion with the diffusion coefficient D_t that does not depend on particle energy (may be estimated as $D_t = u_t L_t/3$, where u_t and L_t are the characteristic random velocity and correlation scale of large-scale turbulent motions of the interstellar gas) and by resonant diffusion with the diffusion coefficient $D_{res} = \beta \kappa_0 R^a$ (here $\kappa_0 = const$, $a = const$) provided by the scattering on hydromagnetic turbulence as was discussed in the previous section. The total diffusion coefficient which appears in equation (14) is equal to

$$D = D_t + D_{res}. \tag{15}$$

Equation (14) may be presented in the following form that is usefull to fit the observations:

$$X_{dif} = \frac{\beta X_0}{1 + \beta\left(R/R_0\right)^a}. \tag{16}$$

Here $X_0 = (\mu c H)/(2D_t)$, and $R_0 = (D_t/\kappa_0)^{1/a}$.

4 Stochastic Reacceleration.

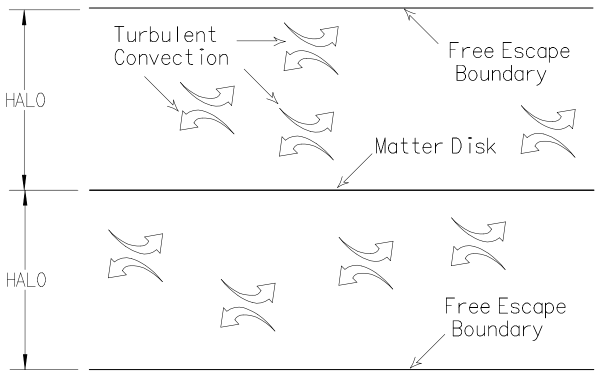

Figure 2: Turbulent Diffusion Model

This model (Seo and Ptuskin 1994) assumes no convective/wind motion in the system. The spatial diffusion is provided by scattering on random hydromagnetic waves with a Kolmogorov spectrum that results into $D \propto vR^a$ with $a = 1/3$. (In principle, the exponent a may be different. For example, the Kraichnan spectrum gives $a = 1/2$. Actually, the value of a may be considered as a free parameter, in the same way as in the leaky-box model and in the models discussed above. This determines the behavior of escape length $X_e = X_0 R^{-a}$ (see equation (9)) at all energies. However, stochastic acceleration with diffusion coefficient on momentum $K \sim p^2 v_A^2 / D$ (v_A is the Alfven velocity) modifies essentially the spectra of primaries and secondaries below about 10 GeV/n and may produce the characteristic peak in B/C ratio at few GV. The acceleration becomes not efficient at high energies and the model is reduced to a simple leaky box without reacceleration and with the escape length $X_e = X_0 R^{-a}$ at $E > 20 - 30$ GeV/n.

We use the same notations as Seo and Ptuskin and use their equation (12) for the calculations:

$$\frac{I}{X_e} + \frac{\sigma}{m} I + \frac{d}{dE}\left[\left(\frac{dE}{dx}\right)_{ion} I\right] + \alpha \left\{ \left[\frac{A}{Z}\frac{(E_k + E_0)}{\beta}\frac{dX_e}{dR} + X_e\right] I - \frac{A}{2Z}\beta(E_k+E_0)^2 \frac{dX_e}{dR}\frac{dI}{dE} - \frac{\beta^2}{2}(E_k+E_0)^2 X_e \frac{d^2 I}{dE^2}\right\} = \frac{q_0(p) p^2 A}{\mu v}. \tag{17}$$

Here the parameter α, which is defined as

$$\alpha = \frac{32}{3a(4-a^2)(4-a)} \frac{h_a}{H} \left(\frac{v_a}{\mu c}\right)^2 \tag{18}$$

(its dimension is (g/cm^2)$^{-2}$), determines the efficiency of reacceleration; h_a is the height of reacceleration region.

The parameters we have to find from fitting the data are X_0 and α. The value of a should be considered as free parameter if the spectrum of the interstellar turbulence is not prescribed.

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