Mean free path of solar electrons and nucleons

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Abstract

The quasilinear scattering mean free path of solar flare electrons and nucleons is calculated for interplanetary plasma wave turbulence consisting of a mixture of slab shear Alfven waves and isotropic fast magnetosonic waves. It is shown that for well-sampled electron and nucleon mean free paths over a wide and overlapping energy range the underlying frequency power spectra of left-handed and right-handed slab Alfven waves can be uniquely determined. Moreover, we demonstrate that the flatness and magnitude problems of quasilinear mean free paths of solar flare particles can be resolved if the interplanetary slab Alfven magnetic power spectrum varies in wavenumber as $\propto |k|^{-s}$ with $s = 5/3$ below $|k| \leq k_c$ and $s = 3$ above $|k| > k_c$ where k_c is the inverse of the ion skin length.

1 Introduction:

Cosmic ray particles in space plasmas are confined and accelerated by resonant interactions in weakly random electromagnetic fields. In the presence of low-frequency magnetohydrodynamic plasma waves, whose magnetic field component is much larger than their electric field component, the particle's phase space distribution function adjusts rapidly to a quasi-equilibrium through pitch-angle diffusion, which is close to the isotropic distribution. The isotropic part of the phase space distribution function $F(z,p,t)$ obeys the *diffusion-convection-equation* including diffusion and convection terms both in momentum (p) and position (z) space, where the space coordinate z is parallel to the uniform background magnetic field $\vec{B}_0$.

Of particular interest is the parallel spatial diffusion coefficient $\kappa = v\lambda/3$ which conventionally is expressed in terms of the mean free path λ and particle speed v. Within quasilinear theory the mean free path results from the pitch-angle ($\mu = p_{\|}/p$) average of the inverse of the pitch-angle Fokker-Planck coefficient $D_{\mu\mu}$ as

$$\lambda = \frac{3v}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}. \tag{1}$$

Schlickeiser and Miller (1997, 1998 – hereafter referred to as SM) have calculated the quasilinear Fokker-Planck coefficient $D_{\mu\mu}$ in a turbulent electromagnetic field consisting of a mixture of slab shear Alfven waves and isotropic fast magnetosonic waves in a low β-plasma. They discussed possible new solutions to the well-known "flatness" and "magnitude" problems (Palmer 1982, Bieber et al. 1994) of the quasilinear mean free path. Here we extend this work and derive the interplanetary plasma turbulence conditions that allow the resolution of the mean free path discrepancy.

2 Quasilinear mean free path in mixed plasma wave turbulence

According to SM the Fokker-Planck coefficient $D_{\mu\mu}$ is the sum of contributions from transit-time damping (T) and gyroresonant interactions with the shear Alfven (A) and fast mode (F) branch. At particle pitch-angles outside the interval $|\mu| \geq \epsilon = V_A/v$ transit-time damping provides the dominant contribution to the pitch-angle scattering, so that the value of the mean free path is primarily fixed by the small but finite scattering due to gyroresonant interactions in the interval $|\mu| < \epsilon$. It has been noted by SM that in this interval the gyroresonance fast mode contribution is much smaller than the shear wave contribution for comparable total wave intensities $(\delta B)_A^2$ and $(\delta B)_F^2$. Consequently, we may approximate the quasilinear mean free path (1) as

$$\lambda \simeq \frac{3v}{8}\int_{-\epsilon}^{\epsilon} d\mu (1-\mu^2)^2 \left[D_{\mu\mu}^A(\mu)\right]^{-1} \simeq \frac{3v\epsilon}{4D_{\mu\mu}^A(\mu=0)}, \tag{2}$$

where we approximated $D^A_{\mu\mu}(|\mu|<\epsilon)\simeq D^A_{\mu\mu}(\mu=0)$. The quasilinear Fokker-Planck coefficient of a cosmic ray particle of gyrofrequency $\Omega=\Omega_0/\gamma$ for the parallel propagating undamped slab Alfven wave branch is related to the one-dimensional wave number power spectrum of magnetic fluctuations $I(k)$ as

$$D^A_{\mu\mu}(\mu)=\frac{\pi\Omega^2(1-\mu^2)}{2B_0^2}\int_{-\infty}^{\infty}dk\; I(k)\delta(\omega(k)-vk\mu-\Omega), \tag{3}$$

where we use the notation $\omega>0$ and $\omega<0$ to refer to left-handed and right-handed circularly polarised plasma waves, respectively. Note that the non-relativistic gyrofrequency is positive for nucleons and negative for electrons. For $\mu=0$ we immediately obtain the frequency resonance

$$D^A_{\mu\mu}(\mu=0)=\frac{\pi\Omega^2}{2B_0^2}\int_{-\infty}^{\infty}dk\; I(k)\delta(\omega(k)-\Omega)=\frac{\pi\Omega^2}{2B_0^2}\int_{-\infty}^{\infty}d\omega\; J(\omega)\delta(\omega-\Omega)=\frac{\pi\Omega^2 J(\Omega)}{2B_0^2}, \tag{4}$$

where we use the relation

$$I(k)dk=J(\omega)d\omega \tag{5}$$

between the frequency ($J(\omega)$) and wavenumber ($I(k)$) power spectra.

Inserting Eq. (4) into Eq. (2) we obtain for the mean free path

$$\lambda(\gamma)\simeq\frac{L_0}{\Omega_0^2}\frac{\gamma^2}{J(\frac{\Omega_0}{\gamma})}, \tag{6}$$

where the constant L_0 is given by

$$L_0\equiv\frac{3V_AB_0^2}{2\pi}. \tag{7}$$

The slab Alfven wave branch has frequencies $|\omega|\le|\Omega_{0,e}|$ and the dispersion relation

$$\omega=V_Ak\Big[\pm\sqrt{1+\frac{k^2}{4k_c^2}}-\frac{k}{2k_c}\Big], \tag{8}$$

where $k_c=\Omega_{0,p}/V_A=(c/\omega_{p,p})^{-1}$ is the inverse ion skin length and $\omega_{p,p}$ the proton plasma frequency. The dispersion relation (8) comprises the right-handed polarised Alfven-Whistler branch at negative frequencies $\Omega_{0,e}<\omega<0$ and the left-handed polarised Alfven-ion-cyclotron branch at positive frequencies $0<\omega<\Omega_{0,p}$, whereas no waves exist in the positive frequency rande $\Omega_{0,p}<\omega<|\Omega_{0,e}|$. Solar electrons with $\mu=0$ only interact with right-handed polarised waves, and Eq. (6) yields

$$\lambda_e(\gamma)\simeq L_0\frac{\gamma^2}{\Omega_{0,e}^2}\frac{1}{J(\frac{\Omega_{0,e}}{\gamma})}. \tag{9}$$

Alternatively, solar flare nucleons with $\mu=0$ interact only with left-handed polarised waves, and Eq. (6) yields

$$\lambda_p(\gamma)\simeq L_0\frac{\gamma^2}{\Omega_{0,p}^2}\frac{1}{J(\frac{\Omega_{0,p}}{\gamma})}. \tag{10}$$

Obviously, using the frequency resonance condition (see Eq. (4)) $\omega=\Omega$, we can invert Eqs. (9) and (10) and determine the frequency power spectra of the slab plasma waves for known electron and proton mean free paths:

(1) For the frequency power spectrum of right-handed ($\omega<0$) polarised waves we find

$$J_{\rm RH}(\omega)=\frac{L_0}{\omega^2}\Big[\lambda_e(\frac{\Omega_{0,e}}{\omega})\Big]^{-1}, \tag{11}$$

which will lie in the Whistler frequency range because measured solar flare electrons have Lorentz factors well below 1836.
(2) In case of protons we obtain for the left-handed ($\omega > 0$) polarised waves

$$J_{\rm LH}(\omega) = \frac{L_0}{\omega^2}\Big[\lambda_p(\frac{\Omega_{0,p}}{\omega})\Big]^{-1} \tag{12}$$

which lies in the Alfvenic frequency range because $\Omega_{0,p}/\gamma < |\Omega_{0,p}|$.
For well-sampled electron and proton mean free paths over a wide and overlapping energy range we may thus calculate the underlying frequency power spectrum of the slab plasma waves. Unfortunately, existing measurements of the particle mean free paths (Dröge 1994) do not have this quality.

3 Plasma turbulence spectra to resolve the mean free path discrepancy:

Turbulence measurements in the interstellar medium (e.g. Dröge et al. 1993) suggest a Kolmogorov-type frequency power spectrum of the form

$$J(\omega) = J_0|\omega|^{-s}, \tag{13}$$

with constants J_0 and s. According to Eqs. (9) and (10) we then obtain

$$\lambda_p(\gamma) \simeq \frac{L_0}{J_0}(\frac{\gamma}{\Omega_{0,p}})^{2-s}, \quad \lambda_e(\gamma) \simeq \frac{L_0}{J_0}(\frac{\gamma}{\Omega_{0,e}})^{2-s} \quad . \tag{14}$$

First, we note that for *any* value of s this yields a constant mean free path of protons

$$\lambda_p \simeq \lambda_0 = \frac{L_0\Omega_{0,p}^{s-2}}{J_0} = \frac{24}{s-1}(\frac{B_0}{\delta B_A})^2\,\frac{c}{\omega_{p,p}}\,(\frac{\Omega_{0,p}}{\omega_0})^{s-1} \tag{15}$$

at non-relativistic kinetic energies because

$$\gamma = 1 + \frac{E_{\rm kin}}{mc^2} \simeq 1. \tag{16}$$

In Eq. (15) we use

$$\frac{(\delta B)_A^2}{8\pi} = 2\int_{\omega_0}^{\infty} d\omega J(\omega) \simeq \frac{2J_0}{s-1}\,\omega_0^{1-s} \quad , \tag{17}$$

where ω_0 denotes the lowest frequency of the plasma waves.

Secondly, using $J_2|\omega|^{-2} = J_0|\omega|^{-s}$ at $\omega = -\Omega_{0,p}$, a constant mean free path of electrons,

$$\lambda_e = \frac{L_0}{J_2} = \frac{L_0}{J_0}\Omega_{0,p}^{s-2}, \tag{18}$$

equal to the non-relativistic proton value (15), results if the special value $s = 2$ is taken for the frequency power spectrum in the *Whistler* frequency range. In this frequency range the full dispersion relation (8) can be approximated as

$$\omega \simeq -V_A k^2/k_c, \tag{19}$$

so that the relation (5) yields the corresponding wavenumber power spectrum

$$I(k) = J(\omega)|\frac{d\omega}{dk}| = I_0 k^{-3}, \tag{20}$$

with $I_0 = 2J_0 k_c/V_A$. Note that this special form of the high wavenumber power spectrum is the limit of allowable dissipation range spectra in order that the mean square curl of the turbulent magnetic field be finite (Bieber et al. 1990, 1994).

To conclude: the flatness and magnitude problems of quasilinear mean free paths of solar flare particles can be resolved if the interplanetary slab Alfven wavenumber magnetic power spectrum varies $\propto |k|^{-s}$ below $|k| \leq k_c$ with $s = 5/3$, and $\propto |k|^{-3}$ above $|k| > k_c$.

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