The Transport of CIR Accelerated Particles

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Abstract

We discuss the transport of energetic particles accelerated at Corotating Interaction Regions (CIRs). The model of Fisk & Lee (1980) is extended to low energies where the diffusive description does no longer apply. A numerical code has been developed to solve the Fokker-Planck equation, which allows low particles velocities and long scattering mean free path. Numerical results are compared to the predictions of the diffusive model.

1 Introduction:

Energetic ions accelerated at the forward and reverse shocks of Corotating Interaction Regions (CIRs) undergo convection, diffusion, and adiabatic cooling or acceleration during their transport in the heliosphere. The combination of all these processes determine the resulting energy spectrum at the shock. Fisk & Lee (1980) proposed an elegant model which allowed to obtain an analytical approximative solution. This approximation serves as a valuable theoretical base in interpreting corotating particle events (Desai et al., 1999). Giacalone & Jokipii (1997) discussed general aspects of the acceleration of pickup ions ar CIRs.

The Fisk & Lee (1980) theory (referred as FL80 hereinafter) assumes a specific form of the diffusion coefficient, which is a choice of convenience to reach an analytical solution. The physical limitation of the FL80 model is that it is based on a diffusive description, which assumes that particle speed is considerably larger than the plasma speed and scattering is frequent enough to produce near isotropic distributions. These conditions are not met at pickup energies. Also there is evidence (Gloeckler et al., 1995) suggesting that scattering mean free path for low rigidity particles may be as large as several AU.

It is the purpose of the present work to extend the FL80 theory to low energies, including pickup energies and below. We consider numerical solutions of the Fokker-Planck equation that remains applicable for both low and high particle speed and for both strong and weak scattering. Also the numerical schemes can readily accommodate various types of pitch-angle scattering, including hemispherical scattering (Isenberg, 1997; Schwadron, 1998).

2 Transport Equations:

Diffusive shock acceleration is described by the fundamental equation of Parker (1965), which includes diffusion, convection, and adiabatic cooling and acceleration. The FL80 model assumes that low-rigidity particles are tied to their field lines. They are convected together with their field lines, and undergo additional diffusive motion along the field lines.

The Fokker-Planck equation (Skilling, 1971) was applied for low particle speeds by Isenberg (1997) and by Kóta & Jokipii (1997). This equation considers the full directional distribution, $f(z, w, \mu, t)$, as function of position along the field line, z, particle velocity, w, cosine of pitch angle, μ, and time, t. w and μ are measured in the solar wind frame. The rate of adiabatic cooling does, in general, depend on μ. The Fokker-Planck equation is based on an adiabatic approach, averaging over gyro-phase. Thus the adiabatic invariant, $w^2(1-\mu^2)/B$, remains strictly conserved until random scattering is included.

2.1 Corotating Shocks: Corotating shocks represent a special class when steady state prevails in the frame corotating with the sun around the axis, $\mathbf{\Omega}$. This implies that the fluid speed taken in corotating frame, $\mathbf{V}_{cor} = \mathbf{V} - \mathbf{\Omega} \times \mathbf{r}$ is parallel to the magnetic field, $\mathbf{B}$, and magnetic field lines appear to be standing in the corotating frame. The Fokker-Planck equation for this case (Kóta & Jokipii, 1997) is

$$\frac{\partial f}{\partial t} + (V_{cor} + w\mu)\frac{\partial f}{\partial z} + \Big\langle \frac{\Delta\mu}{\Delta t}\Big\rangle_{cor}\frac{\partial f}{\partial \mu} + \Big\langle \frac{\Delta w}{\Delta t}\Big\rangle_{cor}\frac{\partial f}{\partial w} = \frac{\partial}{\partial \mu}\Big(\frac{D_{\mu\mu}}{2}\frac{\partial f}{\partial \mu}\Big) + q \quad (1)$$

where the time variation is to be taken in the rotating frame, and z refers to lenght measured along the field line. The right hand side accounts for random pitch-angle scattering, and for sources, $q(z, w, \mu, t)$. The rates of adiabatic focusing and cooling, respectively can be expressed as:

$$\left\langle\frac{\Delta\mu}{\Delta t}\right\rangle_{cor} = -\frac{w(1-\mu^2)}{2}\Big[(1+\frac{\mu V_{cor}}{w})\frac{\partial ln\, B}{\partial z} + \frac{1}{2w^2}\frac{\partial}{\partial z}({V_{cor}}^2 - (\mathbf{\Omega}\times\mathbf{r})^2) - \frac{\mu}{w}\frac{\partial V_{cor}}{\partial z}\Big] \tag{2}$$

$$\left\langle\frac{\Delta w}{\Delta t}\right\rangle_{cor} = -\frac{\mu}{2}\frac{\partial}{\partial z}({V_{cor}}^2 - (\mathbf{\Omega}\times\mathbf{r})^2) - w\mu^2\frac{\partial V_{cor}}{\partial z} + V_{cor}\frac{w(1-\mu^2)}{2}\frac{\partial ln\, B}{\partial z} \tag{3}$$

These expresssions include but are not restricted to the standard Archimedean spiral field. For the standard spiral field $V_{cor} = V/cos\psi$, with ψ denoting the hose angle, and the coeficients (2) and (3) are identical with the coefficients derived by Ruffolo (1995).

2.2 Boundary Conditions: Shocks, similarly as in the FL80 theory, are treated as boundaries. The boundary conditions, however, are more subtle. We follow an adiabatic approach thus particles are either reflected or transmitted so that their adiabatic invariant is conserved. The FL80 model assumes that scattering is frequent in the downstream region so that the diffusive streaming vanishes and the particle population is convected with the fluid speed downstream of the shock. Following the same concept, we presribe $f(-\mu) = f(\mu)$ downstream of the shock. This corresponds to mirror-reflecting the transmitted particles. While this is still a crude approximation, it contains the essential physics. It can be shown that this jump condition becomes equivalent to that of the diffusion theory, if the particle speed is much larger than the fluid speed.

3 Numerical Results and Discussion:

Our numerical code is applied to model the acceleration and transport of energetic ions at corotating reverse and forward shocks in scenarios similar to those considered by Fisk & Lee (1980). Here we present simulation results for a reverse shock placed 4 AU from the sun. An Archimedean spiral field is adopted with a fast wind speed of 700 km/s and a shock ratio of 3. 10 keV/n particles are injected at the shock isotropically in the solar wind frame. We assume isotropic pitch-angle scattering, with $D_{\mu\mu} = w(1-\mu^2)/2\lambda_{||}$. In accord with the FL80 model, we choose a rigidity independent parallel scattering mean free path, $\lambda_{||}$, so that the resulting radial diffusion coefficient, $\kappa_{rr} = w\lambda_{||}cos^2\psi/3$ be proportional to r. One should remember that the diffusion coefficient in the FL80 model is $\kappa_{rr} = \kappa_{||}cos^2\psi$. Hence, we assume $\lambda_{||} = \lambda_0 r/cos^2\psi$. The value of $\lambda_0 = 0.1$ corresponds to the 0.1 AU mean free path used by Fisk & Lee (1980).

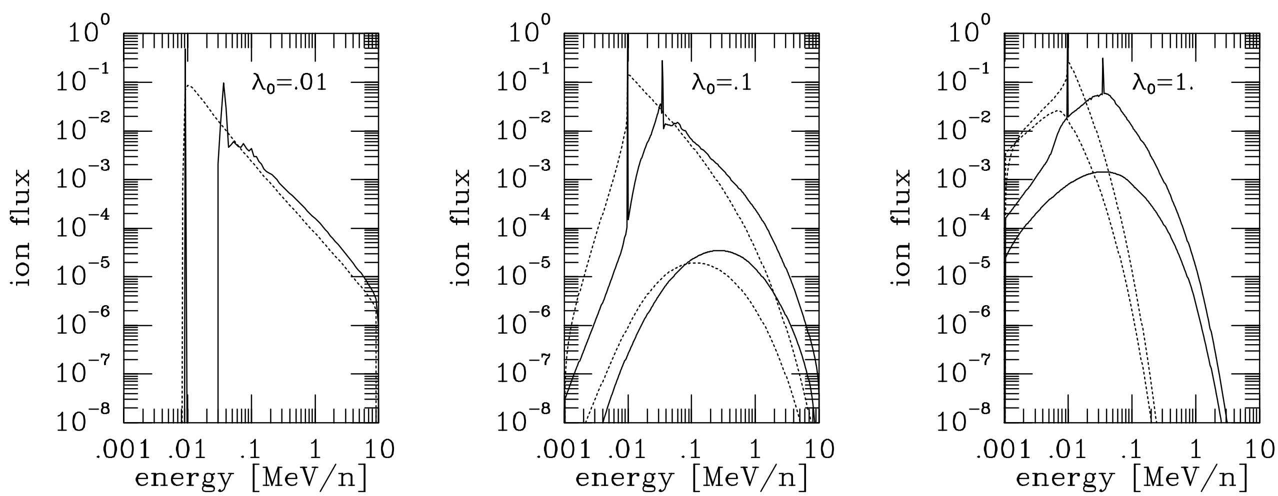

Figure 1: Energy spectra at a corotating reverse shock at 4 AU as obtained from the Fokker Planck equation (solid lines) for scattering rates λ_0= 0.01, 0.1, and 1. Predictions from the diffusive model are shown for comparison (dotted lines). The lower pairs of curves indicate the modulated spectra at 1 AU.

Figure 1 shows the resulting spectra for three different values of the mean free path, together with the predictions obtained from a diffusion model assuming the same isotropic injection at 10 keV/n. Also shown are in Figure 1 the modulated spectra at 1 AU. For strong scattering ($\lambda_0 = 0.01$) the diffusive model gives, as expected, good approximation. Significant difference appears, however, already at the intermediate mean free path, $\lambda_0 = 0.1$. The Fokker-Planck equation gives harder spectrum in the 0.01 - 1 MeV/n region. This may be connected with the fact that diffusion approximation assumes that the relative gain of energy is small in each crossing of the shock. At low energies, however, the relative energy gain in an encounter is quite large. This is demonstrated by the distance of the secondary peak around 30 keV/n from the injection peak at 10 keV/n. The secondary peak contains particles that have been reflected once from the shock.

For the case of weak scattering ($\lambda_0 = 1.$) the results of the Fokker-Planck equation differ quite dramatically from the predictions of the diffusion model, which latter gives much steeper spectrum. Clearly, diffusion models become inaccurate if the scattering mean free path is comparable or larger than the focusing length. Conversely, this implies that fitting observations to a diffusion model may considerably underestimate the scattering mean free path if the low-energy $0.1 - 1$ MeV/n range is considered.

It may be interesting to notice that the diffusion model does significantly underestimate the modulation for weak scattering. In the extreme case, scatter-free propagation would not give any modulation at all in a diffusion model. However, adiabatic cooling and consequent modulation do occur even in scatter-free propagation and this could lead to significant modulation, if the shock is far out (Kóta & Jokipii, 1999). This effect might be responsible for the decreasing magnitude of recurrent ion events at high latitudes (see, Simnett & Roelof, 1998).

We find that the Fokker-Planck equation predicts harder spectra for weak and intermediate scattering. One possible explanation is that the diffusion models assume near isotropy, which is not valid if scattering is weak. The pitch-angle distributions of 1.5 MeV/n ions at the shock, which are shown in Figure 2., reveal a significant anisotropy even in the case of intermediate rate of scattering. The distributions tend to peak around $\mu = -0.7$ which corresponds to the direction of the maximum energy gain at reflection from the shock.

The characteristic times required for acceleration are of interest. Figure 3 shows the time evolution of the spectrum at the shock, for $\lambda_0 = 0.1$. For this rate of scattering it takes about 50 days to reach steady state. The weaker rate of scattering results in longer characteristic times for acceleration. Spectra may significantly soften if the shock happens to be newly formed, or non-steady, as it can be expected, for instance, in a Fisk field (Fisk, 1996).

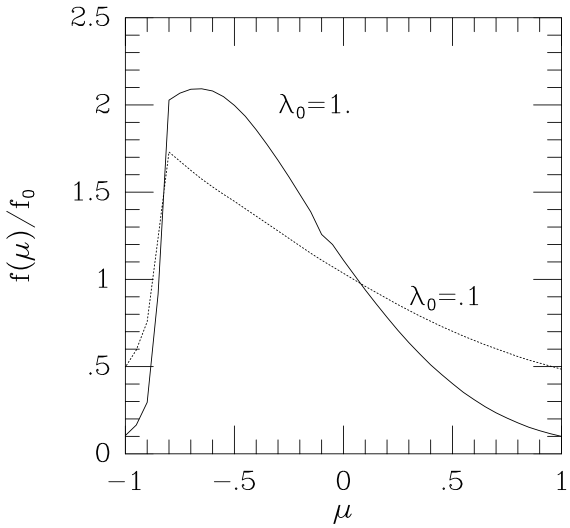

Figure 2: Pitch angle distributions of 1.5 MeV/n ions at a reverse shock, for $\lambda_0 = 0.1$ and 1.0.

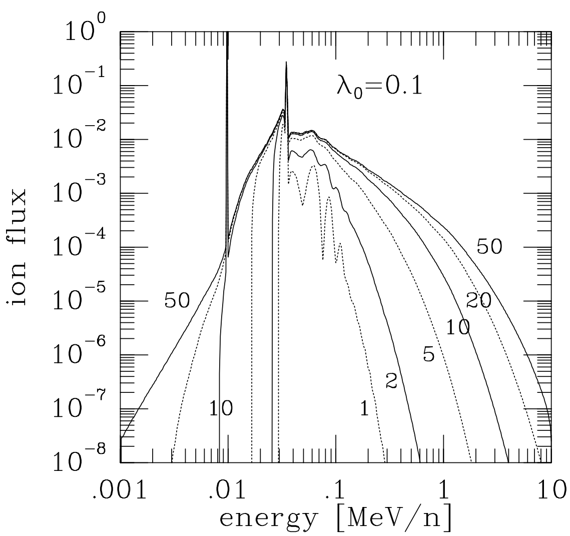

Figure 3: Energy spectra at the shock obtained for $\lambda_0 = 0.1$, at times, 1, 2, 5, 10, 20, and 50 days

3.1 Pickup Ions: The code covers low energies thus it is also capable to model the acceleration and transport of pickup ions, if the condition are met that freshly ionized particles are reflected from the shock.

General aspects of the acceleration of pickup ions at CIRs. has been discussed by Giacalone & Jokipii (1997). Here we present a simulation for the acceleration of freshly ionized particles at a reverse shock. Figure 5 shows the results of such numerical simulations. We inject freshly ionized particles at a corotating reverse shock at 2 AU from the sun. The same shock ratio of 3 and the same form of the scattering mean free path is used as in the previous simulations. We assume a ring-distribution for the source-term, $q(r, w, \mu)$. The 700 km/s value of the fast wind corresponds to an initial energy of ≈ 2.5 keV/n, and an initial value of $\mu_0 \approx -0.7$. The dotted line shows the simulation for intermediate rate of scattering, $\lambda_0 = 0.1$, while the solid line illustrates the resulting spectrum for weak scattering ($\lambda_0 = 1$). The secondary peak at $\sim 17keV/n$ results from particles that have been reflected from the shock one single time.

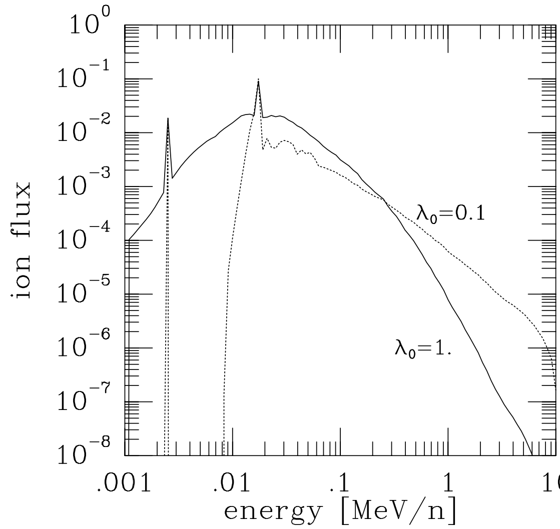

Figure 4: Pickup ions accelerated at a reverse shock 2 AU from the sun

4 Summary

We have reported on the results of a numerical code solving the Fokker-Planck equation and thus extending the model of Fisk & Lee (1980) for particle acceleration at corotating shocks to low energies and large scattering mean free path. We find significant deviations from the diffusion model when the scattering is weak.

The scheme is capable to handle more complex and time-dependent situations like travelling shocks or a Fisk field configuration where field lines are not standing but are slowly rotating in the corotating frame (Fisk, 1996). These will be explored in future work.

5 Acknowledgements

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