

# Influence of Alfvén Wave Nonlinear Interaction on Ion Acceleration at the Earth's Bow Shock

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## Abstract

Influence of Alfvén wave nonlinear interaction on the solar wind ion acceleration at the Earth's bow shock is studied on the basis of numerical solution of selfconsistent transport equations. It is shown that nonlinear interaction of selfexcited Alfvén waves due to the induced scattering and two-quantum absorption essentially restricts the growth of wave amplitudes, whereas a quasilinear approach predicts the generation of Alfvén waves with amplitudes, which significantly exceed the interplanetary magnetic field value. It also leads to a wave energy transform towards the low frequencies that provides an essential increase of the particle acceleration rate.

## 1 Introduction:

An application of the diffusive shock acceleration theory (e.g. Berezhko & Krymsky, 1988) to the Earth's bow shock provides the possibility for the most detailed study of the energetic particle acceleration at collisionless shocks. According to numerous measurements of so-called diffuse energetic particle population with energies from just above solar wind energy to about 200 keV per charge observed at the quasiparallel portion of the Earth's bow shock is almost always accompanied by the intense Alfvén wave generation (e.g. Trattner et al., 1994). Calculations performed in a quasilinear approach show that at some sets of solar wind parameters the amplitude of Alfvén waves  $\delta B$  excited by the accelerated particle streaming can exceeds the interplanetary magnetic field (IMF) value  $B$  (Lee, 1982; Berezhko & Taneev, 1991; Berezhko, Taneev & Petukhov, 1997). One have to take into account the nonlinear Alfvén wave interaction in this case.

We study here an influence of the nonlinear Alfvén wave interaction on the ion acceleration at the Earth's bow shock.

## 2 Model:

We use one-dimensional model for the diffusive ion acceleration at the quasiparallel Earth's bow shock developed in a quasilinear approach (Berezhko & Taneev, 1991; Berezhko, Taneev & Petukhov, 1997). It is based on the diffusive transport equation which is selfconsistently solved together with the Alfvén-wave transport equation and includes an assumption about the injection at the shock front of some small fraction  $\eta \ll 1$  of suprathermal solar wind protons with energy  $\varepsilon_0 = mv_0^2/2 = 5$  keV into the acceleration process, where  $m$  is the proton mass. Besides protons, whose are the main kind of solar wind ions, we also take into account the acceleration of  $\alpha$ -particles, which number density is 5 % of protons. The model effectively takes into account the particle escape from the acceleration region due to their diffusion across the magnetic field lines.

The nonlinear Alfvén wave interaction due to the induced scattering and two-quantum absorption has been studied by Fedorenko et al. (1990, 1995) at physical parameters typical for the solar wind plasma. It is described by the last two terms in the wave transport equation

$$\frac{\partial E_w^\pm}{\partial t} + u \frac{\partial E_w^\pm}{\partial x} = \mp 2\Gamma E_w^\pm + \Gamma_{NL}^\pm E_w^\pm + Q^\pm, \quad (1)$$

where  $E_w^+(E_w^-)$  is the energy density of waves propagated from (towards) the Sun per logarithm of wave number  $k$ ;  $u$  is the solar wind speed;  $\Gamma$  is the growth rate due to the accelerated particle streaming;

$$\Gamma_{NL}^\pm = -\frac{kc_a^2}{E_B} \int_0^\infty S(k, k') E_w^\mp(k') dk'$$

is the wave growth rate due to their nonlinear interaction (Fedorenko et al., 1990, 1995); the x-axis is directed along the Sun-Earth line. At the plasma parameter  $\beta = (v_{Ti}/c_a)^2 \lesssim 1$  the kernel has a form

$$S(k, k') = a_0 \left\{ \frac{k - k'}{k + k'} \exp \left[ -\frac{1}{2\beta} \left( \frac{k - k'}{k + k'} \right)^2 \right] + \left| \frac{k + k'}{k - k'} \right| \exp \left[ -\frac{1}{2\beta} \left( \frac{k + k'}{k - k'} \right)^2 \right] \right\},$$

where  $v_{Ti}$  is the thermal ion speed,  $c_a = B/\sqrt{4\pi\rho}$  is the Alfvén wave speed,  $\rho$  is the solar wind density,  $a_0 = \pi^{3/2}/(32\beta\sqrt{2\beta})$ ,  $E_B = B^2/8\pi$ . The source term

$$Q^\pm = kc_a^2 \frac{E_w^\pm}{E_B} \int_0^\infty S(k, k') E_{w0}^\mp(k') dk'$$

is introduced to provide the necessary condition that the background Alfvén wave spectrum in the solar wind  $E_{w0}^\pm(k)$  is a solution of the wave transport equation when accelerated particles are absent.

Alfvén waves provide the scattering of energetic ions due to their resonant interaction that is described by the parallel ( $\parallel$ ) and perpendicular ( $\perp$ ) diffusion coefficients

$$\kappa_{\parallel}(v) = \frac{v^2 B^2}{32\pi^2 \omega_B E_{w0}(k = \rho_B^{-1})}, \quad \kappa_{\perp}(v) = \frac{\rho_B^2 v^2}{3\kappa_{\parallel}(v)},$$

where  $\rho_B = v/\omega_B$  and  $\omega_B$  are gyroradius and gyrofrequency respectively.

Eq. (1) is solved numerically together with the particle diffusive transport equation at the appropriate boundary conditions (see Berezhko & Taneev, 1991 for details).

### 3 Results and Discussion:

Results presented in Fig. 1 are performed at the proton injection rate  $\eta = 7.6 \times 10^{-3}$  and at typical solar wind parameters: proton number density  $N = 4 \text{ cm}^{-3}$ , solar wind speed  $u = 400 \text{ km s}^{-1}$ , magnetic field  $B = 6 \times 10^{-5} \text{ G}$ , shock compression ratio  $\sigma = 3.5$ , plasma parameter  $\beta = 0.1$ . Background Alfvén wave spectrum is taken in the form  $E_w^\pm(k, t = 0) \equiv E_{w0}^\pm(k) = E_0^\pm (k/k_0)^{-1/2}$ ,  $E_0 = E_0^+ + E_0^- = 6.9 \times 10^{-14} \text{ erg cm}^{-3}$  consistent with the observations. Alfvén waves propagated in antisunward direction is assumed to be twice more than the opposite propagated waves ( $E_0^+ = 2E_0^-$ ).

We assume that the acceleration process starts at  $t = 0$ , when the shock abruptly became quasiparallel. The time is measured in a scale units  $t_0 = \kappa_{0\parallel}(v_0)/u^2 = 0.48 \text{ h}$ , which is determined by the proton diffusion coefficient  $\kappa_{0\parallel}(v_0)$  provided by the background Alfvén wave field  $E_{w0}^\pm(k)$ .

We present in Fig. 1 the intensity of accelerated protons  $J(\varepsilon)$  as a function of kinetic energy  $\varepsilon$ , the spectra of Alfvén waves  $E_w(\nu) = E_w(k = 2\pi\nu/u)/\nu$  as a function of frequency  $\nu = ku/2\pi$  measured in a shock frame, the overall energy density of the excited Alfvén waves

$$W = \frac{1}{E_B} \int_0^\infty [E_w(\nu, t) - E_{w0}(\nu)] d\nu, \quad E_w(\nu, t) = E_w^+(\nu, t) + E_w^-(\nu, t)$$

and the pressure of accelerated particles  $P$  relative to the solar wind ram pressure  $\rho u^2$  as a function of time. All these quantities correspond to the shock front position.

One can see that the nonlinear wave interaction (Fig. 1b,d,f) essentially changes the time-development of the acceleration process as compared to a quasilinear approach (Fig. 1a,c,e). It restricts the wave amplitude at intermediate time moments  $1 \lesssim t/t_0 \lesssim 10$ : the wave energy density  $E_w(\nu)$  near the spectral maximum ( $\nu \simeq 2 \times 10^{-2} \text{ Hz}$ ) and the overall wave energy  $W$  are lower as compared with quasilinear case by a factor of  $10^4$ . The peak value of the wave energy is about  $W = 10$  that corresponds to the average wave amplitude  $\delta B = 3B$ .

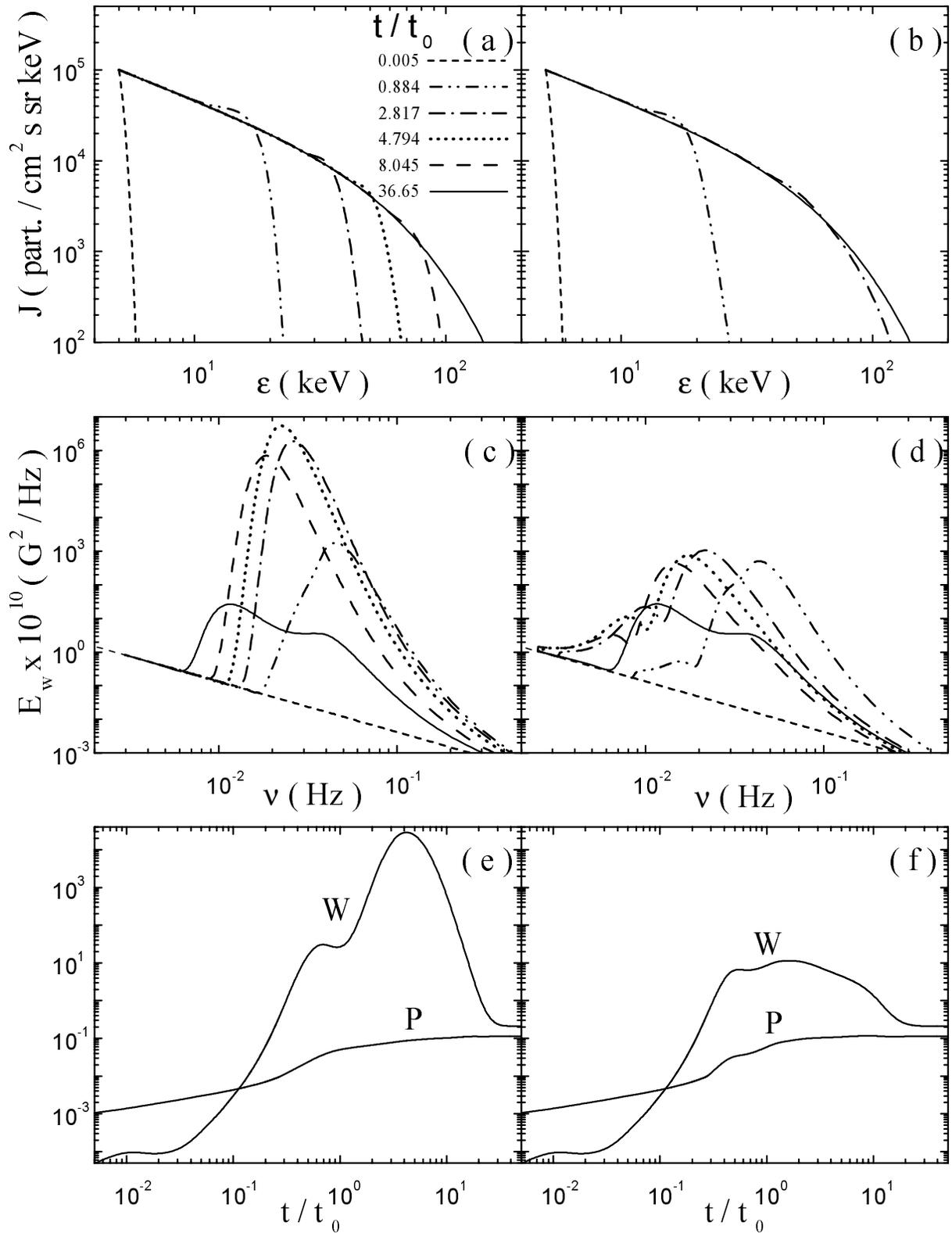


Figure 1: The spectra of accelerated protons (a, b) and Alfvén waves (c, d); energy content of excited Alfvén waves  $W$  and accelerated particle pressure  $P$  (e, f) at the Earth's bow shock calculated in quasilinear (a, c, e) and nonlinear (b, d, f) approaches for six different time moments.

During the initial period of the time ( $t/t_0 < 0.3$ ) and with the approaching of the stationary state ( $t/t_0 > 20$ ) the nonlinear wave interaction is negligible because the wave amplitude calculated in a quasilinear approach is relatively small ( $W < 1$ ) due to the small energy content of accelerated particles (see Fig. 1e). Note, that the extremely high Alfvén wave generation at intermediate phases (at  $1 \lesssim t/t_0 \lesssim 10$  in the considered case) is a direct consequence of so-called overacceleration effect, which is a pure nonstationary phenomenon (Berezhko & Taneev, 1991).

The nonlinear interaction of Alfvén waves changes their spectral distribution  $E_w(\nu)$ . It leads to a formation of the additional spectral peak at frequencies  $\nu \sim 10^{-1}$  Hz during the period of the most intense wave excitation (Fig. 1d).

This wave energy transform towards low frequencies provides an essential increase of particle acceleration rate. As one can see from Fig. 1a and Fig. 1b the spectrum of accelerated protons in nonlinear case is formed much faster in comparison with quasilinear one.

Note, that according to the quasilinear approach the acceleration process is almost insensitive to the relation between  $E_0^+$  and  $E_0^-$ . At the same time, the nonlinear interaction at  $\beta \lesssim 1$  takes place between the opposite propagating waves. As a consequence, the additional low frequency spectral peak includes the waves propagating towards the shock (i. e. in the antisunward direction), whereas the main peak always consists of waves propagating towards the Sun. Therefore, opposite to the quasilinear case the accelerated particle and excited wave dynamic is very sensitive to the relation between  $E_0^+$  and  $E_0^-$ .

In order to study the sensitivity of the nonlinear Alfvén wave interaction to the solar wind parameters we have performed a calculations at different values of parameter  $\beta$ . They demonstrate that the efficiency of wave amplitude restriction decreases with increasing  $\beta$ . At  $\beta = 1$  the energy content of the excited waves reaches the value  $W = 300$  which is about 30 times higher than at  $\beta = 0.1$ . At the same time, the acceleration rate is almost insensitive to  $\beta$ .

At  $\beta \ll 0.1$  the peak value of the wave energy content remains at the level  $W \simeq 3$  which is almost independent on  $\beta$ .

## 4 Conclusion:

Our calculations demonstrate that the nonlinear Alfvén-wave interaction essentially influences on the diffusive acceleration of solar wind ions at the Earth's bow shock. It restricts the wave energy content and provides wave energy transform towards the low frequencies that leads to an increase of the particle acceleration rate.

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