On "box" models of shock acceleration and electron synchrotron spectra

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Abstract

We examine the simplified "box" models of shock acceleration and present a more physical version. We determine simple criteria for the conditions under which "pile-ups" can occur in shock accelerated electron spectra subject to synchrotron or inverse Compton losses (the latter in the Thompson limit). An extension to nonlinear effects is proposed.

1 Introduction

Many authors (Bogdan & Völk, 1983; Moraal & Axford, 1983; Lagage & Cesarsky, 1983; Schlickeiser, 1984; Völk & Biermann, 1988; Ball & Kirk, 1992; Protheroe & Stanev, 1998) have used, under various guises, a simplified but physically intuitive treatment of shock acceleration, sometimes referred to as a "box" model. In this paper we present an alternative more physical interpretation of the "box" model which can be significantly different when additional loss processes, such as synchrotron or inverse Compton losses, are included. A fuller account with more extensive discussion will appear in Astronomy and Astrophysics.

The main features of the "box" model, as presented in the literature (see references above) and exemplified by Protheroe and Stanev (1998) can be summarised as follows. The particles being accelerated (and thus "inside the box") have differential energy spectrum $N(E)$ and are gaining energy at rate $r_{\rm acc}E$ but simultaneously escape from the acceleration box at rate $r_{\rm esc}$. Conservation of particles then requires

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial E}\left(r_{\rm acc} E N\right) = Q - r_{\rm esc} N \tag{1}$$

where $Q(E)$ is a source term combining advection of particles into the box and direct injection inside the box. In essence this approach tries to reduce the entire acceleration physics to a "black box" characterised simply by just two rates, $r_{\rm esc}$ and $r_{\rm acc}$. These rates have of course to be taken from more detailed theories of shock acceleration (eg Drury, 1991).

2 Physical interpretation of the box model

We prefer a very similar, but more physical, picture of shock acceleration which has the advantage of being more closely linked to the conventional theory. For this reason we also choose to work in terms of particle momentum p and the distribution function $f(p)$ rather than E and $N(E)$. If we have an almost isotropic distribution $f(p)$ at the shock front where the frame velocity changes from $\mathbf{U_1}$ to $\mathbf{U_2}$, then it is easy to calculate that there is a flux of particles upwards in momentum associated with the shock crossings of

$$\Phi(p,t) = \int p\frac{\mathbf{v}\cdot(\mathbf{U_1}-\mathbf{U_2})}{\mathbf{v^2}} p^2 f(p,t)\mathbf{v}\cdot\mathbf{n}\, d\Omega = \frac{4\pi p^3}{3} f(p,t)\mathbf{n}\cdot(\mathbf{U_1}-\mathbf{U_2}) \tag{2}$$

where $\mathbf{n}$ is the unit shock normal and the integration is over all directions of the velocity vector $\mathbf{v}$. Notice that this flux is localised in space at the shock front and is strictly positive for a compressive shock structure; in our description it replaces the acceleration rate $r_{\rm acc}$.

The other key element is the loss of particles from the shock by advection downstream. We note that the particles interacting with the shock are those located within one diffusion length of the shock. Particles

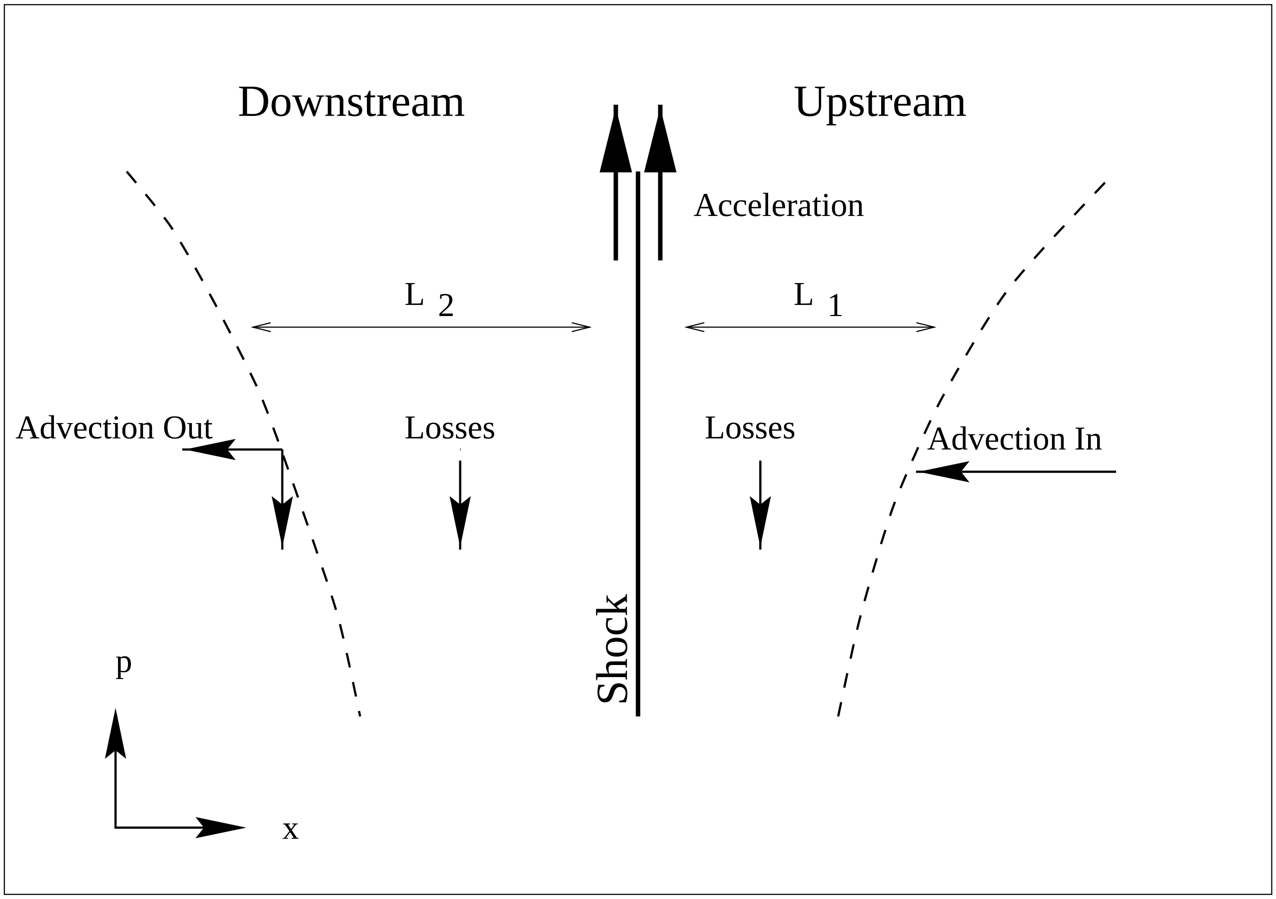

Figure 1: A graphical representation in the x, p plane of our model.

penetrate upstream a distance of order $L_1 = \mathbf{n} \cdot \mathbf{K_1} \cdot \mathbf{n}/\mathbf{n} \cdot \mathbf{U_1}$ where $\mathbf{K}$ is the diffusion tensor and the probability of a downstream particle returning to the shock decreases exponentially with a scale length of $L_2 = \mathbf{n} \cdot \mathbf{K_2} \cdot \mathbf{n}/\mathbf{n} \cdot \mathbf{U_2}$. Thus in our picture we have an energy dependent acceleration region extending a distance L_1 upstream and L_2 downstream. The total size of the box is then $L(p) \equiv L_1(p) + L_2(p)$. Particles are swept out of this region by the downstream flow at a bulk velocity $\mathbf{n} \cdot \mathbf{U_2}$.

Conservation of particles then leads to the following approximate description of the acceleration,

$$\frac{\partial}{\partial t}\left[4\pi p^2 f L\right] + \frac{\partial \Phi}{\partial p} = Q - \mathbf{n} \cdot \mathbf{U_2} 4\pi p^2 f. \tag{3}$$

3 Inclusion of additional loss processes

It is relatively straightforward to include losses of the synchrotron or inverse Compton type (Thomson regime) in the model. These generate a downward flux in momentum space, but one which is distributed throughout the acceleration region. Combined with the fact that the size of the “box” or region normally increases with energy this also gives an *additional* loss process because particles can now “fall” through the back of the “box” as well as being advected out of it. Note that particles which “fall” through the front of the box are advected back into the acceleration region and thus this process does not work upstream. This is shown schematically in Fig 1.

If the loss rate is $\dot{p} = -\alpha p^2$ (the generalisation to different loss rates upstream and downstream is trivial) the basic equation becomes

$$\frac{\partial}{\partial t}\left[4\pi p^2 f L\right] + \frac{\partial}{\partial p}\left[\Phi - 4\pi p^4 f(p)\alpha L\right] = Q - U_2 4\pi p^2 f(p) - 4\pi\alpha p^4 f(p)\frac{dL_2}{dp} \tag{4}$$

In the steady state and away from the source region this gives immediately the remarkably simple result for the logarithmic slope of the spectrum,

$$\frac{\partial \ln f}{\partial \ln p} = -3\frac{U_1 - 4\alpha p L - \alpha p^2 \dfrac{dL_1}{dp}}{U_1 - U_2 - 3\alpha p L}. \tag{5}$$

The denominator goes to zero at the critical momentum

$$p^* = \frac{U_1 - U_2}{3\alpha L} \tag{6}$$

where the losses exactly balance the acceleration. If the numerator at this point is negative, the slope goes to $-\infty$ and there is no pile-up. However the slope goes to $+\infty$ and a pile-up occurs if

$$U_1 - 4U_2 + 3\alpha p^2 \frac{dL_1}{dp} > 0 \qquad \text{at} \qquad p = p^*. \tag{7}$$

In early analytic work (Webb et al, 1984; Bregman et al, 1981) the diffusion coefficient was taken to be constant, so that $dL_1/dp = 0$ and this condition reduces to $U_1 > 4U_2$. However if, as in the work of Protheroe and Stanev, the diffusion coefficient is an increasing function of energy or momentum, the condition becomes less restrictive. For a power-law dependence of the form $K \propto p^\delta$ the condition for a pile-up to occur reduces to

$$U_1 - 4U_2 + \delta\left(U_1 - U_2\right)\frac{L_1}{L_1 + L_2} > 0 \tag{8}$$

(The equivalent criterion for the model used by Protheroe and Stanev is slightly different, namely

$$U_1 - 4U_2 + \delta\left(U_1 - U_2\right) > 0 \tag{9}$$

because of their neglect of the additional loss process.)

For the case where $L_1/L_2 = U_2/U_1$ and with $\delta = 1$ this condition predicts that shocks with compression ratios greater than about $r = 3.45$ will produce pile-ups while weaker shocks will not.

4 Nonlinear effects

At the phenomenological and simplified level of the “box” models it is possible to allow for nonlinear effects by replacing the upstream velocity with an effective momentum-dependent velocity $U_1(p)$, reflecting the existence of an extended upstream shock precursor region sampled on different length scales by particles of different energies. With a momentum-dependent U_1 the logarithmic slope of the spectrum is

$$\frac{\partial \ln f}{\partial \ln p} = -3\frac{U_1 - 4\alpha p L + \dfrac{p}{3}\dfrac{dU_1}{dp} - \alpha p^2 \dfrac{dL_1}{dp}}{U_1 - U_2 - 3\alpha p L} \tag{10}$$

with a pile-up criterion of,

$$U_1(p) - 4U_2 - p\frac{dU_1}{dp} + 3\alpha_1 p^2 \frac{dL_1}{dp} > 0 \qquad \text{at} \qquad p = p^* \tag{11}$$

We see that whether or not the nonlinear effects assist the formation of pile-ups depends critically on how fast they make the effective upstream velocity vary as a function of p. By making $U_1(p^*)$ larger they make it easier for pile-ups to occur. On the other hand, if the variation is more rapid than $U_1 \propto p$, the derivative term dominates and inhibits the formation of pile-ups. If the electrons are test-particles in a shock strongly modified by proton acceleration, and if the Malkov (1998) scaling $U_1 \propto p^{1/2}$ holds even approximately, then a strong synchrotron pile-up appears inevitable (unless the maximum attainable momentum is limited by other effects to a value less than p^*).

5 Conclusion

A major defect of all "box" models is the basic assumption that all particles gain and loose energy at exactly the same rate. It is clear physically that there are very large fluctuations in the amount of time particles spend in the upstream and downstream regions between shock crossings, and thus correspondingly large fluctuations in the amount of energy lost. The effect of these variations will be to smear out the artificially sharp pile-ups predicted by the simple "box" models. However our results are based simply on the scaling with energy of the various gain and loss processes together with the size of the acceleration region. Thus they should be relatively robust and we expect that even if there is no sharp spike, the spectrum will show local enhancements over what it would have been in the absence of the synchrotron or IC losses in those cases where our criterion is satisfied.

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