

# Study of Primary Energy Reconstruction through Fluorescence Light Detection with Corsika

C. Song<sup>3</sup>, Z. Cao<sup>4</sup>, B. Dawson<sup>1</sup>, B. Fick<sup>2</sup>, W. Lee<sup>3</sup>, P. Sokolsky<sup>4</sup>, X. Zhang<sup>3</sup>

<sup>1</sup> *University of Adelaide, Adelaide, South Australia 5005, Australia*

<sup>2</sup> *Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA*

<sup>3</sup> *Nevis laboratories, Columbia University, Irvington, NY 10533, USA*

<sup>4</sup> *High Energy Astrophysics Institute, University of Utah, Salt Lake City, UT 84112, USA*

## Abstract

The energy estimation in electromagnetic cascade is reviewed with air shower simulation program Corsika. The reconstructed energy of an incident particle is found to be about 10 % lower than the true value for electromagnetic shower. Therefore, we propose the new energy reconstruction method and  $E_{em} - E_0$  conversion for hadronic shower in fluorescence light detection experiment.

## 1 Introduction

One of the goals in a cosmic ray experiment is to determine the energy of incident particles. Unfortunately the primary energy cannot be measured directly by using electromagnetic calorimeter technique. The energy in an electromagnetic shower is estimated by means of a formula given by Rossi[2].

$$E_{em} = \frac{E_c}{X_0} \int_0^\infty N_e(X) dX, \quad (1)$$

where  $X_0$  is the radiation length,  $E_c$  is the critical energy of an electron in air, and  $N_e$  is the number of electrons in the shower. According to Rossi[2], the electromagnetic energy is the track length times the  $dE/dX$  of electrons at the critical energy. The track length means the total distance traveled by all shower electrons in unit of radiation lengths. Therefore, his assumption is that because the  $dE/dX$  varies slowly with energy, the  $E_c$  correctly represents the energy loss of electrons in a wide energy region around  $E_c$ . For  $E \gg E_c$ , energy loss by ionization is not important as shown in Figure 1.a. Therefore, Rossi used the mean energy loss rate of electron to estimate the electromagnetic energy without considering the energy spectrum of shower electrons.

For a given  $E_{em}$ , the primary energy  $E_0$  for hadronic showers is usually determined via Linsley's parametrization [5,6]. However, we have found with Corsika simulations that the procedure involving Eq.(1) and Linsley's parametrizations results in a reconstructed energy about 10% lower than the true value.

In order to understand this discrepancy, we first investigate Eq.(1). Eq.(1) should be checked to ensure that it is proper as a tool to determine electromagnetic energy. One sign that Eq.(1) adequately represent electromagnetic shower is to verify that it returns an energy ratio,  $E_{em}/E_0$ , of 1 for purely electromagnetic showers.

In the simulation, the cut-off energy is 300, 700, 0.1 and 0.1 MeV for hadrons, muons, electrons and photons respectively. Particles below cut-off are not taken into account. The observation level is 300 m above sea level.

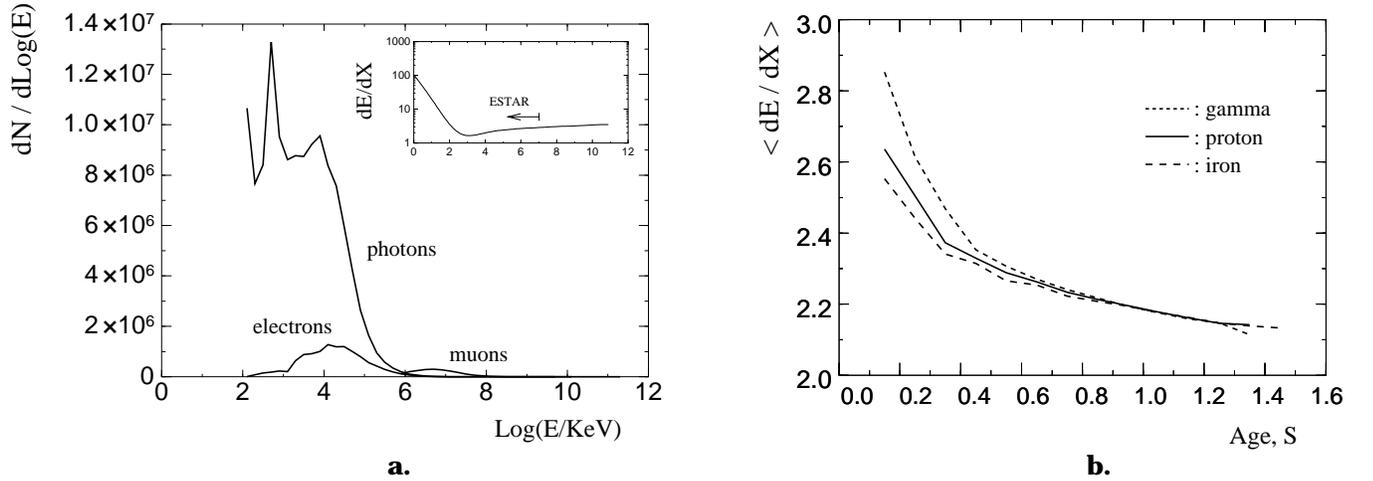


Figure 1: **a.** The mean energy spectrum of  $\gamma$ ,  $e$  and  $\mu$  at 300 m above sea level for 50 proton showers at  $10^{17}$  eV. The inset shows the energy loss rate by ionization of electron in dry air[8]. **b.** The mean  $dE/dX$  as a function of *pseudo age*,  $S$ , for 400 events

## 2 Electromagnetic energy of air shower

For a purely electromagnetic shower, photons with energy above about 1 MeV can produce electron-positron pairs which is the dominant process of photon energy loss in the high energy region. Adding up the energy loss by ionization of the electrons and positrons gives the electromagnetic energy. The electromagnetic shower energy,  $E_{em}$ , can be expressed as

$$E_{em} = \sum_{(k_i > \epsilon)} \mathcal{N}_e(k_i) \cdot \Delta E(k_i), \quad (2)$$

where  $\mathcal{N}_e(k_i)$  is the number of electrons with kinetic energy  $k_i$  and  $\Delta E(k_i)$  is the energy loss by each of those electrons via ionization. However  $E_{em}$  does not take into account the energy loss by low energy photons through photoelectric absorption.

We can express Eq.(2) in integral form for a shower induced by a high energy primary:

$$E_{em} = \int_{\epsilon}^{\infty} \Delta E(k) \mathcal{N}_e(k) dk, \quad (3)$$

where  $k$  is the kinetic energy and  $\epsilon$  is the cut-off energy of electrons. In the simulation, 0.1 MeV is selected as the cut-off energy of photons as well as electrons and positrons. Electrons below cut-off are not taken into account in the simulation. Eq.(3) can be expressed by using the known energy spectrum giving the following relationship:

$$\mathcal{N}_e(k) = \int_0^{\infty} N_e(X) n_e(k, X) \frac{dX}{\Delta X(k)}, \quad (4)$$

where  $\Delta X(k)$  is the mean free path of electrons as a function of  $k$ ,  $N_e(X)$  is the number of electrons, and  $n_e(E, X)$  is the electron energy spectrum normalized to 1.

For this study, the *pseudo age* is defined as;

$$S = \frac{3 \cdot (X - X_1)}{(X - X_1) + 2 \cdot (X_{max} - X_1)} \quad (5)$$

	With $\mu^+\mu^-$ & $\gamma N$			Without $\mu^+\mu^-$ & $\gamma N$		
$E_0$ , eV	$E_{em}/E_0$	$N_\mu$	$N_{max}$	$E_{em}/E_0$	$N_\mu$	$N_{max}$
$10^{15}$	$0.875 \pm 0.016$	$388.5 \pm 223.2$	$(8.885 \pm 0.454)10^5$	$0.889 \pm 0.021$	$0.000 \pm 0.000$	$(8.917 \pm 0.621)10^5$
$10^{16}$	$0.888 \pm 0.026$	$5153. \pm 4459.$	$(8.233 \pm 0.406)10^6$	$0.901 \pm 0.031$	$0.000 \pm 0.000$	$(8.294 \pm 0.445)10^6$

Table 1: The gamma induced showers with and without photo-nuclear interaction and muon pair production. The uncertainties are r.m.s.

where  $X_1$  is the first interaction depth.  $S_{X=X_1} = 0$ ,  $S_{X=X_{max}} = 1$  and  $S_{X=\infty} = 3$ . Even though *age* is defined only for electromagnetic shower, we can define *pseudo age* as Eq.(5) for hadronic shower. Substituting Eq.(4) into Eq.(3) and using the definition of *pseudo age* gives

$$E_{em} = \int_0^\infty N_e(X) \left( \int_\epsilon^\infty \frac{\Delta E}{\Delta X}(k) \tilde{n}_e(k, S) dk \right) dX \equiv \int_0^\infty N_e(X) \alpha(S) dX \quad (6)$$

For comparison with Eq.(1), we calculate the age-averaged coefficient.

$$\langle \alpha \rangle_S = \frac{\sum_i \langle N_e \rangle_{\Delta S_i} \cdot \int_\epsilon^\infty dk \left( \frac{dE}{dX}(k) \right) \langle \tilde{n}_e(k) \rangle_{\Delta S_i}}{\sum_i \langle N_e \rangle_{\Delta S_i}}, \quad (7)$$

where  $\langle N_e \rangle_{\Delta S_i}$  is the mean number of electrons over age bin,  $\Delta S_i$  which is set as 0.1 in the simulation. Figure 1.b shows  $\alpha(S)$  as a function of *pseudo age*. The  $\langle \alpha \rangle_S$  becomes 2.186, 2.193 and 2.189 in MeV/g cm<sup>2</sup> for gamma, proton and iron induced showers respectively at  $10^{17}$  eV. Those numbers are about 7% lower than the ratio of the critical energy of an electron to its radiation length in the air. The  $X_0$  is 36.66 MeV / g cm<sup>-2</sup> in the air and  $E_c$  is 86 MeV, using Rossi's definition[3]. According to the electron energy spectrum shown in Figure 1.a, only a small fraction of the number of electrons falls below the cut-off energy,  $\epsilon$ , of 0.1 MeV. Therefore, this cutoff is safe for this study.

For electromagnetic showers, we simulated gamma induced showers at two energies. The results are found in Table 1 for 500 events. The reconstructed  $E_{em}$  is about 10 % lower than primary energy. The photo-nuclear interaction and  $\mu^+\mu^-$  pair production were switched off to see what has influence on the 10% missing energy. Table 1 shows the results for 200 events. Without the interaction turned on, there is no muon component as expected. Meanwhile, the energy ratio,  $E_{em}/E_0$  is not much changed, which means those interaction do not account for the missing energy.

### 3 Results

The first estimate of missing energy was obtained directly from Linsley[5] who derived estimates of missing shower energy from measurement of electron and muon size and the total assessed energy content of these respective components of the extensive air showers. His corrections were for hadronic showers and aimed to correct for the energy carried by high energy muons, neutrinos and that involved in nuclear excitation. The old Fly's Eyes group had parameterized Linsley's estimates as[6]:

$$E_{em}/E_0 = 0.98995 - 0.078176 \cdot E_0^{-0.175} \quad (8)$$

where  $E_0$  is a total energy in EeV. This parameterization is valid for  $1 \text{ PeV} < E_0 < 100 \text{ EeV}$ . This is compared with the results by Corsika as shown in Figure 2. For practical reason, we express  $E_{em}/E_0$  as function of  $E_{em}$  in EeV.

$$E_{em}/E_0 = a - b \cdot E_{em}^{-c} \quad (9)$$

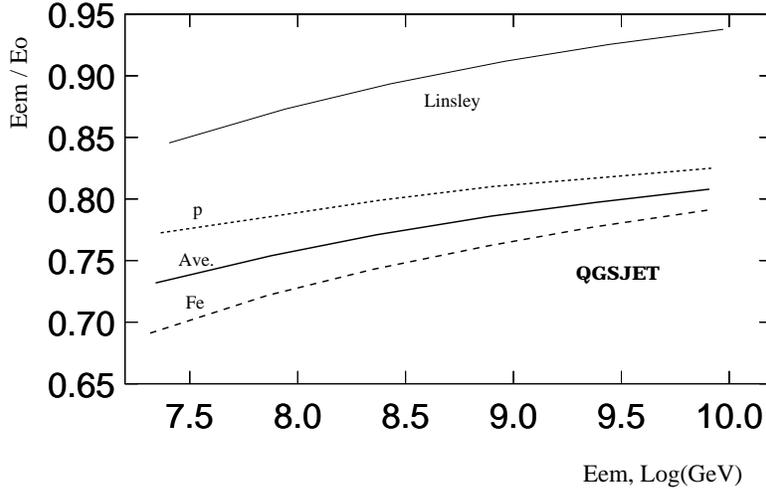


Figure 2:  $E_{em}/E_0$  with the electromagnetic energy by QGSJET model in Corsika

where  $a$  is  $0.85878 \pm 0.00424$ ,  $b$  is  $0.06839 \pm 0.00434$  and  $c$  is  $0.15857 \pm 0.00836$ . This is valid for  $30 \text{ PeV} < E_0 < 10 \text{ EeV}$ . The  $E_{em}$  in Eq.(9) is calculated by Eq.(6). It may suffer the same about 10% missing energy of  $E_{em}$  as that mentioned in the previous section for purely electromagnetic shower.

## 4 Conclusion

We defined electromagnetic energy as total energy loss by electrons. The Eq.(6) is used to determine the electromagnetic energy, instead of approximated formula, Eq.(1). However, the electromagnetic energy misses about 10 % of primary energy for gamma induced showers. We don't know what is the sources of the missing energy. We need to study further.

According to the definition of  $E_{em}$ , the  $E_{em}$  represents the energy which can be estimated by fluorescence light detectors since the number of fluorescence photons produced by an electron is proportional to electron energy loss rate from 1.4 to 1000 MeV[7], and then the energy of incident particle can be determined via Eq.(9).

## References

- [1] FZKA 6019 (1998)
- [2] B.Rossi, High Energy Particles, Prentice Hall, Ch. 5, 1952
- [3] Particle Data Group, Review of Particle Physics, 1998
- [4] M.J.Berger and S.M.Seltzer, "Table of Energy Losses and Ranges of Electrons and Positrons", National Aeronautics and Space Administration Report NASA-SP-3012 (Washington DC, 1964)
- [5] J. Linsley, Proc. 18th ICRC, Bangalore, India, 12, pp159-187 (1983)
- [6] Baltrusaitis et al., Proc. 19th ICRC, La Jolla, USA, 7, pp159-162, (1985)
- [7] F.Kakimoto et al., Nucl. Instr. Meth. A 372, pp527-533, 1996
- [8] <http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html>