Self-consistent Generation of Flat Power-law Particle Momentum Spectra by Diffusive Shock Acceleration

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Abstract

We study the interaction of Alfvén waves with a fast parallel MHD shock wave. We calculate the wave transmission coefficients self-consistently, i.e., taking into account the pressure and energy flux of the waves in the Rankine–Hugoniot conditions when determining the shock's gas compression ratio. We find that downstream waves propagating against the flow are dominating in intensity over the waves propagating along the flow direction for a large set of upstream parameters. At low-Mach-number shocks, this leads to a high scattering-center compression ratio and, therefore, to a flat (spectral index < 2) power-law momentum spectrum of particles accelerated by the first-order Fermi mechanism in the shock.

1 Introduction

The original shock wave acceleration theory in its simplest test-particle form is not in accord with the observed flat particle spectra in shell-type supernova remnants and bright spiral galaxies (e.g., Lerche 1980; Drury 1983; Dröge et al. 1987). According to the standard theory the particle spectral index value $\Gamma_{\rm gas} = (r + 2)/(r - 1)$ is closely tied to the gas compression ratio r. Because this gas compression ratio for a super-Alfvénic shock wave propagating in a nonrelativistic adiabatic medium is limited to values ≤ 4, particle spectral indices less than 2 cannot be accounted for in contrast to the observations.

However, because cosmic shock waves are collisionless, it is not the gas compresssion ratio but rather the scattering center compression ratio r_k, that determines the spectral index of the power law momentum spectrum of the accelerated cosmic ray particles. And, in general these two compression ratios are not the same. The principal difference between the gas compression ratio and the scattering center compression ratio, being equivalent to the difference between the effective plasma wave velocity and the gas velocity, and the possible consequences for the spectral index of the differential momentum spectrum of accelerated particles, has been already noted by Bell (1978), although he did no quantitative calculations of this effect. By calculating the correct transmission coefficients of Alfvén waves through the parallel shock from the Rankine-Hugoniot continuity equations, here we will demonstrate that precisely this effect can account for the generation of particle spectral indices flatter than $\Gamma = 2$.

2 Self-consistent shock equation

We follow earlier work by McKenzie and Westphal (1969) and Scholer and Belcher (1971) to calculate anew the transmission of small-amplitude parallel-moving Alfvén waves through a parallel super-Alfvénic shock. We use the equations for

the continuity of the transverse momentum

$$\left[\rho u_n \vec{u}_t - \frac{B_n \vec{B}_t}{4\pi}\right] = 0, \tag{1}$$

where the shock bracket $[X] \equiv X_1 - X_2$ denotes the difference of the upstream (index 1) and downstream (index 2) value of the physical quantity X, u_n (B_n) and $\vec{u}_t$ ($\vec{B}_t$) are the gas flow velocity (magnetic field) components normal and tangential to the shock plane, respectively;

the continuity of the normal magnetic field

$$B_{n,1} = B_{n,2} = B_0; \tag{2}$$

the continuity of the tangential electric field

$$[u_n \vec{B}_t - B_n \vec{u}_t] = 0; \tag{3}$$

and the continuity of the mass flux

$$[\rho u_n] = 0, \tag{4}$$

with the Alfvén wave properties of the different relation of velocity and magnetic field fluctuations for forward (f) and backward (b) moving waves

$$\delta \vec{u}^{\mathrm{f}} = -\frac{\delta \vec{B}^{\mathrm{f}}}{(4\pi\rho)^{1/2}}, \quad \delta \vec{u}^{\mathrm{b}} = \frac{\delta \vec{B}^{\mathrm{b}}}{(4\pi\rho)^{1/2}}. \tag{5}$$

To arrive at a complete set of equations for the downstream values, i.e., to be able to determine the gas compression ratio $r = \rho_2/\rho_1 = u_{n,1}/u_{n,2}$ of the shock, Eqs. (1–5) must be appended by yet two equations (e.g., Boyd and Sanderson 1969) describing the continuity of the normal momentum

$$\left[\rho u_n^2 + P + \frac{B_t^2}{8\pi}\right] = 0 \tag{6}$$

and energy flux (for an adiabatic equation of state, $P\rho^{-\gamma_g} =$ const.)

$$\left[\frac{1}{2}\rho u_n (u_n^2 + u_t^2) + \frac{\gamma_g P u_n}{\gamma_g - 1} + \frac{u_n B_t^2}{4\pi} - \frac{B_n(\vec{u}_t \cdot \vec{B}_t)}{4\pi}\right] = 0, \tag{7}$$

respectively. For Alfvén waves propagating along the normal of a parallel shock wave the respective normal and tangential components are

$$B_n = B_0, \;\; u_n = u, \;\; \vec{B}_t = \delta\vec{B} = \delta\vec{B}^{\mathrm{f}} + \delta\vec{B}^{\mathrm{b}}, \;\; \vec{u}_t = -(\delta\vec{B}^{\mathrm{f}} - \delta\vec{B}^{\mathrm{b}})/(4\pi\rho)^{1/2}.$$

The combination of these continuity equations (for details see Vainio and Schlickeiser 1999) yields a general cubic shock equation for the gas compression ratio r

$$\begin{aligned} S(r) \;\; \equiv \;\; & (M^2 - r)^2 \left[2r\beta - M^2[\gamma_g + 1 - (\gamma_g - 1)r]\right] \\ & + b^2 M^2 r[(\gamma_g - 1)r^2 + [M^2(2 - \gamma_g) - (\gamma_g + 1)]r + \gamma_g M^2] = 0, \end{aligned} \tag{8}$$

where $M = u_1/V_{A,1}$ is the upstream Alfvénic Mach number of the shock, $\beta = c_{s,1}^2/V_{A,1}^2$ the upstream plasma beta, and $b = \delta B_1/B_0$ the normalised upstream wave magnetic field.

If no Alfvén waves are present ($b = 0$), the shock's cubic immediately reduces to the parallel-shock solution, $r = (\gamma_g + 1)/(\gamma_g - 1 + 2\beta M^{-2})$, and the switch-on shock solution, $r = M^2$. The self-consistent inclusion of finite wave pressure effects ($b \neq 0$) generalises and combines these two special asymptotic shock solutions. The shock's cubic (4) can be solved in parametric form. For $\gamma_g = 5/3$,

$$M^2 = (1 + y) r(y) \tag{9}$$

$$r(y) = \frac{8y^2(y+1) - 6\beta y^2 - b^2(y+1)(5y-3)}{2y^2(y+1) + b^2(y+1)(y+3)}, \tag{10}$$

and the parameter y runs between

$$\frac{\beta - 1 + b^2 + \sqrt{(\beta + 1 + b^2)^2 - 4\beta}}{2} < y < \infty. \tag{11}$$

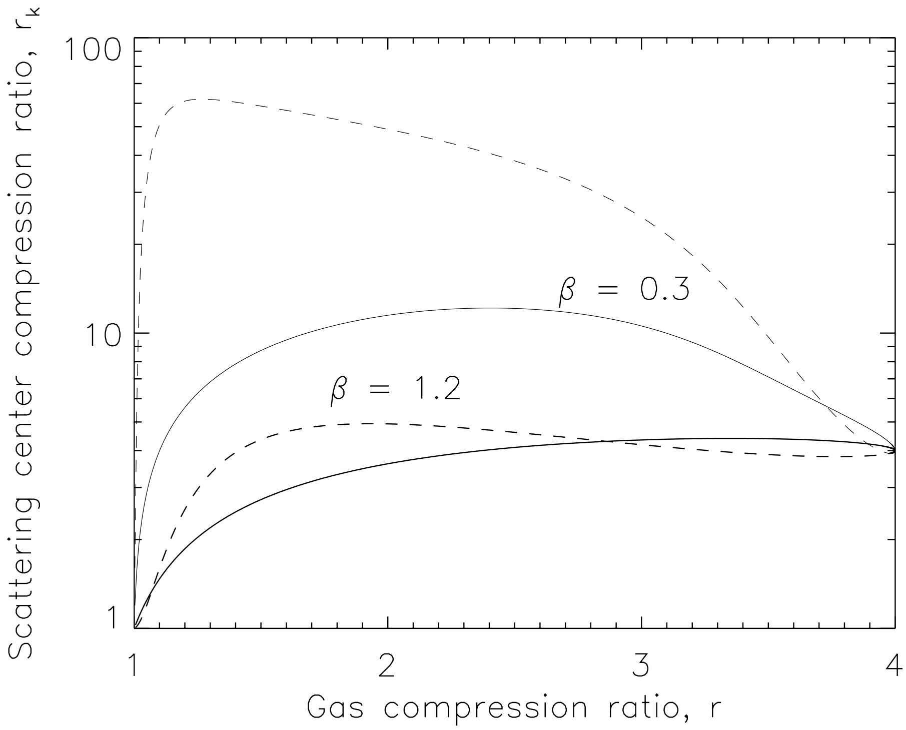

Figure 1: Scattering center compression ratio for an adiabatic shock with a constant upstream beta, where the Alfvén wave normal momentum and energy flux are taken into account in deriving the shock's gas compression ratio. Dashed and solid lines give the results for $H_{c,1} = +1$ and -1, respectively. The magnetic amplitude of the upstream waves is $\delta B_1/B_0 = 0.1$.

3 Results

From this solution the downstream electromagnetic field properties can be calculated from the specified upstream electromagnetic field. In particular, specifying the upstream Alfvén wave cross helicity state $H_{c,1} \equiv [(\delta B^{\rm f})^2 - (\delta B^{\rm b})^2]/[(\delta B^{\rm f})^2 + (\delta B^{\rm b})^2]$ so that the upstream cosmic ray bulk speed is $V_1 = u_1 + H_{c,1}V_{A,1}$, we calculate the resulting downstream cosmic ray bulk speed $V_2 = u_2 + H_{c,2}V_{A,2}$. This immediately yields the scattering center compression ratio

$$r_k \equiv \frac{V_1}{V_2} = r\frac{M + H_{c,1}}{M + r^{1/2}H_{c,2}}, \tag{12}$$

which in general is different from the gas compression ratio r.

In Fig. 1 we show the calculated scattering center compression ratio as a function of the gas compression ratio for four different specified upstream states and an adiabatic index $\gamma_g = 5/3$. In all cases, the scattering center compression ratio r_k differs significantly from the gas compression ratio r. In particular, for low upstream plasma beta very large values $r_k \gg 4$ are possible, whereas the gas compression ratio value is limited to $r \leq 4$ for $M \to \infty$.

When downstream momentum diffusion is neglected, the particle differential energy spectrum at the shock is — up to a cut off determined by losses, particle escape, and finite acceleration time — a power law in momentum

$$dJ/dE \propto p^{-\Gamma}, \quad \Gamma = \frac{r_k + 2}{r_k - 1}, \tag{13}$$

whose spectral index Γ is solely determined by r_k. In Fig. 2 this spectral index is calculated using the results from Fig. 1 and plotted against the spectral index of the conventional theory, $\Gamma_{\rm gas} = (r+2)/(r-1)$. As a

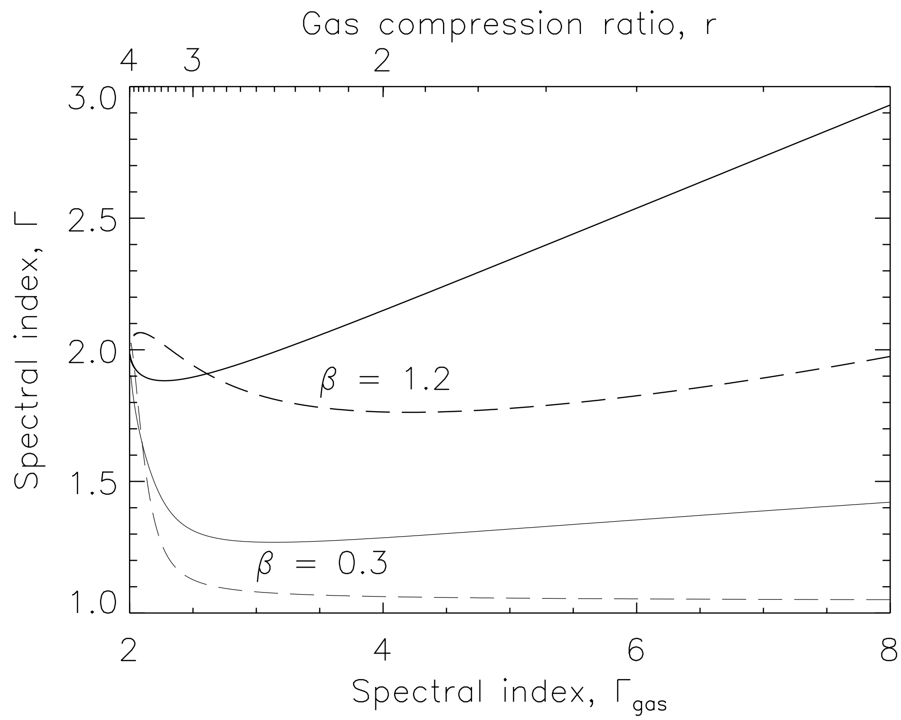

Figure 2: Cosmic ray particle spectral index produced by a shock with a constant upstream plasma beta, neglecting stochastic acceleration in the downstream region. Dashed and solid lines give the results for $H_{c,1} = +1$ and -1, respectively. The magnetic amplitude of the upstream waves is $\delta B_1/B_0 = 0.1$.

consequence of the large scattering center compression ratio values the particle spectral index Γ can become much smaller than 2 approaching the limiting value $\Gamma \to 1$ for very large r_k. Thus, the model is able to generate particle power law spectra harder than the originally limiting value $\Gamma = 2$, and avoids the discrepancy that the original shock wave acceleration theory in its simplest test-particle form is not in accord with the observed flat particle spectra in shell-type supernova remnants and bright spiral galaxies as inferred from the synchrotron radiation of the accelerated relativistic electrons.

References

Bell, A. R. 1978, MNRAS 182, 147

Boyd, T. J. M., & Sanderson, J. J. 1969, Plasma Dynamics (London: Nelson and Son)

Dröge, W., Lerche, I., & Schlickeiser, R. 1987, A&A 178, 252

Drury, L. O'C. 1983, Space Sci. Rev. 36, 57

Lerche, I. 1980, A&A 85, 141

McKenzie, J. K., & Westphal, K. O. 1969, Planet. Space Sci. 17, 1029

Scholer, M., & Belcher, J. W. 1971, Sol. Phys. 16, 472

Vainio, R., & Schlickeiser, R. 1999, A&A 343, 303