

# Cosmic Ray Acceleration by Magnetic Traps

V.N.Zirakashvili<sup>1</sup>

<sup>1</sup>*Institute for Terrestrial Magnetism, Ionosphere and Radiowave Propagation, Russian Academy of Sciences (IZMIRAN), 142092, Troitsk, Moscow Region, Russia*

## Abstract

Cosmic ray acceleration in turbulent interstellar medium is considered. Turbulence is treated as ensemble of moving magnetic traps. We derive equations for particle momentum distribution function that describes acceleration of particles in this case. Rate of acceleration calculated is estimated for our Galaxy and compared with ones given by other acceleration mechanisms.

## 1 Introduction.

Several types of distributed cosmic ray acceleration were investigated by many authors: resonant stochastic acceleration (Tverskoĭ, 1968), stochastic acceleration by large scale turbulence (Bykov & Toptygin, 1979), friction acceleration (Berezhko, 1981, Earl, Jokipii, & Morfill, 1988, Webb, 1989) and in earlier papers of Fermi (Fermi, 1949) and Alfvén (Alfvén & Fälthammar, 1963). The last author introduced so called "magnetic pumping". For such an acceleration mechanism particle changes its perpendicular momentum in varying in time magnetic field, the gain of energy being redistributed due to scattering. In this paper we shall introduce further development of "magnetic pumping" and apply it for cosmic ray acceleration by magnetohydrodynamic (MHD) turbulence.

In our Galaxy interstellar MHD turbulence is mainly created by supernova (SN) explosions. There exists turbulent transfer of kinetic and magnetic energies from the main scale  $L \sim 100$  pc to smaller scales (Ruzmaikin, Sokolov, & Shukurov, 1988). In this paper we shall treat this turbulence as large amount of moving magnetic traps. Energetic particles are trapped inside each trap and change energy due to trap deformation. Weak scattering slowly changes pitch-angle of the particle, determines escape time and hence rate of acceleration.

## 2 Acceleration of particles in magnetic traps.

Assuming that particles gyroradii are much smaller than magnetic trap size we will use drift equation with scattering (Chandrasekhar, 1960)

$$\frac{\partial f}{\partial t} + (\mathbf{u}_E + v\mu\mathbf{b})\nabla f + \frac{1-\mu^2}{2}v\frac{\partial f}{\partial\mu}\nabla\mathbf{b} - p\frac{\partial f}{\partial p}\left(\frac{1-\mu^2}{2}\nabla\mathbf{u}_E + \frac{1-3\mu^2}{2}\mathbf{u}_E(\mathbf{b}\nabla)\mathbf{b}\right) = St_{uf} \quad (1)$$

Here  $f$  is cosmic ray momentum distribution function that is averaged on gyroperiod,  $p$  is particle momentum,  $\mathbf{b} = \mathbf{B}/B$  is the unit vector along the magnetic field  $\mathbf{B}$ ,  $\mu = \mathbf{p}\mathbf{B}/pB$  is the cosine of pitch angle of the particle,  $\mathbf{u}_E = c[\mathbf{E} \times \mathbf{B}]/B^2$  is the electric drift velocity. For frozen magnetic field it is simply perpendicular to  $\mathbf{B}$  component of medium velocity  $\mathbf{u}$ . The right hand side of equation (1) describes scattering of particles by small scale magnetic inhomogeneities moving with medium velocity  $\mathbf{u}$ . Isotropic function  $f_o$  that is independent on time and space coordinates is exact solution of Eq. (1) for  $\mathbf{u} = 0$ . Small deviation  $\delta f$  of cosmic ray momentum distribution function arises for nonzero  $\mathbf{u}$ . Linearizing Eq.(1) we have

$$\begin{aligned} \frac{\partial \delta f}{\partial t} + v\mu(\mathbf{b}\nabla)f + \frac{1-\mu^2}{2}v\frac{\partial \delta f}{\partial\mu}\nabla\mathbf{b} - \frac{\partial}{\partial\mu}\nu\frac{1-\mu^2}{2}\frac{\partial \delta f}{\partial\mu} = \\ = p\frac{\partial f_o}{\partial p}\left(\frac{1-\mu^2}{2}\nabla\mathbf{u}_E + \frac{1-3\mu^2}{2}\mathbf{u}_E(\mathbf{b}\nabla)\mathbf{b}\right) \end{aligned} \quad (2)$$

Left hand side of Eq.(2) describes scattering and movement of particle, the third term being correspond to magnetic mirroring. The terms proportional to  $\nu u$  are neglected since we are going to find solution of this equation in the limit of small scattering frequency when the particle free path length  $\lambda = v/\nu$  is much greater than the trap size  $l$ . In this case it is convenient to use new variable  $q = (1 - \mu^2)/B$  instead of  $\mu$ . Let  $\delta f = \delta f^+$  for  $\mu > 0$  and  $\delta f = \delta f^-$  for  $\mu < 0$ , then assuming quasistationarity of Eq.(2):

$$\pm v \frac{\partial \delta f^\pm}{\partial s} - 2 \frac{\partial}{\partial q} \frac{\nu q}{B(s)} \sqrt{1 - qB(s)} \frac{\partial \delta f^\pm}{\partial q} = \frac{\frac{p}{2} \frac{\partial f_o}{\partial p}}{\sqrt{1 - qB(s)}} [qB(s) \nabla \mathbf{u}_E - (2 - 3qB(s)) \mathbf{u}_E (\mathbf{b} \nabla) \mathbf{b}] \quad (3)$$

Here  $s$  is the coordinate along the magnetic field line. Our aim is to perform averaging of Eq.(1) on ensemble of realizations of random magnetic field and medium velocity. Since mirroring should be included the problem is nonlinear. It will be simplified if one assumes that the medium velocity scale is larger than the magnetic field scale and will treat them as statistically independent quantities. We will also assume that medium is filled by many identical magnetic traps, and magnetic field of each trap varies from  $B_{\min}$  up to  $B_{\max}$ . The same treatment was used for investigation of influence of magnetic traps on the second harmonics of cosmic ray anisotropy (Klepach & Ptuskin, 1995).

For trapped particles  $1/B_{\max} < q < 1/B_{\min}$ . These particles perform oscillations inside a trap and slowly diffuse on  $q$ . Hence function  $\delta f^\pm$  for these particles is approximately independent on  $s$  and can be found from the equation:

$$-2 \frac{\partial}{\partial q} \int_{s_1(q)}^{s_2(q)} ds \frac{\nu q}{B(s)} \sqrt{1 - qB(s)} \frac{\partial \delta f^\pm}{\partial q} = \frac{p}{2v} \frac{\partial f_o}{\partial p} \int_{s_1(q)}^{s_2(q)} \frac{ds}{\sqrt{1 - qB(s)}} [qB(s) \nabla \mathbf{u}_E - (2 - 3qB(s)) \mathbf{u}_E (\mathbf{b} \nabla) \mathbf{b}] \quad (4)$$

Here  $s_1(q)$  and  $s_2(q)$  are roots of equation  $B(s)q = 1$ . Performing one integration on  $q$  one obtains the following equation:

$$2 \int_{s_1(q)}^{s_2(q)} ds \frac{\nu q}{B(s)} \sqrt{1 - qB(s)} \frac{\partial \delta f^\pm}{\partial q} = -\frac{p}{v} \frac{\partial f_o}{\partial p} \int_{s_1(q)}^{s_2(q)} \frac{ds}{B(s)} \left[ \nabla \mathbf{u}_E \left( \frac{1}{3} (1 - qB(s))^{3/2} - \sqrt{1 - qB(s)} \right) + \right. \\ \left. + \left( (1 - qB(s))^{3/2} - \sqrt{1 - qB(s)} \right) \mathbf{u}_E (\mathbf{b} \nabla) \mathbf{b} \right] \quad (5)$$

Substituting the solution of this equation into Eq.(1) and performing ensemble averaging one can find momentum diffusion coefficient (energy changes of trapped particles are taken into account only):

$$D_{pp} = \frac{p^2 w}{4} \int_{1/B_{\max}}^{1/B_{\min}} dq \left[ \int_{s_1(q)}^{s_2(q)} ds' \frac{\nu q}{B(s')} \sqrt{1 - qB(s')} \right]^{-1} \int_{s_1(q)}^{s_2(q)} \frac{ds''}{B(s'')} \cdot \\ \left\langle \left[ \nabla \mathbf{u}_E \left( \frac{1}{3} (1 - qB(s''))^{3/2} - \sqrt{1 - qB(s'')} \right) + \left( (1 - qB(s''))^{3/2} - \sqrt{1 - qB(s'')} \right) \mathbf{u}_E (\mathbf{b} \nabla) \mathbf{b} \right] \cdot \right. \\ \left. \int_{s_1(q)}^{s_2(q)} \frac{ds}{l} \left[ \nabla \mathbf{u}_E \left( \frac{1}{3} (1 - qB(s))^{3/2} - \sqrt{1 - qB(s)} \right) + \left( (1 - qB(s))^{3/2} - \sqrt{1 - qB(s)} \right) \mathbf{u}_E (\mathbf{b} \nabla) \mathbf{b} \right] \right\rangle \quad (6)$$

Here  $w$  is the volume factor of traps and  $\langle \dots \rangle$  means ensemble averaging on  $\mathbf{u}$ . Expression (6) is rather cumbersome. In order to simplify it we assume that the magnetic field disturbances are small:  $\delta B/B \ll 1$ . In this

case trapped particles have pitch angles close to  $\pi/2$ , that is  $\mu \ll 1$  and one can neglect by terms  $(1-qB)^{3/2}$  in the expression (6). We will also neglect by second terms in two last integrals in comparison with first ones. Assuming also that medium velocity scale is larger than trap size  $l$  and using  $\mu$ -independent scattering frequency one can obtain

$$D_{pp} = \frac{p^2}{6} w \langle (\nabla \mathbf{u}_\perp)^2 \rangle \frac{1}{l\nu} \int_0^l ds \sqrt{\left(1 - \frac{B(s)}{B_{\max}}\right)^3} \quad (7)$$

The deviation of momentum distribution function is given by the expression:

$$\delta f = p \frac{\partial f_o}{\partial p} \frac{\nabla \mathbf{u}_\perp}{2\nu} \left( \frac{B_{\max} - B(s)}{B_{\max}} - \mu^2 \right) \theta \left( \frac{B_{\max} - B(s)}{B_{\max}} - \mu^2 \right) \quad (8)$$

where  $\theta(x)$  is step function and  $\mathbf{u}_\perp$  is perpendicular to the mean field component of the medium velocity. The expression (7) can be readily obtained from the following estimation. For trapped particles  $|\mu| < \mu_*$ . If trap is compressed or expands in perpendicular to the mean field direction particle changes its momentum with the rate  $\frac{\Delta p}{\Delta t} \sim p \nabla \mathbf{u}_\perp$ . The escape time is  $\Delta t \sim \mu_*^2/\nu$ . Taking into account that trapped particles are only  $w\mu_*$  part of all particles one can find that

$$D_{pp} \sim \frac{(\Delta p)^2}{\Delta t} \sim p^2 \frac{\langle (\nabla \mathbf{u}_\perp)^2 \rangle}{\nu} w \mu_*^3 \quad (9)$$

If magnetic field disturbances have component parallel to the mean field (magnetosonic wave magnetic disturbances) then  $\mu_* \sim \sqrt{\delta B/B}$  and we obtain expression (7).

### 3 Discussion.

It should be mentioned that expression (9) has the same form as the one for the friction acceleration (Berezhko, 1981, Earl, Jokipii & Morfill, 1988, Webb, 1989). The friction acceleration was derived for the case when medium velocity scale is larger than particle free path length. The acceleration considered here works for opposite relation  $l \ll \lambda$  and thus can be stronger for sufficiently small  $l$ . The same is concerned for comparison with stochastic Fermi acceleration beacose it is of the same order as friction acceleration for  $l \sim \lambda$ . The rate of resonance acceleration is roughly (cf. Berezhinsky et al., 1990)

$$D_{pp}^{res} \sim \Omega \frac{v_a^2}{v^2} \left( \frac{\delta B}{B} \right)^2 p^2 \quad (10)$$

where  $\Omega$  is particle gyrofrequency,  $v_a$  is Alfvén velocity, and  $\delta B$  is the value of random magnetic field in the scale of the order of particle gyroradius  $r_g = v/\Omega$ . In order to compare estimations (9) and (10) one should take into account that  $u \sim v_a \delta B/B$ ,  $\mu_* \sim \sqrt{\delta B/B}$ ,  $w \sim 1$  and take the smallest possible  $l \sim r_g$  when the drift approximation is marginally valid. Taking for the scattering frequency value that is determined by resonance scattering  $\nu \sim \Omega (\delta B/B)^2$  (cf. Berezhinsky et al., 1990) and assuming that velocity scale is of the order of the trap size one can find that

$$D_{pp} \sim \Omega \frac{v_a^2}{v^2} \left( \frac{\delta B}{B} \right)^{3/2} p^2 \quad (11)$$

Therefore resonant acceleration is a factor  $(\delta B/B)^{1/2}$  less effective than considered here trap acceleration. If scattering frequency drops at small  $\mu$  the difference will be larger in favor of trap acceleration.

It is well known now that resonant acceleration is enough for weak reacceleration of galactic cosmic rays with energies about 1 GeV/nucleon in order to explain mean thickness dependence at these energies (Seo & Ptuskin, 1994). Problems can arise for more powerful acceleration. It seems that magnetosonic waves with lengths of the order of 1 GeV proton gyroradius cannot exist in the interstellar medium because strong Landau damping (acceleration rate estimation is obtained for such waves).

In our Galaxy compressible turbulence exists in the main scale  $L \sim 100pc$  and possibly at smaller scales due to nonlinear cascade transfer. The minimum scale of compressible turbulence is determined by dissipative processes and is estimated as  $L_{\min} \sim 0.01 - 1pc$  (Ruzmaikin, Sokolov, & Shukurov, 1988). It should be noted that this minimum scale can be also determined by dissipation related with the acceleration mechanism considered. The acceleration rate (9) is determined by values of velocity and magnetic perturbations for this minimum scale. In this case mechanism of acceleration considered has very attractive feature: for ultrarelativistic particles the rate of acceleration is proportional to space diffusion coefficient. Such a feature gives a possibility of formation of power-low cosmic ray spectrum. This dependence is valid up to the energies when particle gyroradius is approximately equal to minimum scale of turbulence and is estimated as  $Z \cdot 10^{15} eV$  where  $Z$  is the particle charge,  $L_{\min} \sim 0.1pc$  and  $B \sim 10^{-5}Gs$  was assumed. The ratio of diffusive exit time to acceleration time can be estimated as

$$\frac{\tau_{dif}}{\tau_{acc}} \sim \frac{H_h h_{acc}}{L_{\min}^2} \frac{v_a^2}{v^2} \left( \frac{\delta B_{\min}}{B} \right)^{7/2} \quad (12)$$

Here  $H_h$  is the galactic halo height,  $h_{acc}$  is the height of acceleration region,  $\delta B_{\min}$  is the value of magnetic field perturbation for minimum turbulence scale  $L_{\min}$ . This ratio is about 0.1 for  $L_{\min} \sim 0.1pc$ ,  $H_h \sim 10kpc$ ,  $h_{acc} \sim 3kpc$ ,  $v_a \sim 100km/s$ ,  $\delta B_{\min}/B \sim 1/10$ .

For rough comparison with acceleration on SN shocks one should note that both mechanisms use such a source of energy as mechanical energy of SN explosion. Since only about 5 percent of initial mechanical energy of SN explosion goes into mechanical energy of interstellar medium (Ruzmaikin, Sokolov & Shukurov, 1988) one can expect that if more than 5 percent of the initial mechanical energy of SNs goes into accelerated at SN's shock particles, then acceleration mechanism considered here is negligible in comparison with SN shock acceleration. In the opposite case it can be important for our Galaxy.

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