

# Second order Fermi acceleration in pulsar wind

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## Abstract

Large scale regular magnetic and electric fields are believed to exist in pulsar winds confined by SNR. We consider that cosmic rays accelerated preliminary by SN shocks can enter pulsar wind and be subjected to continuous interactions with Alfvén waves in the wind. The rate of this second order Fermi acceleration is estimated.

## 1 Introduction:

Mechanisms of c.r. acceleration to energies higher than these believed to be attainable in SN shocks ( $< 10^{15}$  eV $\cdot$ Z), are not known. According to earlier investigations (Bell 1992, Giller and Michalak 1995,1996) this maximum energy could be increased if a SNR confines a pulsar emitting relativistic magnetized plasma (Rees and Gunn, 1974). Large scale electric fields produced by this wind inside the SNR could accelerate further the most energetic c.r. particles produced at the blast wave in the ISM. We have shown, however, that only a small fraction of SNR accelerated particles could penetrate deep inside the nebula and get a significant energy increase - only those whose strong radial  $\vec{B} \times \text{grad}B$  drifts overcome the outward wind velocity  $\vec{u}$ , so that the resulting radial velocities of their guiding centres are directed inward in the pulsar frame. Most cosmic rays which enter the nebula (i.e. whose radial  $\vec{B} \times \text{grad}B$  drifts are directed inward, so that their guiding centres move inward in wind frame), cannot penetrate deeper into the nebula, which is strongly related with gaining substantial energy. They are captured by large scale toroidal magnetic field  $\vec{B}_0$  of the nebula and stay near its outer boundary at  $R_n$ .

In this work we assume that there should be some level of the wind excitation in a form of the Alfvén waves propagating along  $\vec{B}_0$ . The trapped c.r. particles would interact with them, being accelerated by a second-order Fermi process. Here we examine its effectiveness.

## 2 Model of the Nebula:

The details of the pulsar wind model, adopted here, are described by Bell (1992) and Giller and Michalak (1995), being based on earlier works by Rees and Gunn (1974) and Kennel and Coroniti (1984). We only remind here that according to it, the pulsar wind fills a cavity (nebula) of radius  $R_n = 5 \cdot 10^{18}$  cm (all numerical values correspond to Crab Nebula), forming an inner shock at radius  $R_s = 0.1 \cdot R_n$  due to wind confinement by the SN ejecta. Pulsar rotation produces toroidal magnetic field which frozen in the wind moves with velocity  $\vec{u}$  and increases towards the outer regions, reaching values  $\leq 10^{-3}$  G at  $R_n$ . Associated electric field in the pulsar frame ( $\vec{E} = -\vec{u} \times \vec{B}$ ) is directed along meridians. These fields depend on zenith angle  $\theta$  as  $\sin^2(\theta) \cos(\theta)$ .

Experimental data for the Crab Nebula show that ratio of the magnetic plus electric energy flux to that of the particles ( $e^+e^-$ ) upstream the relativistic shock at  $R_s$  is about 0.003 (Kennel and Coroniti, 1984). It follows that the Alfvén waves velocity  $v_A$  is about 0.14c downstream close to the shock at  $R_s$ . The velocity  $v_A$  should be considerably higher near the nebula boundary  $R_n$ , which follows from models of nebula expansion (Kennel and Coroniti, 1984, Giller and Michalak, 1997). The Alfvén waves in the nebula can be generated by turbulences produced by the shock induced by positronic flow in SN ejecta or by perturbations at the shock at  $R_s$ .

## 3 Model of Cosmic Ray Acceleration by Alfvén waves:

According to our earlier analysis a large fraction of cosmic rays accelerated preliminary by the SN blast wave in the ISM or by the shock that positronic flow induces in SN ejecta can just enter the upper region

(at  $R_n$ ) of the magnetized nebula and be accelerated further by interactions with the Alfven waves.

While moving along helix-like trajectory in regular large scale magnetic field  $B_0$ , a proton undergoes stochastic changes in transverse component of its momentum (denoted as  $p_t$ ) caused by electric fields of Alfven waves, longitudinal component  $p_l$  remaining constant during the process.

Let us consider a helix- type proton trajectory in regular azimuthal magnetic field  $\vec{B}_0$  in local undisturbed wind frame (we neglect here any zenithal dependence of  $\vec{B}_0$ ). In that frame there are electric fields  $\vec{E}_1$  connected with Alfven waves as

$$\vec{E}_1 = -\frac{\vec{v}_1}{c} \times \vec{B}_0 = -\frac{1}{c} \left( -\frac{1}{\sqrt{4\pi\rho}} \vec{B}_1 \times \vec{B}_0 \right) = -\frac{\vec{v}_A}{c} \times \vec{B}_1 \quad (1)$$

where velocity  $\vec{v}_1$  and magnetic field  $\vec{B}_1$  refer to amplitudes of the wave. Energy change  $\delta E$  of a cosmic ray particle (proton), moving with velocity  $\vec{v}$ , after time  $dt$  can be expressed as:

$$\delta E = e\vec{E}_1(t) \cdot \vec{v}(t)dt = e \cdot v_t \cdot E_1(t) \cdot e^{i\omega t} dt = e \left( c \frac{p_t}{p} \right) \cdot \frac{v_A}{c} \cdot B_1(t) \cdot e^{i\omega t} dt \quad (2)$$

where  $\omega = v_t/R = eB_0/p$  is proton Larmour frequency and  $B_1(t)$  is taken at the proton position at time  $t$ . Adding up all  $\delta E$  occurring within time  $\Delta t$  we obtain energy gain  $\Delta E = E' - E$ . If  $\Delta t$  is small enough (such that we can assume that all  $(\delta E)^2$  are on average the same), it is easy to find the variance of  $\Delta E$ ,  $\sigma^2(\Delta E)$ . Expressing  $B_1(t)$  in (2) as a Fourier integral over all circular frequencies  $\omega$ , we find that

$$\sigma^2(\Delta E) = \left( e \cdot v_t \cdot \frac{v_A}{c} \right)^2 \cdot |B_1(\omega)|^2 \cdot \Delta t \quad (3)$$

For small  $\Delta t$   $\sigma^2(\Delta E)$  is proportional to  $\Delta t$  but the coefficient of proportionality depends on particle energy which increases with time. A more convenient quantity here is the rate of increase (growth) in time of the variance of the variable  $\ln(E'/E)$ , denoted by  $dG/dt$ . To calculate it we have to assume power spectrum of irregularities  $B_1$ . We adopt Kolmogorov form of it i.e. that spectral density  $|B_1(k)|^2$  is proportional to  $k^{-\frac{5}{3}}$ , where  $k$  is a wave vector. Taking into account that  $|B_1(k)|^2 = |B_1(\omega)|^2 \cdot v_{\pm}$  where  $v_{\pm} = \left| v_A \pm c \cdot \frac{p_l}{p} \right|$  is the relative longitudinal velocity of the particle and the wave, we obtain

$$\frac{dG}{dt} = \frac{1}{3} \cdot \alpha \cdot \left( \frac{p_t}{p} \cdot \frac{v_A}{c} \cdot \omega \right)^2 \cdot \left[ \left( \frac{\omega}{v_+ \cdot k_0} \right)^{-\frac{5}{3}} \cdot \frac{1}{v_+ \cdot k_0} + \left( \frac{\omega}{v_- \cdot k_0} \right)^{-\frac{5}{3}} \cdot \frac{1}{v_- \cdot k_0} \right] \quad (4)$$

Here we have introduced two free parameters: 1) the ratio of the density energy of irregular magnetic field  $B_1$  (i. e. Alfven waves) to that of the regular one - denoted by  $\alpha$ , and 2) the minimal value of wave number  $k$  in the Kolmogorov spectrum corresponding to the largest irregular structure  $L_0$  ( $k_0 = 2\pi/L_0$ ). Parameter  $\alpha$  cannot be reasonably taken larger than 1 in this picture, as field  $B_1$  is understood as small disturbance of the total magnetic field, and value of  $L_0$  cannot exceed the radius of nebula  $R_n$ .

Putting all the dependence on  $p_t$  and  $p_l$  into the term 'f', defined below, we can write the above formula as:

$$\frac{dG}{dt_{yr}} = 334 \cdot \alpha_1 \cdot \left( \frac{L_0}{10^{15}\text{m}} \right)^{-\frac{2}{3}} \cdot \left( \frac{B_0/10^{-3}\text{G}}{p/10^{12}\text{eV}/c} \right)^{\frac{1}{3}} \cdot \left( \frac{v_A}{c} \right)^{\frac{8}{3}} \cdot f, \quad (5)$$

where time  $t_{yr}$  is in years and

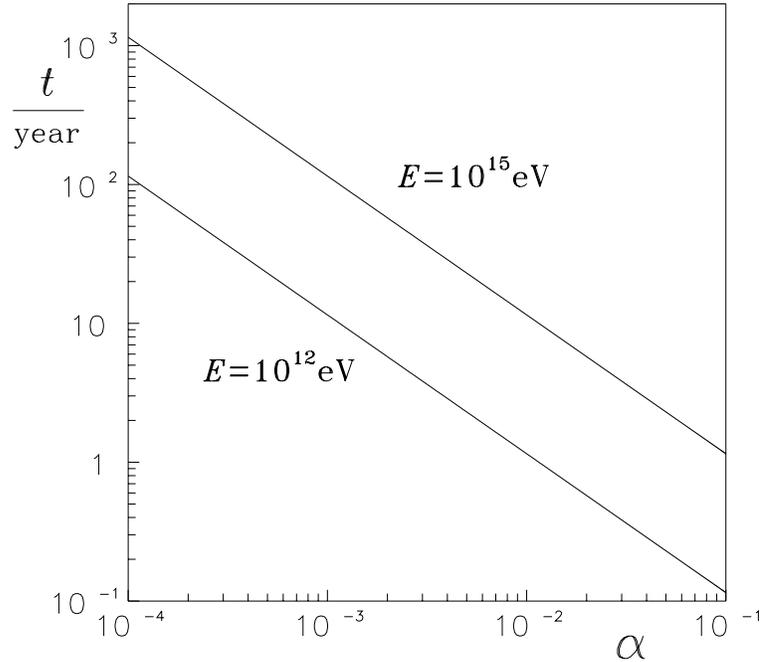
$$f = \left( \frac{p_t}{p} \right)^2 \cdot \left[ \left| 1 + \frac{p_l/p}{v_A/c} \right|^{\frac{2}{3}} + \left| 1 - \frac{p_l/p}{v_A/c} \right|^{\frac{2}{3}} \right].$$

The value of ' $f$ ' is close to 2 for the later times in the acceleration process.

In order to assess the effectiveness of the acceleration let us take the lowest value of  $dG/dt$ , which occurs at highest momentum  $p$ , and extrapolate that value towards lower energies. In the case of  $dG/dt = \text{const}$  and a power spectrum of injection energies at an initial moment, we get such a final energy spectrum as if every initial proton energy was multiplied by the factor  $\exp\left(\frac{\gamma}{2} \frac{d\alpha^2}{dt} t\right)$ , where  $\gamma$  is the power index of the integral spectrum. We should bear in mind that the above factor is valid for energies far enough from the low energy cut-off in the energy spectrum, where it is not distorted by the cut-off. We do not discuss here the fact, that there is also upper limit in Kolmogorov spectrum (resulting from dissipation of Alfvén waves) which would modify our formula on  $dG/dt$ .

#### 4 Discussion and Conclusions:

The results of our investigations are illustrated in Fig.1. On the assumption that the growth rate  $dG/dt$  is constant, we have plotted there the time necessary to accelerate a proton of a given initial energy ( $10^{12}$  eV and  $10^{15}$  eV) by factor of 10 as a function of the relative energy density in the Alfvén waves,  $\alpha$ . The graph is for  $v_A/c = 0.5$ ,  $B_0 = 10^{-3}\text{G}$ ,  $L_0 = 10^{15}\text{m}$ ,  $f = 2$  and  $\gamma = 1$ . We can see that the effect could be quite strong for lower energies even for rather small ( $\alpha = 10^{-4}$ ) irregular fields. Thus, we conclude that the discussed second order Fermi acceleration in pulsar wind seems quite important for the presented parameters of the nebula and our simplified model. Further investigations and numerical calculations should account at least for time-dependent injection spectrum of protons, getting as a result an output spectrum (for steady state nebula). Effect of change in helix position in nebula due to drifts during acceleration, which could cause particles to escape from the nebula, should be discussed and accounted for.



**Figure 1:** Time scale for accelerating c.r. proton by factor of 10 as a function of  $\alpha$  (fraction of electromagnetic energy contained in Alfvén waves). Initial proton energies are shown. For other parameters –see text.

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