# Non-resonant pitch-angle scattering, and parallel mean-free-path

**B.R. Ragot**
*Laboratory for High Energy Astrophysics, NASA/GSFC, Greenbelt, MD 20771, USA*

**Abstract**

A new, non-resonant pitch-angle scattering process has been found, which very efficiently scatters the cosmic ray particles through the zero pitch-angle cosine. Combined with the effects of gyroresonance and transit-time damping, it can solve the "flatness problem" for the parallel mean-free-path as a function of the particles rigidity. The main lines of its derivation, and the implications on the mean-free-path at low rigidities ($< 10^5$ MV), are summarized here.

## 1 Introduction

The scattering of charged particles through the zero pitch-angle cosine, $\mu$, has long remained a challenging problem in the theory of particles transport, in a magnetic turbulence composed of plasma waves superimposed on a larger-scale, regular magnetic field (see *e.g.*, Bieber et al. 1994; Ragot 1999 and references therein). The quasilinear theory (Vedenov, Velikhov & Sagdeev 1962; Jokipii 1966), which usually describes this problem of particles transport, only includes the resonant interactions between the waves and the particles and, when the real spectral shape of the turbulence is taken into account, can fail at producing any significant scattering through $\mu = 0$ at low particles' rigidities. More sophisticated, nonlinear theories (see references in Ragot 1999) require enhanced levels of fluctuations to achieve this scattering through $\mu = 0$, which are not necessarily observed each time the particles are efficiently scattered. The occurence of this problem of pitch-angle scattering at precisely $\mu = 0$ is, however, somewhat surprising. Indeed, why should the point where $\mu = 0$ always be the critical point, when $\mu$ is defined with respect to the main magnetic field $\vec{B}_0$, while locally, the real field lines are not along $\vec{B}_0$? — If there had to be a problem, given that the particle scattering along field lines is expected to be the fastest process in many situations, the problem should occur when the velocity of a particle crosses the normal to the local magnetic field line direction. — The answer to this problem can be formulated in a quite simple form. In order to correctly describe the pitch-angle scattering through $\mu = 0$, *one must take into account the lower-frequency waves even if they are not in resonance with the particles*, because they determine the local variations of the field line direction.

The frequencies at which resonant interactions can take place between waves and particles depend on the particles' rigidity and the dispersion relation of the waves. It appears that the most dramatic effect — extremely weak scattering and resulting divergence of the parallel mean free path — occurs when the resonant frequencies fall in the dissipation range of the turbulence, leaving the waves of the inertial range out of the wave-particle interaction description, despite the fact that these later, lower-frequency waves are responsible for the local variations of the field line direction. This is the case, in particular, for low-rigidity cosmic rays in the solar wind. They cannot gyroresonate with MHD waves in the inertial range of the turbulence, below a few hundreds of MV when their pitch-angle cosine $\mu$ approaches 0, and below $\approx 1$ MV for any $\mu$. — Gyroresonance between waves of frequency $\omega$ (parallel wavenumber $k_{||}$) and particles of gyrofrequency $|\Omega| = |q|B/(mc\gamma) = (m_p/m)(\Omega_{p_0}/\gamma)$ occurs when the condition: $k_{||}z - \omega_j t \pm n|\Omega|t = 0$ is satisfied for some integer $n \neq 0$, $j = \pm 1$ denoting forward and backward propagating waves. — At small $\mu$, less than the ratio Alfvén speed $V_A$ over particle speed $v$, no transit-time damping (TTD) interaction ($n = 0$ resonance) is possible either, with the fast magnetosonic component of the spectrum. As a consequence, the quasilinear theory, which only takes into account the resonant interactions — gyroresonant and TTD —, predicts a very low pitch-angle scattering, and a very long mean-free-path along the direction of the magnetic field lines.

The original quasilinear prediction (Jokipii 1966) gave a short parallel mean-free-path, but this was due to the absence of cut-off in the turbulence spectrum. Latter measurements in the solar wind (Coroniti et al. 1982;

Denskat et al. 1983) showed a strong steepening of the spectrum above the ion gyrofrequency $\Omega_{p_0} \approx 1\,\mathrm{Hz}$, with a spectral index going from $\approx -1.7$ to $\approx -2.9$, which is responsible for the "divergence" of the mean-free-path below about $10^2$ MV. Beside this problem of cut-off and divergence, the original quasilinear mean-free-path in fact never gave the right dependence of $\lambda$ as a function of $R$. $\lambda$ kept decreasing with decreasing $R$, whereas the observational data show little dependence of $\lambda$ on $R$ for rigidities between $10^{-1}$ and $10^3$ MV, which is known as the "flatness problem".

If it seems relatively clear that the problem of scattering through $\mu = 0$ arises from the exclusion of the lower-frequency waves, the precise, quantitative prediction of the parallel mean free path for solar cosmic rays, and the solution of the "flatness problem", require a more detailled description of the process of wave-particle interaction, and of the turbulence. We do *not* know for sure what the real composition of the turbulence is. But it is likely that the wave turbulence is made of Alfvén and fast-magnetosonic waves, because in a magnetized, low — but finite — $\beta$ plasma ($\beta = 2c_s^2/V_A^2$, $c_s$ being the sound speed) like the solar wind, these two types of waves are the less heavily damped. As for the distributions of $\vec{k}$-vectors for these waves, we have to make some assumptions. We will assume in this paper, as in the papers by Schlickeiser & Miller (1998), Ragot & Schlickeiser (1998) and Ragot (1999), a *slab* Alfvén turbulence (parallel propagating, with $\vec{k}$ along $\vec{B}_0$) and *isotropic* fast magnetosonic waves. We only present, here, the main lines of the derivation of the non-resonant pitch-angle scattering process with these waves, and show that it very efficiently scatters the particles through $\mu = 0$. Detailed calculations for this process can be found in the paper by Ragot (1999), and fits of the parallel mean free path as a function of the rigidity, deduced from measurements for solar cosmic rays, are presented in SH 3.1.24. The case of oblique Alfvén waves is also briefly considered in SH 3.1.24. Here the slab Alfvén waves, for reasons of symmetry, do not contribute to the non-resonant interaction process (see also SH 3.1.24). We thus ignore them in the evaluation of this effect.

## 2 Derivation of the non-resonant scattering process

Gyroresonance is very inefficient at scattering low rigidity cosmic rays, because most of the energy is in the waves that have much *too low frequencies* to be in gyroresonance with these particles. The limit between gyroresonant and non-gyroresonant waves is given, for a linear dispersion relation of the waves $\omega_j = jkV_A$, valid well below $k_c = \Omega_{p_0}/V_A$, by:

$$\frac{k}{k_c} \approx \frac{1}{R}\frac{\varepsilon_A}{\max(|\eta\mu|,\varepsilon)}\frac{m_pc^2}{Ze} \tag{1}$$

which can be easily shown, for $\mu < \varepsilon$, to be larger than 1 as soon as $R < 940[1 + (mc^2/(RZe))^2]^{-1/2}$ MV. $Z$ is the number of charge of the particle, $\varepsilon = V_A/v$ and $\varepsilon_A = V_A/c$. Turning this into a positive statement, we can say that the gyroperiod $\Omega^{-1}$ of low rigidity cosmic rays is the *shortest timescale* of this problem of wave-particle interaction. Consequently, the *equation of motion* for a particle of momentum $\vec{p}$, Lorentz factor $\gamma$, mass $m$ and charge $q$:

$$\frac{d\vec{p}}{dt} = q\delta\vec{E} + \frac{q}{mc\gamma}\vec{p} \times (\vec{B}_0 + \delta\vec{B}) \ , \tag{2}$$

*can be averaged* on the short timescale $\Omega^{-1}$ (for a detailed justification of this averaging, see Ragot 1999). For our fast magnetosonic waves (hereafter denoted by FMWs) turbulence:

$$\delta\vec{B} = \sum_j \int d\vec{k}\,\delta B^j_{\vec{k}} \cos\psi_{\vec{k}} \begin{pmatrix} -\eta\cos\phi_{\vec{k}} \\ -\eta\sin\phi_{\vec{k}} \\ \sqrt{1-\eta^2} \end{pmatrix} \quad \text{and} \quad \delta\vec{E} = \sum_j \int d\vec{k}\,\frac{\omega^j}{kc}\delta B^j_{\vec{k}} \cos\psi_{\vec{k}} \begin{pmatrix} -\sin\phi_{\vec{k}} \\ \cos\phi_{\vec{k}} \\ 0 \end{pmatrix} \ , \tag{3}$$

with $\psi_{\vec{k}} = \vec{k}.\vec{x} - \omega_{\vec{k}}t + \alpha_{\vec{k}}$, $k$ the norm of the wavevector, $\eta$ the cosine of the angle between $\vec{k}$ and $\vec{B}_0$, and $\phi_{\vec{k}}$ the angle between $\vec{k}$ and the plane $(x,z)$, the $z$-axis being along $\vec{B}_0$. Once averaged, the equation for the pitch-angle cosine:

$$\dot{\mu} = \frac{\Omega_0}{\gamma} \sin\theta \sum_j \int d\vec{k}\, \frac{\delta B^j_{\vec{k}}}{B_0} \left( \eta - \frac{\mu m \gamma \omega^j}{pk} \right) \cos\psi^j_{\vec{k}} \sin(\varphi - \phi_{\vec{k}}) \tag{4}$$

becomes

$$\dot{\mu} = -\frac{\Omega_0}{\gamma} \sin\theta \sum_j \int d\vec{k}\, \frac{\delta B^j_{\vec{k}}}{B_0} \left( \eta - \frac{\mu \omega^j}{vk} \right) J_1 \left( \frac{v\sqrt{1-\mu^2}\gamma}{|\Omega_0|} k_\perp \right) \sin\left( k\eta v \int dt\, \mu - \omega^j_{\vec{k}} t + \alpha_{\vec{k}} \right) \,, \tag{5}$$

in terms of the "constant" speed $v$ and Lorentz factor $\gamma$. $\varphi$ denotes the gyrophase of the particle, and $p$ and $\Omega_0$, the norm of its momentum and non-relativistic gyrofrequency, respectively. Integrating the time-averaged equation (5) on a timescale short enough to keep the particles rigidity constant, and in the range of pitch-angles $\mu < \varepsilon = V_A/v$ where the scattering by resonant processes is known to be the most deficient, we can show that the particles are in fact linearly pushed out of the small-$\mu$ range by the low-frequency waves. Indeed,

$$\mu - \mu_0 \approx -\frac{\Omega_0}{\gamma}(t-t_0) \sum_j \int d\vec{k} \frac{\delta B^j_{\vec{k}}}{B_0} \eta J_1 \left( \frac{\gamma}{\varepsilon} \frac{m}{m_p} \frac{k}{k_c} \sqrt{1-\eta^2} \right) \sin(\alpha'^j_{\vec{k}}) \,, \tag{6}$$

with $\alpha'_{\vec{k}} = \alpha_{\vec{k}} + (\eta\mu/\varepsilon - j) k V_A t_0$ constant on the time interval $[t_0, t]$, if we only keep in the spectrum the wavenumbers smaller than $K_M = \min(K, \Delta k_F)$, $K$ being the largest wavenumber satisfying the condition: $t - t_0 \ll (\pi/2)(k_c/K)(1/\Omega_{p_0})$, and $\Delta k_F$ the actual width of the FMW spectrum. The contribution to the variation of $\mu$ from the wavenumbers larger than $K$ is negligibly small, because it is given by the integral of an oscillating sine function of time and $k$. We still have to check that there exists a time interval $t - t_0$ such that $K$ is larger than the lower boundary of the spectrum, and $\mu - \mu_0 \sim \varepsilon$, *i.e.*, that the relation (6) holds until the particles leave the small-$\mu$ range. Once they have reached the boundary $\varepsilon$, they are efficiently scattered away by the transit-time damping interaction with the FMWs (see Schlickeiser & Miller 1998, Ragot 1999).

Averaging on many successive passages through this small-$\mu$ region, *i.e.*, over the phases $\alpha'_{\vec{k}}$, we can estimate the average exit time $\tau$ , and an equivalent "diffusion" coefficient $D_{\mu\mu_{nr}} = \varepsilon^2/(2\tau)$. Assuming that the spectrum is a simple power law, *i.e.* $\propto k^{-q}$, above $k_m$, we can write:

$$\tau \approx \frac{\varepsilon\gamma}{\sqrt{q-1}\delta b \Omega_0} \left( \frac{\varepsilon}{\gamma} \frac{m_p}{m} \frac{k_c}{k_m} \right)^{(q-1)/2} (I_M - I_m)^{-1/2} \,, \tag{7}$$

with

$$I_a = \int_0^1 d\eta\, \eta^2 \sqrt{1-\eta^2}^{\,q-1} \int_0^{Z_a\sqrt{1-\eta^2}} dZ\, Z^{-q} J_1^2(Z) \,, \tag{8}$$

and

$$(Z_m, Z_M) = \frac{\gamma}{\varepsilon} \frac{m}{m_p} \frac{(k_m, K_M)}{k_c} = R \frac{Ze}{m_p c^2} \frac{(k_m, K_M)}{\varepsilon_A k_c} \,. \tag{9}$$

The corresponding parallel mean-free-path reads:

$$\lambda_{nr} \approx \frac{3V_A}{4D_{\mu\mu_{nr}}(|\mu| < \varepsilon)} \approx \frac{3}{2} \frac{V_A \tau(K_M(R))}{\varepsilon^2} \approx \frac{3}{2k_c} \frac{\Omega_{p_0} \tau(K_M(R))}{\varepsilon_A^2 \left[ 1 + \left( \frac{mc^2}{RZe} \right)^2 \right]} \,, \tag{10}$$

where we have assumed that the slowest scattering process still occurs at small $\mu$.

The precise expression for $I_a$ can be found in the paper by Ragot (1999). Here, we want to point out that, despite the apparent dependence of the average exit time $\tau$ on the lowest wave number $k_m$, which is very badly known, there is no problem of lower cut-off value. As we show in Fig. 1, the lowest wavenumbers only give a negligible contribution at low rigidities, and the scattering process is dominated by higher and higher frequencies as the rigidity of the particles decreases.

## 3 Resulting mean free path, and conclusion

Comparing the mean-free-path $\lambda_{nr}$ with the one resulting from gyroresonance with slab Alfvén waves, we show in Fig. 2 that the non-resonant scattering process becomes much more efficient than the gyroresonant one below about $10^2$ MV, *even* in the absence of spectral cut-off. Further comparaison with the mean-free-path derived from the smaller pitch-angles indicates that the slowest scattering process does not occur around $\mu = 0$ any longer for rigidities less than $\approx 1$ MV. This again results from the upper steepening of the spectrum. Indeed, when the particles' rigidity is really low, gyroresonance also becomes impossible around $|\mu| = 1$. As the transit-time damping interaction, which is due to the compressive component of the magnetic field — along $\vec{B}_0$ — , is very inefficient at these large $|\mu|$, it produces a relatively large mean-free-path. Note that this mean-free-path is constant below $\approx 10^{-1}$ MV (see Ragot 1999 for the derivation).

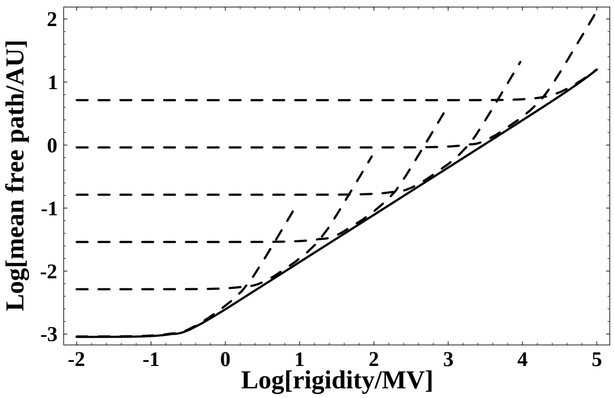


**Figure 1:** *Logarithm of the nonresonant mean free path $\lambda_{nr}$ normalized to 1 AU, for $\Delta k_F = k_c$. The contribution from each "decade" of the spectrum is given in dashed line starting, in the top of the figure, with the wavenumber intervale $10^{-6}k_c$ to $10^{-5}k_c$.*

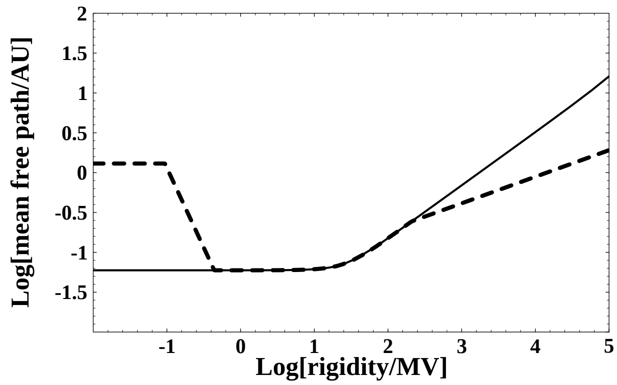


**Figure 2:** *Mean free path $\lambda$, for $\Delta k_F = k_c/100$ and $\Delta k_A \leq k_c$. Continuous line: $\lambda_{nr}$; thick, dashed line: effective $\lambda$. Above a few 100 MV, $\lambda$ results from gyroresonance at $\mu < \varepsilon$ with Alfvén waves. Below 1MV, for electrons, it is determined by TTD at $\mu \geq \varepsilon$.*

We can reproduce the main features of the parallel mean free path as a function of the particles rigidity — for electrons, down to 1 keV, and for protons, down to 1 or 10 MV. The flatness of the curve between 1 MV and $10^3$ MV seems to require a steepening of the fast mode component of the turbulence spectrum above about $10^{-2}k_c$. This is, in presence of low-energy cosmic rays, plausible, given that the fast magnetosonic waves in the range $[10^{-2}k_c, k_c]$ give the main contribution to the transit-time damping acceleration process.

**Acknowledgements:** This work was performed while the author held a National Research Council - NASA / GSFC Research Associateship. B.R.R. gratefully acknowledges interesting discussions with A. Roberts, M. Lee and F. Jones.

## References


Bieber, J., Matthaeus, W., Smith, C., Wanner, W., Kallenrode, M.-B. & Wibberenz, G. 1994, ApJ, 420, 294

Coroniti, F. V., Kennel, C. F., Scarf, F. L. & Smith, E. J. 1982, J. Geophys. Res., 87, 6029

Denskat, K. U., Beinroth, H. J. & Neubauer, F. M. 1983, J. Geophys. Res., 54, 60

Jokipii, J. R. 1966, ApJ, 146, 480

Ragot, B. R. & Schlickeiser, R. 1998a, A&A, 331, 1066; and 1998b, Astrop. Phys., 9, 79

Schlickeiser, R. & Miller, J. A. 1998, ApJ, 492, 352

Ragot, B. R. 1999, ApJ, in press

Vedenov, A. A., Velikhov, E. P. & Sagdeev, R. Z. 1962, Nucl. Fusion Suppl., 2, 465