The Dynamics of Dissipation Range Fluctuations with Application to Cosmic Ray Propagation Theory

Robert J. Leamon, Norman F. Ness and Charles W. Smith
Bartol Research Institute, University of Delaware, Newark, DE 19716, USA

Abstract

Unlike larger spatial scales of the interplanetary magnetic field fluctuation spectrum, study of the smallest scale fluctuations (comparable to the gyroradius of a thermal proton) which form the so-called dissipation range has been somewhat neglected. This spectral range is characterized by a steeply falling power spectrum and frequently nonzero magnetic helicity, features thought to result from the dissipation of magnetic fluctuations by thermal particle populations. Although this range contains relatively little energy, it is relevant to energetic particle scattering because low-rigidity particles and all particles at large pitch angles become resonant with these fluctuations. Analyzing power and helicity spectra of WIND data, we deduce the orientation of the wavevectors and find that most of the wave energy is associated with wavevectors at large angles to the mean magnetic field. We place these observations within an existing framework for turbulent scattering of cosmic rays.

1 Introduction

Relatively few studies of the dissipation range of interplanetary magnetic turbulence exist when compared to the inertial range at lower frequencies. Figure 1a shows an example of a high-resolution spectrum taken by the WIND spacecraft in near-Earth orbit, and its associated reduced magnetic helicity spectrum. The inertial range spectrum terminates at 0.44 Hz in a spectral break to a steeper spectral index. This break marks the onset of the dissipation range.

The possible involvement of ion cyclotron activity in the observed onset of steepening has been discussed for some time (Behannon 1976; Denskat, Beinroth, & Neubauer 1983); we too observe such a correlation. In all of our events, this steepening of the dissipation range sets in at $f_{\rm sc} > \Omega_p/2\pi$, where Ω_p is the proton cyclotron frequency, but the $\mathbf{k} \cdot \mathbf{V}_{SW}$ Doppler shift makes it likely that $\omega < \Omega_p$. In the spacecraft frame, we find that as a reasonable first approximation, the break frequency $f_{\rm sc}$ is about 4 times the gyrofrequency $\Omega_p/2\pi$.

Although the dissipation range contains very little energy, it is important because low-rigidity particles and all particles at large pitch angles become resonant with fluctuations at those scales. Magnetostatic, quasilinear scattering by the "slab" geometry that omits consideration of the dissipation range gives too much scattering, especially at low rigidity. To counter this, Bieber, Smith & Matthaeus (1988) and Smith, Bieber & Matthaeus (1990) argue that incorporation of a dissipation range in magnetostatic scattering significantly alters the mean-free-paths of energetic

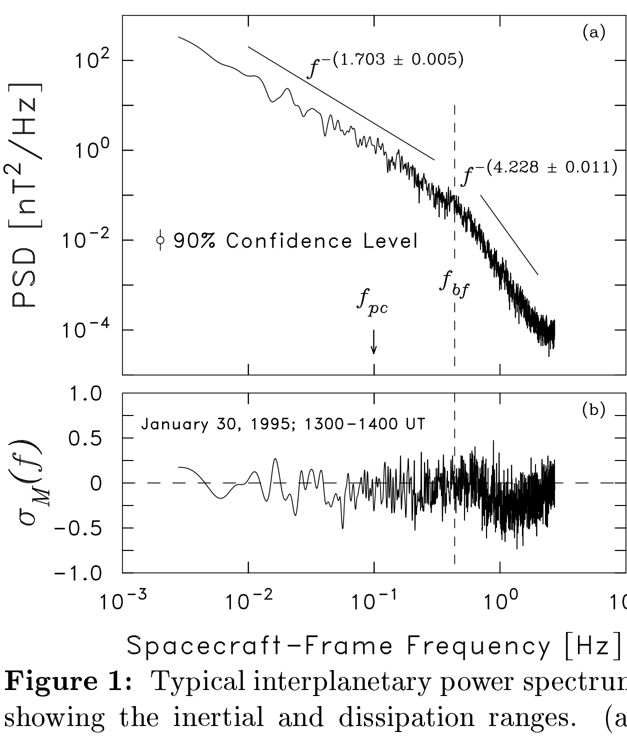

Figure 1: Typical interplanetary power spectrum showing the inertial and dissipation ranges. (a) Trace of the spectral matrix with a break at $\sim$ 0.4 Hz where the dissipation range sets in. (b) The corresponding magnetic helicity spectrum.

particles. Bieber et al. (1994) employ the dissipation range, together with magnetodynamic effects, to produce mass-dependent mean-free-paths that are distinct from the usual rigidity-dependent forms. This leads to differing mean-free-paths for protons and electrons of equal rigidity, in general agreement with a large class of solar energetic particle observations.

Understanding suprathermal particle scattering therefore requires better determination of the turbulence geometry; *i.e.*, direction of $\mathbf{k}$. Traditionally, the reported observation of magnetic fluctuations perpendicular to the mean magnetic field (Belcher & Davis 1971) has been used to motivate $\mathbf{k} \parallel \mathbf{B}$. However, the possibility that an energetically significant fraction of the wave vectors could be nearly at $\mathbf{k} \perp \mathbf{B}$ was shown by Matthaeus, Goldstein & Roberts (1990). Bieber, Wanner & Matthaeus (1996) assumed a composite two-dimensional (2D)/slab model for the magnetic turbulence and determined that in the inertial range there is a dominant ($\sim 85\%$ by energy) 2D component. The 2D component does not contribute to resonant scattering of very energetic particles (cosmic rays) and can explain the observed problem of "too small" cosmic ray mean free paths (Bieber et al. 1994). Whereas Bieber et al. (1996) considered solar particle events, we shall extend their methods to the undisturbed solar wind and in frequency to the high-frequency end of the inertial range (~ 0.02 to ~ 0.2 Hz) and the low-frequency end of the dissipation range (~ 0.5 to $\lesssim 2$ Hz).

2 Magnetic Helicity

The results presented here are based on the analysis of 33 1-hour intervals of quiet solar wind data, from the magnetic field and thermal plasma instruments of the WIND spacecraft. This data set and the method of analysis is described in detail by Leamon et al. (1998).

For the 33 quiet solar wind intervals, the spectral indices of the inertial range were between -1.46 and -1.93, with an average of -1.67. The dissipation range indices range from -1.93 to -4.43, with the average being -3.01. No clear correlation between the fitted indices of the two ranges is observed.

Figure 1b shows the reduced magnetic helicity spectrum for that interval. Note the negative signature at dissipation frequencies, averaging -0.275 over those frequencies used to compute the dissipation range spectral slope. If there is finite magnetic helicity, the sign of the particle's charge can enter into the rigidity dependence of mean free path as a second order effect (Goldstein & Matthaeus 1981; Bieber, Evenson & Matthaeus 1987; Bieber et al. 1994). This is accomplished by changing the amount of energy available for resonant scattering by adjusting the net polarization of the power spectrum within any given range.

Perhaps more importantly, nonzero magnetic helicity can lead to charge-sign dependent resonant scattering. Dissipation of outward propagating Alfvén waves leads to $\langle B_R \rangle \langle \sigma_M \rangle$ effects that modify scattering of energetic particles when in resonance with the small scales. Of the 33 events studied, 27 have dissipation range helicity signatures of $|\sigma_M| > 0.1$, and only 3 events have $\langle B_R \rangle \langle \sigma_M \rangle > 0$.

The apparent depletion of outward propagating Alfvén waves at frequencies comparable to the proton gyrofrequency naturally suggests resonant cyclotron damping of such Alfvén waves as the leading candidate for the formation of the dissipation range. However, such a theory cannot predict the onset of the dissipation range (Leamon et al. 1998).

3 Anisotropy

The classic study of inertial range magnetic fluctuations is that of Belcher & Davis (1971), They defined a coordinate system relative to the mean magnetic field direction, $\hat{\mathbf{B}}$, and radial direction, $\hat{\mathbf{R}}$, according to: $\{\hat{\mathbf{B}} \times \hat{\mathbf{R}}, \hat{\mathbf{B}} \times (\hat{\mathbf{B}} \times \hat{\mathbf{R}}), \hat{\mathbf{B}}\}$ and showed that the average variances for these three components are in the ratio $5 : 4 : 1$. We note that this implies a ratio for the total variance transverse to and aligned with the mean field of $9 : 1$. This high level of anisotropy is consistent with the fluctuations consisting of Alfvén waves, especially considering the high degree of $\langle \mathbf{v} \cdot \mathbf{b} \rangle$ correlation observed.

We define $P_{||}$ to be the power in fluctuations parallel to $\hat{\mathbf{B}}$ and $P_\perp$ to be the total power in both components perpendicular to the mean field. We acknowledge that there is a subtle difference between our method and that of Belcher and Davis who use average variances. For the high-frequency end of the inertial range, our results find a mean $P_\perp : P_{||}$ ratio of 14 : 1, with a range $3.0 \leq P_\perp/P_{||} \leq 53.2$. Taking into account that the above mean may be unduly biased by several samples with unusually large values, we note that the geometric mean of $P_\perp/P_{||}$ is only 10.4, which is in closer agreement with the result of Belcher and Davis. For the dissipation range we find a mean ratio of 5.4 : 1 with a range $2.36 \leq P_\perp/P_{||} \leq 12.8$ and a geometric mean ratio of 4.9 : 1. The dissipation range ratios $P_\perp/P_{||}$ are consistently less than inertial range ratios, implying a decreased importance of transverse fluctuations in the dissipation range and an increase in the compression of the plasma at these scales.

4 Geometry

The Belcher & Davis anisotropy is usually taken as evidence of slab waves, even though it is consistent with 2D turbulence. By 2D turbulence we mean fluctuations that have wave vectors that are nearly transverse to **B**. Most people interpret Belcher & Davis' 5 : 4 : 1 anisotropy as a $P_\perp : P_{||} = 9 : 1$ ratio; the 5 : 4 part is considered physically unimportant. However, there is physical meaning to the ratio of the power in the two perpendicular directions (*i.e.*, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ in the mean-field coordinate system outlined in section 3), and reason to expect that they should not be equal. Following Bieber et al. (1996), in a test based on the analysis of Oughton (1993) we use this ratio as a direct link to the percentage of slab waves and 2D modes in the fluctuations. Bieber et al. use a coordinate system that is a 90° right-handed rotation away from Belcher & Davis (around the $\hat{\mathbf{z}}$ or $\hat{\mathbf{B}}$ axis). In the analysis that follows, we use Bieber's conventions, such that $\hat{\mathbf{y}} = \hat{\mathbf{B}} \times \hat{\mathbf{R}}$.

We assume that the magnetic fluctuations consist of a mixture of slab and 2D geometries and compute their relative strengths from the ratio of transverse spectral powers. C_S and C_2 are the amplitudes of the slab and 2D components, respectively; *i.e.*, the slab spectrum in the range of interest is parameterized by $C_S k^{-q}$ and the 2D spectrum by $C_2 k^{-q}$. We further assume that the two components obey the same power law (that is, they have the same spectral index $-q$). This is equivalent to the statement that P_{xx} and P_{yy} obey the same power law, which is not strictly obeyed, at least not within our data set, but is approximately true.

The "slab fraction," r, is the contribution of the slab component to the energy spectrum, relative to the total energy,

$$r \equiv \frac{C_S}{C_S + C_2} = \frac{1}{1 + r'}, \tag{1}$$

where $r' = C_2/C_S$. Equations (16) and (17) of Bieber et al. (1996) and the above definition leads to the following formula for the ratio of power between components:

$$\begin{aligned} \frac{P_{yy}(f)}{P_{xx}(f)} &= \frac{C_S\left(\frac{2\pi f}{V_{SW}\cos\theta}\right)^{1-q} + C_2\frac{2q}{1+q}\left(\frac{2\pi f}{V_{SW}\sin\theta}\right)^{1-q}}{C_S\left(\frac{2\pi f}{V_{SW}\cos\theta}\right)^{1-q} + C_2\frac{2}{1+q}\left(\frac{2\pi f}{V_{SW}\sin\theta}\right)^{1-q}} \\ &= \frac{\left(\frac{2\pi}{V_{SW}\cos\theta}\right)^{1-q} + r'\frac{2q}{1+q}\left(\frac{2\pi}{V_{SW}\sin\theta}\right)^{1-q}}{\left(\frac{2\pi}{V_{SW}\cos\theta}\right)^{1-q} + r'\frac{2}{1+q}\left(\frac{2\pi}{V_{SW}\sin\theta}\right)^{1-q}} \end{aligned} \tag{2}$$

The ratio P_{yy}/P_{xx} (which under our assumptions becomes independent of frequency in the relevant range) and the parameters V_{SW}, θ, the angle between the magnetic field and solar wind velocity, and q are derivable from observations by a single spacecraft. Thus the only unknown in equation (2) is r', which, in turn, gives us the slab fraction r.

For the "middle" of the inertial range, Bieber et al. conclude that IMF geometry is $\sim 85\%$ 2D and only $\sim 15\%$ slab waves. Our results provide an essentially identical result for the high-frequency end of the inertial range, with $\sim 89\%$ of the energy in 2D fluctuations. In the dissipation range, on the other hand, the 2D component falls to $\sim 55\%$, which we may explain by preferential dissipation of 2D structures.

In terms of application to scattering theory, the large 2D component reduces the overall scattering rate by the same percentage. Perpendicular wavevectors are inefficient scatterers of particles, essentially making their percentage of the total energy unavailable for particles.

5 Discussion

In recent work (Leamon et al. 1998, 1999) we have shown that there is both observational and theoretical evidence to support the claim that the dissipation range forms as the result of dissipating energy associated with wave vectors at large angles to the mean magnetic field. This is consistent with inertial range studies (Matthaeus, Goldstein & Roberts 1990; Bieber et al. 1996) that indicate the same geometry at these larger scales and cosmic ray mean-free-path analyses (Bieber et al. 1994). The results described here are expected to aid in the refinement of ongoing cosmic ray propagation analyses.

Additionally, we hope to examine the possible role of magnetic helicity within the dissipation range in determining cosmic ray propagation. Since resonant scattering of large pitch-angle particles by the dissipation range is balanced against magnetodynamic effects and other considerations, the possible role of helicity at the small scales is unclear.

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