Cosmic ray spectrum above the Greisen-Zatsepin-Kuzmin cutoff in a model with fractal distribution of sources

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Abstract

The detection of events with energies above the GZK cutoff energy $E_c \sim 5{\cdot}10^{19}$ eV implies the non-uniform source distribution for ultra high energy cosmic rays. This may be in accordance with the studies of galaxy distribution in the Universe that exhibits the fractal structure on scales less than about 100 Mpc.

1 Introduction.

The sources of observed ultra high energy cosmic rays remain unidentified. Most probably, the particles with energies $E > 10^{19}$ eV have extragalactic origin since their angular distribution does not show any significant excess from the direction of galactic disk where potential galactic sources are distributed. The Galactic origin for these particles is possible if they have considerable fraction of heavy nuclei with the Larmor radius $r_g = 10(E/10^{20}Z$ eV$)(B/1\mu$G$)$ Kpc that is less than the height of the galactic magnetic corona 3-10 Kpc (Zirakashvili *et al*., 1998). Here Z is the particle charge, and B is the galactic magnetic field. Another possible galactic model employs the slow decay of superheavy relic particles produced in the post-inflationary Universe and concentrated at a present epoch in the extended galactic halo (Berezinsky, Kachelriess, & Vilenkin, 1997).

The GZK cutoff (Greisen, 1966; Zatsepin & Kuzmin, 1966) in cosmic ray spectrum at about $5{\cdot}10^{19}$ eV is characteristic of the models with extragalactic sources homogeneously distributed on the Hubble scale $c/H_0 \sim 3{\cdot}10^3/h$ Mpc. Here c is the velocity of light, $H_0 = 100h$ km/(s·Mpc) is the Hubble constant and we specify $h = 0.75$ in the present work. The data (Takeda *et al*., 1998) do not show the cutoff and this is considered as a decisive argument against universal extragalactic models. However, the galaxy distribution in the Universe is not uniform (Peebles, 1980) and can be described as a fractal (Mandelbrot, 1998). This can remove the cut off constraint from extragalactic models.

The attenuation length for a proton with energy 10^{20} eV moving through the 2.7 K microwave radiation is equal to $L_{at} \approx 160$ Mpc, and $L_{at} \approx 20$ Mpc at $E = 3{\cdot}10^{20}$ eV, the maximum particle energy detected in cosmic rays. The clumpiness of galaxies continues over this range of scales up to about $L_{max} \sim 100/h$ Mpc and thus is essential for cosmic ray propagation. The number of galaxies within a distance R from an observer scales as $N(>R) \sim R^D$ for a fractal of correlation dimension D. The value $D = 3$ corresponds to uniform, not fractal, distribution. Some authors (Sylos Labini, Montuori, & Pietronero, 1998) give $D \sim 2$, $L_{max} > 100/h$ Mpc and found that galaxy density decreases around our Galaxy according to the expected law, while others (Wu, Lahov, & Rees, 1999) found smaller $L_{max} \sim 60/h$ Mpc with dependent on distance $D(r)$.

The problem of nonhomogeneous source distribution and its influence on the observed spectrum of ultra high energy cosmic rays was studied by Berezinsky & Grigoreva (1979) in terms of the density contrast between the Local Supercluster and surrounding background galaxies. The visible distribution of galaxies at distances less than 100 Mpc taken from CfA redshift catalog was used for the same purpose by Medina Tanco (1999). In the present work we calculate cosmic ray spectrum for a few source distributions with different fractal dimensions.

We do not specify here the acceleration mechanisms and the nature of extragalactic sources. The candidate astronomical objects include jets from active galactic nuclei, gamma ray bursts (double star merging), and colliding galaxies. Top-down mechanism (decays of topological defects and relic particles) presents an alternative.

2 Cosmic ray spectrum.

Transport of ultra relativistic protons produced by a homogeneous distribution of sources in a flat expanding Universe without lambda term is described by the equation

$$-\frac{\partial}{\partial z}\left(\frac{J(E,z)}{(1+z)^3}\right)-\frac{\partial}{\partial E}\left[\left(1+\frac{(1+z)^{3/2}}{H_0 t_0((1+z)E)}\right)\frac{EJ(E,z)}{(1+z)^4}\right]=H_0^{-1}q(E)(1+z)^{m-\frac{5}{2}}. \qquad (1)$$

Here *JdE* is the cosmic ray intensity, *z* is the redshift, *qdE* presents the cosmic ray production per unit volume per unit time, *m* describes the evolution of sources (*m* = 0 implies no evolution). We use the approximation of continuous energy loss on pair and pion photo production, $dE/dt = -E/t_0(E)$ at $z = 0$, and take expressions for $t_0(E)$ from the works of Chodorowski, Zdziarski, & Sikora (1992) and Waxman (1995). The scaling of energy loss rate on *z* in Eq. (1) is dictated by the evolution of the black body microwave radiation in the expanding Universe. The solution of Eq. (1) is

$$J(E,z)=H_0^{-1}(1+z)^3\int_z^{z_{\max}} dz_1(1+z_1)^{m-\frac{5}{2}}q(E_1(z_1;z,E))\frac{\partial E_1(z_1;z,E)}{\partial E}. \qquad (2)$$

Here z_{max} is the maximum redshift at which sources exist (in the following, we assume that z_{max} = 2), $E_1(z_1;z,E)$ is the energy at which a proton should be injected at an epoch *z* in order to be observed with energy *E* at redshift *z*.

The homogenous model (1), (2) works on scales larger than L_{max}. Let us consider now the distances $r < L_{max}$ where the source distribution is not homogeneous at $D < 3$. A proton with energy $E = 10^{20}$ eV moving in a random extragalactic magnetic field with strength $B = 10^{-9}$ G and correlation scale L_B = 3 Mpc experiences a small angle scattering. This scattering results in spatial diffusion with a mean free path of the order of $l \sim r_g^2/L_B \sim 3\cdot 10^3(E/10^{20}$ eV) Mpc. Hence the approximation of rectilinear trajectories can be used on scales less than $L_{max} \sim 100/h$ Mpc for calculations of cosmic ray intensity at $E > 10^{19}$ eV. One can also ignore the Hubble expansion in this nearby region where $r << c/H_0$. (Recall that $r = 3\cdot 10^3 z/h$ Mpc at small $z<<1$.) Thus the transport equation and its solution $J_0(E)$ at $r = 0$ are respectively:

$$-c\frac{\partial J(E,r)}{\partial r}-\frac{\partial}{\partial E}\left[\frac{E}{t_0(E)}J(E,r)\right]=q(E)S(r), \qquad (3)$$

and

$$J_0(E)=J_m(E_m)\frac{E_m t_0(E)}{E t_0(E_m)}+t_0(E)\int_1^{E_m/E} dx\,q(xE)S\left(c\int_E^{xE} dE' t_0(E')/E'\right). \qquad (4)$$

Here *S(r)* describes the spatial distribution of sources, so that $S \propto r^{-3}\exp\left(\int^r dr_1 D(r_1)/r_1\right)$ for fractal distribution with depending on distance dimension; the intensity $J_m(E) = J(E,r=L_{max})$ has to be calculated from Eq. (2) to provide the continuity of cosmic ray density at $r = L_{max}$. The energy $E_m(E)$ is determined by the transcendental equation $L_{\max}=c\int_E^{E_m} dE' t_0(E')/E'$.

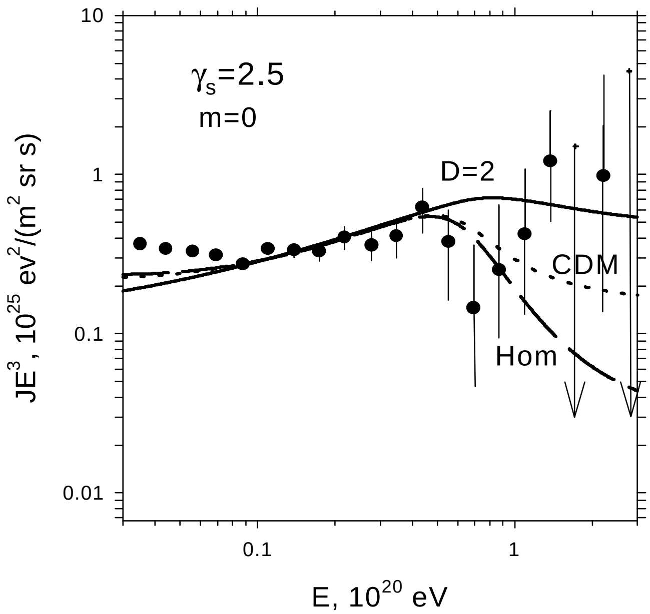

Figure 1: Predicted spectra of extragalactic cosmic rays: fractal source distribution with dimension D = 2 described in the text (curve D=2), homogeneous source distribution (curve Hom), and galaxy distribution derived by Wu *et al.* (1999) in the Cold Dark Matter theory (curve CDM). The data points are taken from Takeda *et al.* (1998). The curves are fitted to observations at energy E = 10^{19} eV. The shape of the source spectrum is qdE μ $E^{-2.5}dE$. No evolution of the individual source strength is assumed, m = 0.

Figure 1 illustrates the results of our calculations of particle energy spectra. It is assumed that the extragalactic component dominates over the galactic component in observed cosmic ray spectrum at $E > 10^{19}$ eV. Three models of the distribution of extragalactic sources were used in the calculations. The solid curve corresponds to the fractal distribution with dimension D = 2 for scales $3 < r \leq 100/h$ Mpc and D = 3 (the homogeneous distribution) at $r > 100/h$ Mpc. The dashed curve corresponds to the homogeneous distribution. The dotted curve corresponds to the distribution of galaxies in the Cold Dark Matter theory (Wu *et al.*, 1999) where $D(r)$ is varied with distance from D = 2 at $r \leq 10/h$ Mpc to D = 3 at $r > 100/h$ Mpc. The power law source spectrum of the form $q \propto E^{-g_s}$, g_s = const is assumed in the calculations.

3 Conclusion.

Figure 1 demonstrates clearly the impact of source clustering on cosmic ray spectrum. The fractal distribution of sources may explain the extension of particle spectrum beyond the Greisen-Zatsepin-Kuzmin cutoff characteristic of the uniform source distribution in the Universe.

The approximation of a continuous source distribution used in the present calculations needs the justification because of the discrete nature of cosmic ray sources. Point instant sources randomly distributed in space and time generate cosmic rays with fluctuating intensity. The relative amplitude of fluctuations can be estimated as $dJ / J \sim r_g \left(L_{at} L_B^{1/2} (Q/c)^{1/8} \right)^{-1}$, where Q is the rate of occurrence of sources per unit volume. These fluctuations are small if the frequency of source occurrence is high enough. Assuming that the sources are associated with normal galaxies, one can find that $dJ / J < 1$ at $E = 10^{20}$ eV if cosmic ray bursts occur every 10^8 yr in every galaxy. This frequency is probably typical for the gamma ray bursts (Paczynski, 1993). The discrete nature of cosmic ray source distribution may be not essential when the condition $dJ / J << 1$ is fulfilled. The problem of cosmic ray propagation from local extragalactic sources was recently studied by Blasi & Olinto (1999) and Sigl *et al.* (1999).

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