The Modified Weighted Slab Technique: Results

V.S. Ptuskin[1,2], A. Lukasiak[2], F.C. Jones[3], and W.R. Webber[4]

[1]*Institute for Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences (IZMIRAN), Troitsk, Moscow Region 142092, Russia*
[2]*Institute for Physical Science and Technology, University of Maryland, College Park, MD 20742, USA*
[3]*Laboratory for High Energy Astrophysics, NASA/Goddard Space Flight Center, Greenbelt, MD 20771, USA*
[4]*Astronomy Department, New Mexico State University, Las Cruces, NM 88003, USA*

Abstract

We have computed the B/C and Sub-Fe/Fe ratios for the standard diffusion model, the dynamical halo model, the turbulent diffusion model, and the diffusion model with minimal reacceleration. We have fit recent data with a variety of model parameters, including energy dependent transport coefficients and different source spectra, to reproduce the observed secondary to primary ratios. Attention focusses on the interpretation of the dependence of energetic particle transport in the Galaxy on energy.

1 Introduction.

The presence of secondary nuclei in cosmic rays produced in the course of propagation and nuclear fragmentation of primary cosmic rays in the interstellar gas allows studying the nature of cosmic ray transport in the Galaxy. Under some assumptions about the character of cosmic ray transport, the set of equations for all primary and secondary species involved in a chain of sequential nuclear spallations can be solved exactly with the use of the modified weighted slab method (Ptuskin, Jones, & Ormes, 1996). This method and the detailed description of the propagation models investigated in the present work are discussed in our Paper I at this Conference (Jones *et al*., 1999). Here we present some results of the numerical simulations where the most recent set of spallation cross sections (Webber *et al*., 1998) was used.

2 Standard diffusion model.

The dependence of sec/prim isotope ratios on energy gives the key to understanding how cosmic rays propagate in the interstellar medium. One can introduce the escape length of cosmic rays $X(E)$, the mean thickness of matter traversed by cosmic rays before their exit from the Galaxy. In the popular leaky box model, the escape length X_{lb} is the only characteristic, which describes the galactic transport of cosmic rays. Our present calculations show that observed ratios B/C and (Sc+Ti+V)/Fe (Sub-Fe/Fe) may be fitted by the following escape length, see Figure 1 (solid lines):

$$X_{lb} = 11.3b \text{ g/cm}^2 \text{ at } R < 5 \text{ GV}, \; X_{lb} = 11.3b(R/5\text{GV})^{-0.54} \text{ g/cm}^2 \text{ at } R \geq 5 \text{ GV}. \quad (1)$$

This parameterization is valid for particles in the interstellar medium with energies from about 0.4 GeV/n to 300 GeV/n where data on secondary isotopes are available. The escape length given by Eq. (1) is close to that presented by Webber *et al*. (1996). It is worth noting that the same escape length (1) satisfactorily reproduces both B/C and Sub-Fe/Fe ratios (no need for path length "truncation").

The physical interpretation of the empirical equation (1) can be given in a framework of the diffusion model. The approximation of a flat-halo diffusion model assumes that cosmic rays diffuse with a diffusion coefficient $D(E)$ in a halo with a half thickness H (Ginzburg, Khazan, & Ptuskin, 1980; Berezinskii *et al*. 1990). The cosmic ray sources and the interstellar gas are concentrated in a relatively thin galactic disk with half thickness $h \ll H$. The flat-halo diffusion model is equivalent to the leaky box model for calculation of abundance of stable nuclei in cosmic rays if the diffusion coefficient is determined by the following equation:

$$D = mbcH / (2X_{lb}) \,, \quad (2)$$

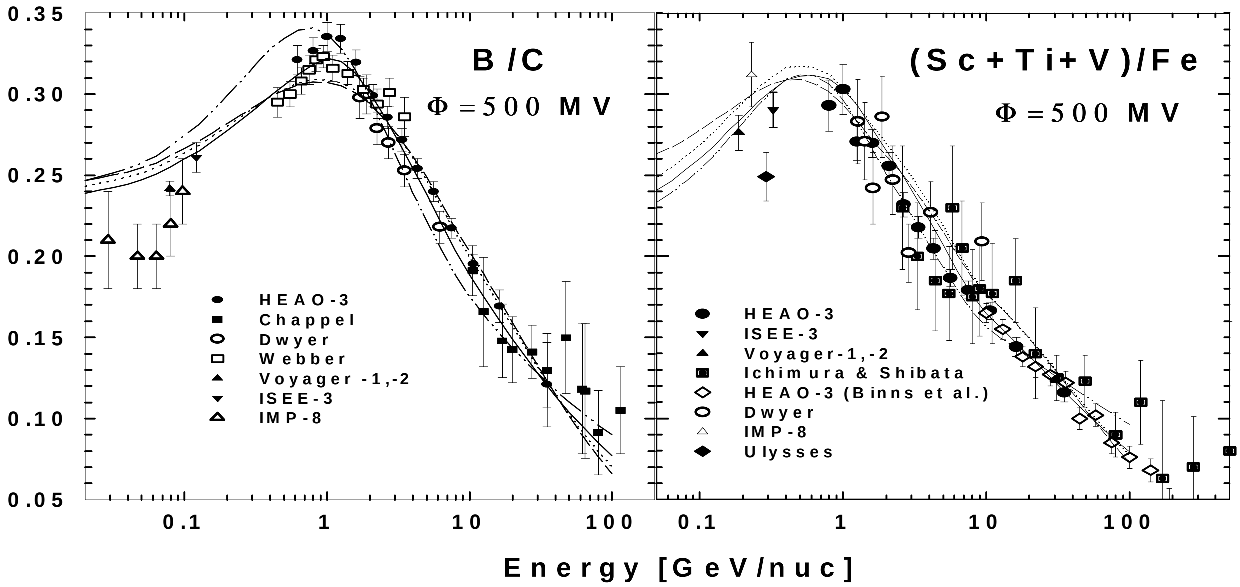

Figure 1: The least squares fit to observed B/C and Sub-Fe/Fe ratios in four propagation models: turbulent diffusion (dashed lines), wind (dotted lines), reacceleration (dash-dotted lines), and the standard diffusion model with the diffusion coefficient given by Eq. (2) (solid lines). Data are from a comprehensive compilation by Stephens & Streitmatter (1998).

where $m \approx 1.9$ mg/cm^2 is the surface mass density of the galactic disk.

The theory of particle resonant scattering and diffusion in the turbulent interstellar medium predicts the scaling of the diffusion coefficient $D_{res} = kbR^a$, where constant k is determined by the level of hydromagnetic turbulence with the spectrum $W_k \propto k^{-2+a}$, a = const. The scattering of particles with Larmor radius r_g is mainly due to the interaction with inhomogeneities of the scale $1/k \sim r_g$.

Eqs. (1), (2) gives $D_{res} = 3.0\cdot10^{28}(H/5\text{kpc})\,(R/1\text{GV})^{0.54}$ cm^2/s at $R > 5$ GV, $b \approx 1$. This implies a particle scattering on the interstellar turbulence with a power law spectrum $W_k \propto 1/k^{1.46}$ at wave numbers $k \geq 1/(3\cdot10^{12}$ cm) assuming that the interstellar magnetic field $B = 5$ μG. Eqs. (1), (2) implies D = const at R < 5GV.

3 Turbulent diffusion.

The energy independent transport at low energies can be naturally provided by the large-scale convective motions of the interstellar medium in the form of turbulent diffusion or regular (wind) flow, see Paper I. The model of turbulent diffusion assumes particle diffusion with the composite diffusion coefficient $D = D_t + D_{res}$. Here D_t is the turbulent diffusion coefficient, which is due to the large scale random gas motions and does not depend on energy; the "microscopic" diffusion coefficient is D_{res}. The escape length is now

$$X_t = bX_0\left(1 + b(R/R_0)^a\right)^{-1}, \qquad X_0 = mcH/(2D_t), \qquad R_0 = \left(D_t/k\right)^{1/a}. \qquad (3)$$

The fit of Eq. (3) to the data on B/C and Sub-Fe/Fe ratios allows us to find $X_0 = 14.1$ g/cm^2, $R_0 = 15$ GV, and $a = 0.85$, see Figure 1 (dashed lines). Thus $D_t = 3.2\cdot10^{28}(H/5\text{kpc})$ cm^2/s and $D_{res} = 3.2\cdot10^{27}(H/5\text{kpc})b(R/1\text{GV})^{0.85}$ cm^2/s. The turbulent diffusion coefficient can be estimated as $D_t = u_t L/3$, where u_t is the characteristic velocity and L is the characteristic scale of the turbulent motions. The magnitude of velocity u_t can hardly exceed 100 km/s that leads to the very large value of $L \geq 2.5(H/5\text{kpc})$ kpc. Thus the model of cosmic ray turbulent transport necessitates D_t that is difficult to reconcile with parameters of the interstellar turbulence.

4 Wind model.

The wind model (Jokipii, 1976; Jones, 1979) with constant wind velocity *u* also gives energy independent particle transport at low energies. The effective escape length in this model is

$$X_w = bX_1\left(1-\exp\left(-b^{-1}\left(R/R_1\right)^{-a}\right)\right), \qquad X_1 = mc/2u, \qquad R_1 = \left(uH/k\right)^{1/a}, \qquad (4)$$

and the specific adiabatic energy loss term is described in Paper I. The set of parameters X_1 = 12.5 g/cm^2, R_1 = 12.0 GV, and a=0.73 gives the fit to B/C and Sub-Fe/Fe ratios, see Figure 1 (dotted lines). This gives u = 24 km/s, D_{res} = 6.1·10^{27}(H/5kpc)b(R/1GV)$^{0.73}$ cm^2/s, the typical values for such kind of models.

5 Minimal reacceleration model.

The distributed reacceleration of cosmic rays after their exit from the compact sources (supernovae remnants) changes the shape of the particle spectra and presents an alternative explanation of the decrease of secondary/primary ratios at small energies (Simon *et al*., 1986). We consider the minimal model where stochastic reacceleration occurs as a result of scattering on randomly moving waves responsible for spatial diffusion (Osborne & Ptuskin, 1988; Seo & Ptukin, 1994). The efficiency of reacceleration is determined by the particle diffusion coefficient on momentum $D_{pp} \sim p^2V_a^2/D_{res}$, where V_a is the Alfven velocity. The parameter $a = C(a)\left(V_a/mc\right)^2$ characterizes the strength of reacceleration (C(1/3) ≈ 2.2 for a Kolmogorov spectrum, see Paper I). To reduce the number of free parameters, we assume that the turbulence has a Kolmogorov type spectrum and thus a = 1/3. The escape length is determined by the particle resonant scattering and is parameterized as

$$X_{re} = X_0\,(R/1GV)^{-1/3}. \qquad (5)$$

The fit to B/C and Sub-Fe/Fe ratios gives a = 1.4·10^{-3} (g/cm^2)$^{-2}$ and X_{re} = 11.5 g/cm^2, see Figure 1 (dashed-dotted lines). This implies the values of V_a = 21 km/s and D_{res} = 3.8·10^{28}b (H/5kpc)(R/1GV)$^{1/3}$ cm^2/s. These parameters are close to ones found by Seo & Ptuskin (1994) and Heinbach & Simon (1995).

6 Source spectrum.

Using the above determined propagation models, we tested different source spectra to fit observed spectra of primary nuclei C, O, and Fe at energies 0.5-100 GeV/n. The best fit is provided by the source spectrum $Q \propto R^{-2.35}$, see Figure 2.

One can expect that asymptotically at very high energies, E > 100 GeV/n, the relation $g = g_s + a$, (g is the exponent of the observed differential spectrum $I(E) \propto E^{-\gamma}$) is fulfilled. For observed g = 2.5 – 2.7 this relation can be provided only in the model with reacceleration where g_s+a = 2.68 (see also Seo & Ptuskin, 1994).

7 Conclusion.

The objective of the present work was the modeling of cosmic ray propagation and nuclear fragmentation in the interstellar medium with the use of the most recent set of spallation cross sections and with the employment of the modified weighted slab method which allows us to obtain an exact solution of the problem. The standard flat-halo diffusion model, the models with turbulent diffusion and wind, and the diffusion model with reacceleration were tested. All these models are able to explain the bend of secondary/primary ratios at energies below a few GeV/n. The turbulent diffusion and the wind transport work simultaneously with resonant diffusion. The last process dominates in cosmic ray transport at high energies. In order to reproduce a sufficiently sharp bend, the diffusion must have strong dependence on rigidity $D_{res} \propto bR^a$, where a = 0.73 - 0.85. However, this leads to a serious problem. The extrapolation of such strong rigidity dependence of diffusion to energies 10^3-10^5 GeV leads to an anisotropy of galactic cosmic rays that is larger than observed anisotropy at these energies. The difficulty arises if a > 1/3 (Ptuskin, 1998). The model with reacceleration is free of this defect and can reproduce the secondary/primary ratio on energy is decreasing with energy and is becoming insignificant when compared

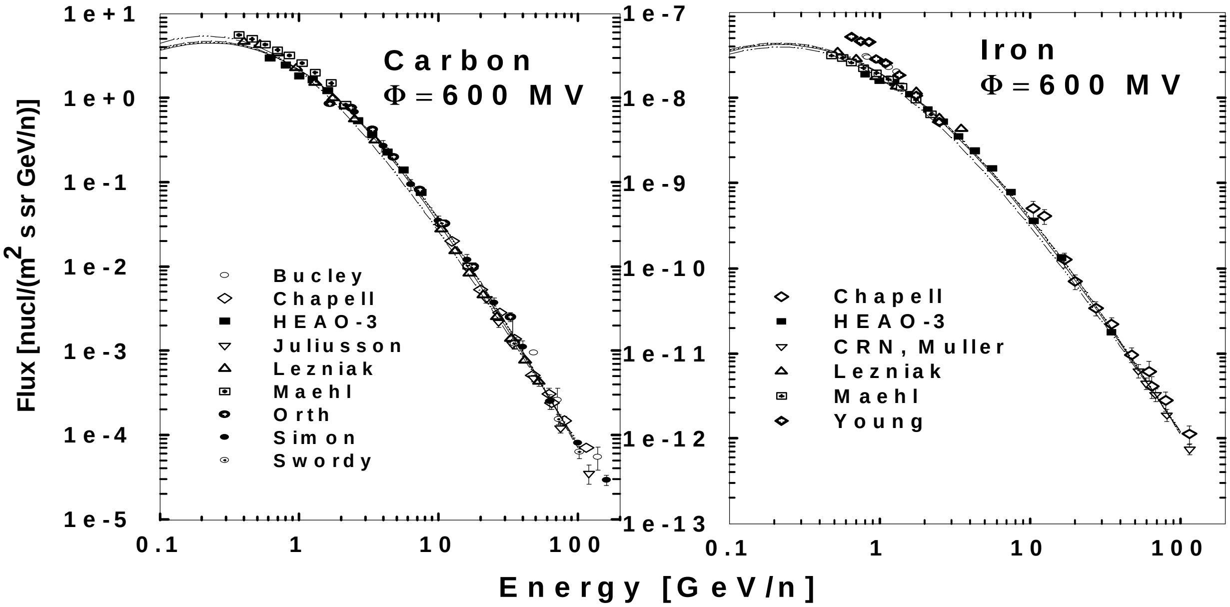

Figure 2: The least squares fit to C and Fe spectra in the same propagation models as indicated in Figure 1.

with the escape from the Galaxy at E > 10-15 GeV/n. The escape with the escape length (5) dominates over reacceleration at high energies.

Our conclusion is that reacceleration model is favoured as regards natural explanation of the energy dependence of particle transport in the Galaxy.

References

Berezinskii, V.S., Bulanov, S.V., Dogiel, V.A., Ginzburg, V.L., & Ptuskin, V.S. 1990, Astrophysics of Cosmic Rays, North Holland

Ginzburg, V.L., Khazan, Y.M., & Ptuskin, V.S. 1980, Astrophys. Space Sci. 68, 295

Heinbach, U. & Simon, M. 1995, ApJ 441, 209

Jokipii, J.R. 1976, ApJ 208, 900

Jones, F.C. 1979, ApJ 229, 747

Jones, F.C., Ptuskin, V.S., Lukasiak, A., & Webber, W.R.1999, this Conference, OG 3.2.07 (Paper I)

Osborne, J.L. & Ptuskin, V.S. 1988, Sov. Astron. Lett. 14, 132

Ptuskin, V.S. 1997, Adv. Space Sci. 19, 697

Ptuskin, V.S., Jones, F.C., &. Ormes, J.F. 1996, ApJ 465, 972

Seo, E.S. & Ptuskin, V.S. 1994, ApJ 431, 705

Simon, M., Heinrich, W., & Mattis, K.D. 1986, ApJ 300, 32

Stephens, S. A. & Streitmatter, R.E. 1998, ApJ 505, 266

Webber, W.R. 1997, Space Sci. Rev. 81, 107

Webber, W.R., Lukasiak, A., McDonald, F.B., & Ferrando, P. 1996, ApJ 457, 435

Webber, W.R., Kish, J.C., Rockstroh, J.M. *et al*. 1998, ApJ 508, 940