Bulk Speeds of Cosmic Rays Resonant with Parallel Plasma Waves

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Abstract

The cold-plasma resonance conditions of cosmic rays with waves propagating parallel to the mean magnetic field are studied. We calculate the phase speeds of the waves resonant with particles of fixed type as a function of particle velocity. This allows us to determine the mean scattering-center speed for given particle species and wave modes as a function of particle momentum. As an application, we discuss the resulting scattering-center compression ratio of a low-Mach-number parallel shock wave for electrons and ions at different energies.

1 Introduction

The quasi-linear interaction of cosmic-ray particles with transverse, parallel-propagating plasma waves occurs via gyro resonance. To interact efficiently with a circularly polarized wave, the particle must gyrate around the mean magnetic field in the same sense and with the same frequency as the electric field of the wave when viewed in the rest frame of the particle's guiding center (GC frame). Augmented with the dispersion relations of the relevant wave modes, this condition determines the wavenumbers and frequencies of the waves resonant with particles of a given type (charge/mass) and velocity. The intensity of the waves at these wavenumbers, in turn, determines how fast the given particle is diffusing in momentum space.

If the particle's guiding center moves much faster than the waves relative to the plasma, one may neglect the plasma-frame wave frequency in the (Doppler-shifted) GC-frame wave frequency and make the so-called magneto-static approximation (e.g., Jokipii 1966). This approximation, however, does not give correct results for particles with pitch angles close to 90°. Since the description of particle scattering in this region determines the fundamental cosmic-ray transport parameter, the spatial diffusion coefficient (Schlickeiser & Miller 1998), one has to abandon the magneto-static approximation at least when computing this parameter from the assumed/observed spectrum of magnetic fluctuations. In this paper, we study the effect of finite phase speeds of the waves on another transport coefficient, the bulk speed of the cosmic rays, which is the effective speed of the waves that scatter the cosmic-ray particles. Dispersive waves, therefore, can give rise to bulk speeds that are dependent on cosmic-ray charge/mass and momentum. We shall study how this affects the scattering-center compression ratio in low-Mach-number parallel shock waves.

2 Dispersion Relation and Resonance Condition

The dispersion relations of parallel transverse waves in a cold electron–proton plasma can be described with the equation (e.g., Steinacker & Miller 1992)

$$k^2\, c^2 = \omega^2 \left[1 + \frac{c^2}{V_{\rm A}^2} \, \frac{\Omega_{\rm e}\Omega_{\rm p}}{(\Omega_{\rm p}-\omega)(\Omega_{\rm e}-\omega)} \right]. \tag{1}$$

where k is the wavenumber and ω the wave frequency, $\Omega_{\rm e[p]} = q_{\rm e[p]}B/(m_{\rm e[p]}c)$ is the signed non-relativistic electron [proton] gyro-frequency, $q_{\rm e(p)}$ and $m_{\rm e[p]}$ are the electron [proton] charge and mass, B is the background magnetic field magnitude, c is the speed of light, and $V_{\rm A} = B[4\pi n_{\rm e}(m_{\rm p}+m_{\rm e})]^{-1/2}$ is the non-relativistic Alfvén speed, $n_{\rm e}$ is the electron density of the plasma. Negative (positive) frequencies denote right (left) handed polarization and the sign of ω/k fixes the propagation direction of the wave relative to the

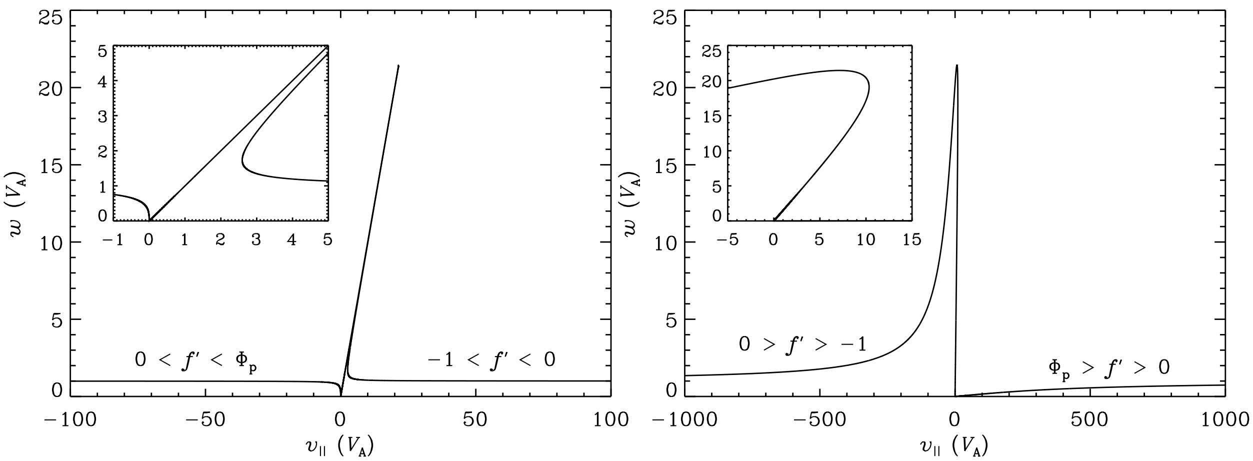

Figure 1: Phase speed, as a function of parallel particle velocity, of parallel-propagating transverse waves resonant with particles having constant dimensionless gyro-frequency of $\Phi = \Phi_{\rm p}$ (left) and $-1/3$ (right).

background magnetic field direction. Assuming that $(V_{\rm A}/c)^2 \ll 4\,\Omega_{\rm p}/|\Omega_{\rm e}| = 0.00218$, we may write the dispersion relation (1) in the dimensionless form

$$\kappa \simeq \pm f \sqrt{\frac{\Phi_{\rm p}}{(\Phi_{\rm p} - f)(1+f)}}, \qquad (2)$$

where $\kappa = kV_{\rm A}/|\Omega_{\rm e}|$, $f = \omega/|\Omega_{\rm e}|$, $\Phi_{\rm p} = \Omega_{\rm p}/|\Omega_{\rm e}| = m_{\rm e}/m_{\rm p} = 1/1836$; the wave frequency takes values between $-1 \leq f \leq \Phi_{\rm p}$; and the sign fixes the wave-propagation direction relative to the background magnetic field. When $|f| \ll \Phi_{\rm p}$, the dispersion relation (2) describes Alfvén waves. At positive frequencies, the Alfvén waves are converted to proton-cyclotron waves as $f \to \Phi_{\rm p}$. At negative frequencies they are first converted to whistlers at $f \sim -\Phi_{\rm p}$ and finally to electron-cyclotron waves as $f \to -1$.

Finally, the gyro-resonance condition between the cosmic rays and the parallel/anti-parallel waves is

$$f' - \kappa' v_{||}/V_{\rm A} = \Phi, \qquad (3)$$

where κ' and f' are the dimensionless resonant wavenumber and frequency, v is the particle speed and $v_{||}$ the particle velocity parallel to the background magnetic field (parallel velocity), $\Phi = qB/(\gamma mc|\Omega_{\rm e}|) = m_{\rm e}q/(\gamma m|q_{\rm e}|)$ is the (signed) dimensionless gyro-frequency, q is the charge, γ is the Lorentz factor, and m is the mass of the cosmic ray.

3 Effective Wave Speed

In the following, we will treat $\Phi_0 = \Phi\gamma = m_{\rm e}q/(m|q_{\rm e}|)$ as constant. Combining equations (2) and (3) allows us to write down an equation for the phase speed $w = V_{\rm A} f'/\kappa'$ of the waves resonant with particles of fixed v as a function of $v_{||}$ in a parametric form

$$w(f') = \pm V_{\rm A}\sqrt{\frac{(\Phi_{\rm p} - f')(1+f')}{\Phi_{\rm p}}} \qquad (4)$$

$$v_{||}(f') = w(f')\left(1 - \frac{\Phi}{f'}\right), \qquad (5)$$

from which an implicit form, i.e., $v_{||} = v_{||}(w)$, may be derived straight-forwardly. In Figure 1 we have plotted the solution (4–5) for two values of Φ corresponding to non-relativistic protons and mildly relativistic electrons. We have indicated what values the wave frequency f' takes in each branch of the curves. The curves are plotted for parallel-propagating waves; for anti-parallel waves, both $v_{||}$ and w change signs for constant f' and a complete figure would include the negative w axes with curves obtained by rotating the plots in Figure 1 about their origins by 180°.

Scattering by waves that all move with the same phase speed, e.g., parallel Alfvén waves, tends to make the particle distribution isotropic in the wave frame, i.e., the coordinate system moving with the phase speed of the waves relative to the plasma. This results in a plasma-frame bulk motion of the cosmic rays with the phase speed of the waves. If waves with several speeds are present the situation is a bit more involved, but the use of quasi-linear theory with the diffusion approximation for cosmic-ray propagation gives the plasma-frame bulk speed of the particles in form (Schlickeiser 1989)

$$V(p) = \frac{1}{3p^2}\frac{\partial}{\partial p}\left[p^3 D(p)\right] \quad (6)$$

$$D(p) = \frac{3v}{4p}\int_{-1}^{+1}(1-\mu^2)\frac{D_{\mu p}}{D_{\mu\mu}}\,d\mu, \quad (7)$$

where p and $\mu = v_{||}/v$ are particle momentum and pitch-angle cosine, and $D_{\mu p} = \frac{1}{2}\left\langle \Delta\mu\,\Delta p\right\rangle/\Delta t$ and $D_{\mu\mu} = \frac{1}{2}\left\langle(\Delta\mu)^2\right\rangle/\Delta t$ are components of the momentum diffusion tensor in the cosmic-ray kinetic equation.

In general, the diffusion tensor components $D_{\mu\mu}(\mu,p)$ and $D_{\mu p}(\mu,p)$ are obtained by taking ensemble averages of the first-order corrections due to wave fields to the helical particle orbit. For parallel cold-plasma waves, they have been calculated by Steinacker & Miller (1992). Let us, however, study how the bulk speed V in equation (6) may be estimated without knowing the detailed form of these coefficients. Let us study the interaction of the particle with a single resonant wave mode with phase speed w. The interaction between the particle and the wave component can be viewed in the wave frame, where the wave's magnetic field is static making the scattering elastic. Thus, we may write the equation $\Delta p' = 0$ for the wave-frame momentum, $p' = p[1 - 2\mu\, w/v + (w/v)^2]^{1/2}$, which leads to

$$\Delta p = \frac{wp}{v-\mu w}\,\Delta\mu,$$

if terms of the order (w/c) are neglected. Thus, in this case, we may write for the ratio of the momentum-diffusion-tensor components

$$\frac{D_{\mu p}}{D_{\mu\mu}} = \frac{\langle\Delta\mu\Delta p\rangle}{\langle(\Delta\mu)^2\rangle} = \frac{wp}{v-\mu w}. \quad (8)$$

If several waves, numbered by α, are scattering the particle with given μ and p, we may write

$$\frac{D_{\mu p}}{D_{\mu\mu}} = \sum_\alpha a_\alpha \frac{w_\alpha p}{v-\mu w_\alpha}, \quad (9)$$

where $a_\alpha = D^\alpha_{\mu\mu}(\sum_\alpha D^\alpha_{\mu\mu})^{-1}$ and $D^\alpha_{\mu\mu} = A_\alpha(1-\mu^2)(v-\mu w_\alpha)^2/|\mu v - w_{\mathrm{g},\alpha}|$ is the pitch-angle diffusion coefficient related to the wave α, w_α and $w_{\mathrm{g},\alpha}$ are the phase and group speeds of the wave α, and $A_\alpha(p)$ is proportional to the power in the magnetic field fluctuations of the wave α. Note that this result agrees with Steinacker & Miller (1992), where both coefficients were calculated using quasi-linear theory directly. Combining the results (8–9) with equations (4–5) allows us to calculate the bulk speed $V(p)$ in (6–7) of the cosmic rays, if the scattering frequencies as a function of wave frequency are specified. In particular, if the spectrum of waves as a function of wavenumber is steep enough, we may approximate that a_α is unity for the resonant wave with the lowest wavenumber and zero for the others.

4 Discussion

In cosmic shock waves, particles can gain energy through first and second order Fermi mechanisms by multiple shock crossings and stochastic downstream acceleration, respectively. When first-order Fermi acceleration dominates, the spectral index of the shock accelerated particles is $\Gamma = (r_k+2)/(r_k-1)$ and, thus, determined by the scattering-center compression ratio of the shock, $r_k(p) = [u_1 + V_1(p)]/[u_2 + V_2(p)]$, where $u_{1[2]}$ is the upstream [downstream] shock-frame flow speed of the plasma and $V_{1[2]}$ is the respective relative bulk speed of particles due to finite phase speed of the waves. Let us examine the effects of the non-zero wave speeds on the first-order Fermi acceleration of cosmic rays. In the upstream region of the shock, we may assume that all waves are propagating against the flow (backward waves, $w < 0$) if they are self-generated by the accelerated particles through the streaming instability. Let us assume that backward waves are generated at all frequencies $-1 < f' < \Phi_{\rm p}$. All upstream waves that have $-w < u_1$ will then be convected to the shock and become downstream waves. In the downstream region, due to the interaction of the upstream waves with the shock, waves propagating in both directions will be present. For Alfvén waves, Vainio & Schlickeiser (1999a, 1999b) showed that the dominating downstream wave components are the backward ones. However, since we assume that the waves are generated in the upstream region, we can not have downstream backward waves propagating faster than the shock relative to the downstream plasma. This implies that we do not have backward waves at frequencies $\frac{1}{2}(-1 + \Phi_{\rm p} - f_0) < f' < \frac{1}{2}(-1 + \Phi_{\rm p} + f_0)$, where $f_0^2 = (1 + \Phi_{\rm p})^2 - 4\Phi_{\rm p} M^2/r$, M is the Alfvénic Mach number of the shock and r is the gas compression ratio of the shock. When the downstream Alfvénic Mach number $M/r^{1/2} > w_{\rm max}/V_{\rm A} = (1 + \Phi_{\rm p})(4\Phi_{\rm p})^{-1/2} \approx 21.4$ all waves are able to propagate in the downstream region. For cold upstream plasma this means that $M > 42.8$ but since we are considering also the downstream modes in the cold-plasma approximation, we must restrict ourselves to small gas compression ratios and shocks with $r^{1/2} \lesssim M < 2$.

As an illuminating example, we shall consider downstream turbulence consisting of (i) Alfvén waves with $|f| \ll \Phi_{\rm p}$ being dominated by the backward propagating waves, (ii) forward whistler waves with $\Phi_{\rm p} \ll f \ll 1$, and (iii) equal intensities of forward and backward waves near the cyclotron frequencies. The last assumption is made since we do not really know how waves with high frequencies and wave numbers interact with shocks and since other wave-generation processes may also be important at high frequencies. For upstream waves we make the assumption that waves at low wavenumber dominate in intensity over the waves at high wavenumber. We assume that all upstream waves are backward waves. Using these assumptions for turbulence near the shock and the results of previous section we conclude the following: (i) up- and downstream bulk speed of the energetic ($v \gg V_{\rm A}$) ions relative to the plasma is close to local Alfvén speed, $V \gtrsim -V_{\rm A}$; (ii) upstream bulk speed of energetic electrons is decreasing with momentum from $V_1 \sim -9\, V_{\rm A}$ at $v \sim 2\, w_{\rm max}$ to $V_1 \gtrsim -V_{\rm A}$ at ultra-relativistic ($\gamma > 200$) energies; (iii) downstream bulk speed of energetic electrons is $V_2 > 0$ at non-relativistic energies and $V_2 \sim -V_{\rm A}$ at ultra-relativistic energies.

Our study reveals that (i) for ions and ultra-relativistic ($E > 100$ MeV) electrons $r_k = r(M-1)/(M + H_{\rm c,2} r^{1/2})$, where the downstream cross-helicity state is close to $H_{\rm c,2} = -1$ and, thus, $\Gamma \sim 1$ (see Vainio & Schlickeiser [1999a, 1999b] for a more detailed discussion); (ii) for less energetic electrons, the first-order Fermi process will be less efficient and will, in fact, turn to deceleration at mildly relativistic or non-relativistic energies. Thus, we expect stochastic acceleration in the downstream region to determine the spectrum of these electrons at the shock.

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