Non-diffusive SCR propagation in the radial IMF

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Solar cosmic ray propagation in the interplanetary medium is considered in terms of the kinetic equation describing both the scattering of charged particles by magnetic inhomogeneities and their focusing by the regular interplanetary magnetic field (IMF). The analytical expression for the distribution function of energetic particles is obtained and it is shown that temporal profiles of cosmic ray (CR) intensity vary significantly in dependence on a value of pitch angle of detected particles.

1. Cosmic ray distribution function

High energy charged particles being accelerated during solar flares are focusing by the regular IMF and also scattered by magnetic inhomogeneities. Larmor radius of the energetic particles in IMF is small in comparison with a characteristic scale of regular magnetic field variations, therefore, one can use the drift approximation of kinetic equation to describing the propagation of CR (Earl,1976;Toptygin,1985):

$$\frac{\partial f}{\partial t} + v\ \cos\theta\ \frac{\partial f}{\partial r} - \frac{v\ \mathrm{div}\mathbf{h}}{2}\sin\theta\ \frac{\partial f}{\partial \theta} = Stf\,. \tag{1}$$

Here f is CR distribution function, θ is the particle pitch angle and $\mathbf{h}$ is a unit vector in the direction of the regular IMF. The right hand side of the kinetic equation (1) presents the collision integral which describe the change in the particle number caused by their scattering by magnetic inhomogeneities.

Let us examine CR propagation in the radial IMF $\mathbf{h} = \mathbf{r}/r$. In this case Eq.(1) has the form

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial r} + \frac{v(1-\mu^2)}{r}\frac{\partial f}{\partial \mu} = Stf\,, \tag{2}$$

where $\mu = \cos\theta$. Following Toptygin (1985), Fedorov et al. (1995), the collision integral describing CR scattering can be written as

$$Stf = \frac{\nu_s}{4\pi}\int d\Omega f - \nu_s f\,, \tag{3}$$

ν_s is the collision frequency, and the integration is performed over the angles of the particle velocity vector.

We introduce the dimensionless quantities $\rho = r/\Lambda$, $\tau = vt/\Lambda$, $\Lambda = v/\nu_s$, and an instanteneous isotropic particle source situated at r_0:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{v}}{v}\frac{\partial f}{\partial \mathbf{r}} + f - \frac{1}{2}\int_{-1}^{1} d\mu\ f = \frac{\delta(\rho-\rho_0)\delta(\tau)}{16\pi^2\Lambda^3\rho^2}\,. \tag{4}$$

Let us introduce a new function, which is proportional to the product of CR distribution function $f(\rho,\mu,\tau)$ and $\exp(\tau)$ and perform Fourier transform in the space variable and Laplace transform in time (Fedorov et al.,1995). The obtained solution is the superposition of distribution functions of unscattered and scattered particles (f_0 and f_s, respectively). Their Fourier-Laplace transforms are

$$f_0(\mathbf{k},\omega) = \frac{\sin k\rho_0}{4\pi\Lambda^3 k\rho_0(\omega + i\mathbf{kg})}; \quad (5)$$

$$f_s(\mathbf{k},\omega) = f_0(\mathbf{k},\omega)\frac{\arctan(k/\omega)}{k - \arctan(k/\omega)}. \quad (6)$$

Performing inverse Laplace and Fourier transforms of Eq.(5) one obtains the following relation for the unscattered particles distribution function

$$f_0(\rho,\mu,\tau) = \frac{\delta\{\zeta(\tau) - \rho_0\}\exp(-\tau)}{16\pi^2\Lambda^3\rho_0\zeta(\tau)}, \quad (7)$$

$\zeta(\tau) = \sqrt{\rho^2 + \tau^2 - 2\rho\mu\tau}$.

In accordance with Eq.(7) in a fixed point ($\rho > \rho_0$) one can observe the unscattered particles with pith angles situated only in the limited interval of Θ , so that $\mu > \mu_{\min} = \sqrt{1 - (\rho_0/\rho)^2}$. In case of prolonged injection of particles the expression for distribution function of unscattered particles in the external region ($\rho > \rho_0$) has been derived by integration Eq.(7) over the time for a constant particle source:

$$f_0(\rho,\mu) = \frac{\exp(-\rho\mu)\ \mathrm{ch}\sqrt{\rho_0^2 - \rho^2(1-\mu^2)}}{8\pi^2\Lambda^3\sqrt{\rho_0^2 - \rho^2(1-\mu^2)}}. \quad (8)$$

The distribution function (8) increases unlimitedly when Θ approaches its maximum value $\Theta_{\max} = \arccos\sqrt{1 - (\rho_0/\rho)^2}$, however, the integration of Eq.(8) over μ leads to the limited value of unscattered particle density.

To calculate the distribution function of scattered particles f_s we perform at first the inverse Laplace transform of Eq.(6) and then the inverse Fourier transform. In result,

$$\begin{aligned} f_s(\rho,\mu,\tau) &= \frac{\exp(-\tau)}{8\pi^2\Lambda^3\rho_0}\int_0^{\pi/2} dk\ \frac{k^2\sin k\rho_0}{\sin^2 k}\int_0^{\tau} d\xi\frac{\exp[k\ \mathrm{ctg}\ k(\tau-\xi)]\ \sin k\varsigma(\xi)}{\varsigma(\xi)} \\ &+ \frac{\exp(-\tau)}{64\pi^3\Lambda^3\rho_0}\int_0^{\tau}\frac{d\xi}{\zeta(\xi)}\int_0^1 d\eta\{\Phi_1(\xi,\eta) - \Phi_2(\xi,\eta)\} \end{aligned} \quad (9)$$

where

$$\Phi_i(\xi,\eta) = \Theta(z_i)\exp\left(\frac{\kappa}{2}z_i\right)\{2\pi\kappa\ \cos\frac{\pi}{2}z_i + (\kappa^2 - \pi^2)\sin\frac{\pi}{2}z_i\},$$

$$\kappa = \ln\frac{1-\eta}{1+\eta}; \qquad z_{2,1} = \mid\zeta(\xi) \pm \rho_0\mid - (\tau - \xi)\eta$$

and Θ(x) stands for the Heaviside step-function.

2. Discussion

The dependence of distribution function of scattered particles (9) on dimensionless time in the $\rho = 1$ position is shown in Fig.1. Numbers near the curves stand for μ values and ρ_0

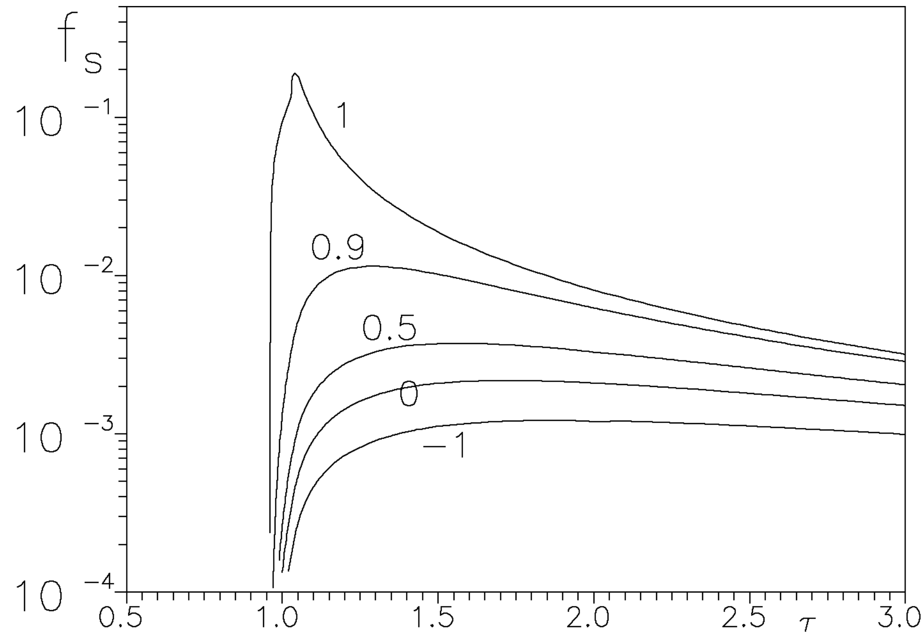

Fig.1. The scattered particle distribution function dependence on the time in the position ρ=1.ρ_0=.04.Numbers correspond to μ=−1. 0. .5. .9. 1.

equals to 0.04. One can see that the temporal profiles of scattered particles intensity depend significantly on the direction of their motion. The maximal values of distribution functions for $\mu = 1$ and $\mu = -1$ differ from each other on more than two orders of magnitude and the variance in the initial phase of the event is even more strong.

Thus, the CR distribution function consist of unscattered and scattered particles if CR scattering is described by collision integral corresponding to the large angle scattering. The unscattered particles propagate as a strongly anisotropic beam ($\Theta < \Theta_{\rm max}$) and they cause impulsive CR intensity peak. It is worth to note that in the case of small angle scattering the separation of particles on scattered and unscattered is not true any more and CR distribution function is described by quite different expression (Earl, 1996).

References

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