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On the cosmic ray cross-field diffusion in the presence of highly perturbed magnetic fields

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Abstract

A process of particle cross-field diffusion in high amplitude Alfvénic turbulence is considered using the Monte Carlo particle simulations. We derive the cross-field diffusion coefficient $\kappa_\perp$ in the presence of different 1-D, 2-D and 3-D turbulent wave field models. Vanishing of $\kappa_\perp$ in 1-D turbulence models is used as an accuracy check for our numerical computations. We note substantial differences in the cross-field diffusion efficiency at the same perturbation amplitude, depending on the detailed form of the considered turbulent field. We reproduce the expected increase of $\kappa_\perp$ with the growing power of waves propagating perpendicular to $\mathbf{B}_0$. Substantially larger values of $\kappa_\perp$ appear in the presence of long compressive fast-mode waves in comparison to the Alfvén waves. We interpret these results in terms of particle drifts in non-uniform magnetic fields. In some cases an initial regime of sub-diffusive transport appears in the simulations.

1 Introduction

Investigation of cosmic ray transport in highly perturbed magnetic fields rises a number of issues which are poorly understood. In particular, an analytic theory enabling derivation of particle diffusion across the magnetic field is still not available. To date the quantitative analytical derivations of the cross-field diffusion coefficient, $\kappa_\perp$, in turbulent magnetic fields are limited to small perturbation amplitudes, $\delta B \ll B_0$ (cf. Jokipii 1971, Achterberg & Ball 1994). A significant result in this respect was achieved in last years by Giacalone & Jokipii (1994; see also Jones et al. 1998), who provided a proof that the cross-field diffusion requires a three dimensional nature of the turbulent field. In the present paper we review our present numerical Monte Carlo simulations of the particle cross-field diffusion in different magnetic turbulent fields.

2 Description of simulations

Let us consider (cf. Michałek & Ostrowski 1997, 1998) an infinite region of tenuous plasma with a uniform mean magnetic field along the z-axis. It is perturbed with the described below propagating MHD waves. Test particles are injected at random positions into this turbulent magnetized plasma and their trajectories are followed by integrating particle equations of motion in space and momentum. By averaging over a large number of trajectories one derives the required diffusion coefficients for turbulent wave fields. In the simulations we usually used 500 relativistic particles with the same initial velocity $v_{in} = 0.99c$ in an individual run.

2.1 Wave field models For high amplitude waves, there are no analytic models available reproducing the turbulent field structure. Because of that, approximate models representing such fields are considered, with turbulence represented as a superposition of Alfvén or fast-mode waves. The wave parameters – wave vectors k , wave amplitudes δB_0 and initial phases Φ – are drawn in a random manner from the flat ($F(k) \propto k^{-1}$) or the Kolmogorov ($F(k) \propto k^{-5/3}$) wave spectra. The wave vectors are expressed in units of the 'resonance' wave vector $k_{res} = 2\pi/r_g(< B >, p_0)$ for the injected particle with momentum $p = p_0$ in the mean magnetic field $< B >= \sqrt{B_0^2 + \delta B^2}$. The wave vectors are selected from the range $0.08k_{res} < k < 8.0k_{res}$. Integration time is expressed in units of gyrofrequency $\Omega_0 = eB_0/\gamma mc$. The magnetic field fluctuation vector related to the wave 'i', $\delta\mathbf{B}^{(i)}$, is given in the form:

$$\delta\mathbf{B}^{(i)} = \delta\mathbf{B}_0^{(i)} \sin(k^{(i)}\vec{r} - \omega^{(i)}t - \Phi^{(i)}) \quad . \tag{1}$$

We consider the following turbulence models:
(i) Linearly polarized plane waves-model (A)
In the model we consider superposition of plane Alfvén waves propagating with the same intensity along the z-axis, in the positive (forward) and the negative (backward) direction.
(ii) 'Wave-packets' models (B1 & B2)
We propose a simple extension of the above model to three dimensions by considering wave packets, involving wave modulation in one direction perpendicular to the propagation direction. In the present case one can use the formula (1) for $\delta\mathbf{B}^{(i)}$, where the phase parameter is subject to sinusoidal modulation. Two types of modulation – presented for the x-components in Eq. (1) – are considered: **B1.)** the 'smooth' sinusoidal modulation with $\Phi_x^{(i)}(y) = \sin(k_y^{(i)} y)$ and **B2.)** the 'sharp-edged' modulation with $\Phi_x^{(i)}(y) = y \; mod \; (1/k_y^{(i)})$. The y-components can be obtained from the above formulae by interchanging x and y. Vectors $k_x^{(i)}$ and $k_y^{(i)}$ are drawn in a random manner from the respective wave-vector range for $k^{(i)}$.
(iii) Oblique MHD waves
We consider a superposition of plane MHD waves propagating obliquely to the average magnetic field $\vec{B}_0 \equiv B_0\hat{\mathbf{e}}_z$. The wave propagation angle with respect to $\vec{B}_0$ is randomly chosen from a uniform distribution within a cone ('wave-cone') along the mean field. For a given simulation two symmetric cones are considered centered along $\vec{B}_0$, with the opening angle 2α, directed parallel and anti-parallel to the mean field direction. The same number of waves is selected from each cone in order to model the symmetric wave field.

For the model (iii) we consider four different turbulent fields labeled as follows:
— Alfvén waves with the flat wave spectrum - AF,
— Alfvén waves with the Kolmogorov spectrum - AK,
— Fast-mode magnetosonic waves with the flat spectrum - MF,
— Fast-mode magnetosonic waves with the Kolmogorov spectrum - MK,
and characterized with parameters α and δB.

3 Results of simulations and discussion

3.1 Alfvénic turbulence models (A, B1, B2) Examples of the derived *formal*[1] cross-field diffusion coefficients versus the integration time are presented for the considered Alfvénic turbulence models A, B1, B2 at Fig. 1 . For the one dimensional plane wave model A one can note that – as required by Giacalone & Jokipii (1994) – the cross-field diffusion coefficient falls off as $\propto t^{-1}$, as expected for a particle dispersion constant in time. An important feature seen in Fig. 1 and Fig. 2 is that the value of $\kappa_\perp$ depends substantially on the assumed shape of the magnetic field perturbations. For the same amplitude and a 'similar' form of modulation applied in models B1 and B2 the diffusion coefficient values can differ by more than an order of magnitude. For the model B2 (sharp-edge modulated waves), the regime of sub-diffusive transport across the mean magnetic field is discovered on a short time scale, with $\kappa_\perp$ slowly decreasing in the beginning and it approaches a constant value at large t. This time evolution of particle spatial cross-field dispersion differs from the one expected for the ordinary diffusion, with a short initial free-streaming followed by the phase with $\kappa_\perp$ fluctuating near some constant value. The observed behaviour reflects the restraining influence of stochastic particle trapping by large amplitude magnetic waves. When inspecting the initial part of the curve

[1]E.i., the derived particle dispersion squared and divided by the integration time multiplied by two.

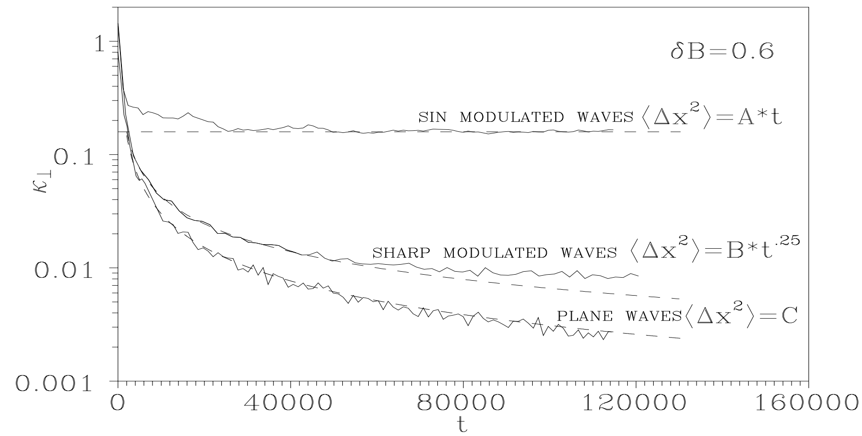

Figure 1: Examples of simulated $\kappa_\perp$ versus the integration time t. The power-law fits of the cross-field particle dispersion $<\Delta x^2>$ are presented for the model A and for the initial part of the curve for the model B2 (A, B and C – constants). A constant fit is provided for the model B1.

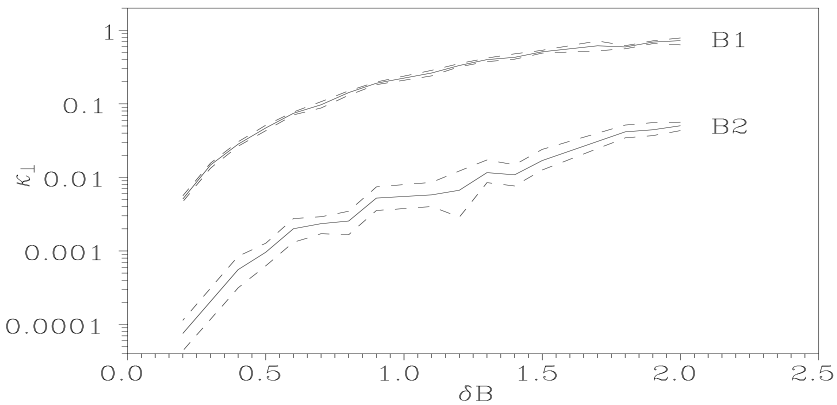

Figure 2: The simulated values of $\kappa_\perp$ versus δB for the wave models B1 and B2. Solid lines join the results obtained using our fitting procedure. The adjacent dashed lines provide information about errors as they join the maximum (or respectively minimum) values of the quantity measured within the range used for fitting.

for the model B1, one observes in a narrow time range an analogous sub-diffusive evolution of particle distribution. In this case the ordinary particle cross-field diffusion is much larger than in the case B2 and particles decorelate from any given 'trap' much earlier. Numerical experiments performed by us proved that the phenomenon is caused by the long distance correlations introduced by the longest waves. In Fig. 2 we present the simulated values of $\kappa_\perp$ versus the wave amplitude δB. For the considered 3-D turbulence models we proved the possibility of substantial – by more than one order of magnitude at the same δB – difference in $\kappa_\perp$ between at first glance similar turbulence models. Such difference does not disappear for $\delta B \geq 1$. The reason for this difference is a more uniform modulation pattern in our model B2 with respect to B1. The value of $\kappa_\perp$ is closely related to the value of the magnetic field line diffusion coefficient D_m (Michałek & Ostrowski 1997), but the growth of the wave amplitude is accompanied by a slight increase in the ratio of $\kappa_\perp / D_m$. This corresponds to the relative increase of the particle cross-field scattering due to particle-wave interactions relative to the diffusion caused by magnetic field line wandering.

3.2 Oblique MHD waves The derived values of $\kappa_\perp$ for different wave-cone opening angles and for different turbulence amplitudes are presented in Fig. 3. For the flat spectrum turbulence a

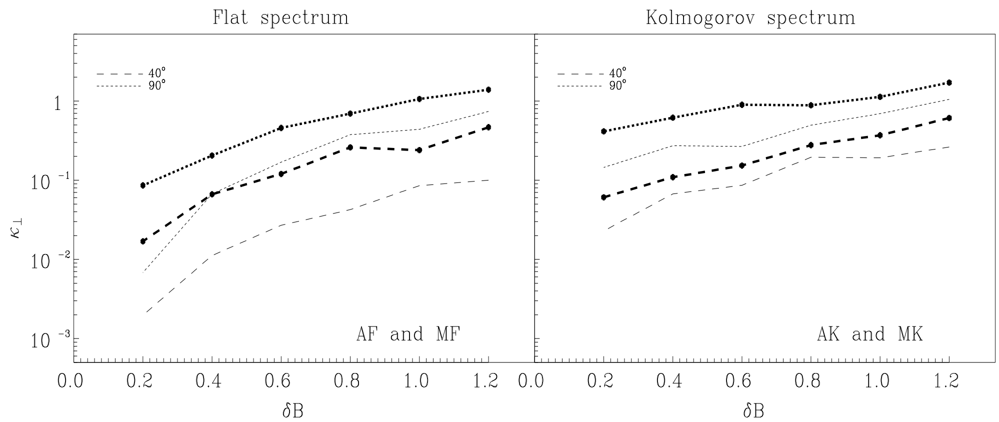

Figure 3: Variation of the cross-field diffusion coefficient $\kappa_\perp$ versus the perturbation amplitude δB and the wave propagation anisotropy (angle α) for the flat spectrum and the Kolmogorov spectrum. Results for the Alfvén turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed on the same panels.

systematic increase of $\kappa_\perp$ with amplitude occurs and the rate of this increase roughly scales as δB^2. The value of $\kappa_\perp$ at any given δB is a factor ~ 10 larger for the fast-mode waves in comparison to the Alfvén waves. It grows substantially with the increasing wave cone opening α, i.e. with increasing power of waves perpendicular to the mean magnetic field. For the Kolmogorov spectrum a dependence of $\kappa_\perp$ on the perturbation amplitude is flatter, the values of the cross-field diffusion coefficient at small δB are larger and there is a smaller difference between the fast-mode waves and the Alfvén waves. The characteristic features seen in Fig. 3 can be qualitatively explained with the use of simple physical arguments discussed by Michałek & Ostrowski (1998). Results presented there show much larger increases of respective D_m than $\kappa_\perp$. It proves that in the range of δB considered here $\kappa_\perp$ is in a substantial degree controlled by the cross-field drifts and the resonance cyclotron scattering, and not by the field line diffusion. Let us stress that the substantial cross-field shifts accompany wave particle interaction involving the so called 'transit time damping resonance', where for the effective cross-field drift, the particle velocity $v_{||}$ and the wave phase velocity $V_{||}$ along the mean field are approximately equal ($V_{||} = V_A$ for the Alfvén waves and $V_{||} = V_A(k/k_{||})$ for the magnetosonic fast-mode ones). For $V_A = 10^{-3}$ and $v = 0.99$ considered in our simulations a noted difference between $\kappa_\perp$ for the Alfvén and the fast-mode waves occurs as a result of satisfying the resonance condition in a wider range of $v_{||}$ by oblique fast-mode waves. Another difference arises from the fact that the linear compressive terms occur only in fast-mode waves. It enables for gradient drifts at small perturbation amplitudes and enables particle cross-field transport when interacting with long waves.

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References

Achterberg A., Ball L., 1994, A&A, 284, 687
Giacalone J., Jokipii J.R., 1994, ApJ Lett., 430, L137
Jokipii J.R., 1971, Rev. Geophys. Space Phys., 9, 27
Jones F.C., Jokipii J.R., Baring M.G., 1998, ApJ, 509, 238
Michałek G., Ostrowski M., 1997, A&A, 326, 793
Michałek G., Ostrowski M., 1998, A&A, 337, 558