Simulation of Cosmic Ray Transport and Acceleration Near an Oblique, Spherical Shock

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Abstract

Simulations of pitch angle transport and acceleration of cosmic rays nearby an oblique, spherical shock can be performed using the well-known total variation diminishing method for the spatial advection terms and treating how particles cross the shock. We aim to examine the effects of adiabatic focusing and deceleration on the resulting particle spectrum, acceleration time and pitch angle distribution.

1 Introduction:

Curved shocks with oblique magnetic field configurations are known to accelerate particles at various locations in the solar system, including planetary bow shocks, traveling interplanetary shocks driven by coronal mass ejections, and corotating interaction regions, all of which are in the context of the laterally diverging solar wind. Therefore, in addition to diffusive shock acceleration, there is also the competing process of adiabatic deceleration due to the solar wind.

The model of a spherical shock has been a prototype for understanding the effects of a curved geometry or a diverging wind on the shock acceleration of energetic particles ever since the pioneering work of Jokipii (1968), and this arguably represents the simplest shock configuration after the plane-parallel case. Webb, Forman, & Axford (1985) have examined particle acceleration at a spherical shock in the diffusion approximation, and recently Vandas (1995) has made progress in analytically examining electron transport and acceleration a spherical shock front. Here, we aim to examine the effects of adiabatic focusing and deceleration by numerically simulating the transport of energetic charged particles in space, momentum, and pitch angle near an oblique, spherical shock by solving a Fokker-Planck equation for the evolution of the particle distribution function, using methods developed for the plane-parallel configuration (Ruffolo 1999). That work, and the preceding report (Ruffolo & Chuychai 1999) have shown that interesting effects can emerge when treating first-order Fermi acceleration in the framework of pitch-angle transport, i.e., beyond the diffusion approximation. Here we will consider the configuration of a spherical shock front at constant radius r from a central wind source, which approximates a traveling interplanetary shock due to a coronal mass ejection. The wind speed is taken to be in the radial direction, with one constant speed upstream and another downstream. This configuration permits one to assume Archimedean spiral magnetic fields upstream and downstream of the shock, and has the nice property that a reference frame corotating with the Sun serves as a de Hoffmann-Teller frame (de Hoffmann & Teller 1950) in which the magnetic field and fluid flow are parallel.

2 Theoretical Background:

We consider a theoretical framework similar to that of Ruffolo (1995), with a radial wind of speed u and an Archimedean spiral magnetic field on either side of a spherical shock. We modify the transport equation of that work (see also the preceding paper, Ruffolo & Chuychai 1999) to obtain

$$\begin{aligned}\frac{\partial F}{\partial t} &= -\frac{\partial}{\partial r}\left(\mu v\cos\psi + u - \frac{\mu^2 v^2 u\cos^2\psi}{c^2}\right)F \\ &-\frac{\partial}{\partial \mu}\left[\frac{v}{2L}\left(1-\frac{\mu v u\cos\psi}{c^2}\right)+\frac{\mu u}{r}\left(1-\frac{3}{2}\sin^2\psi\right)\right]\cdot(1-\mu^2)F \\ &+\frac{\partial}{\partial \mu}\frac{\varphi}{2}\frac{\partial}{\partial \mu}\left(1-\frac{\mu v u\cos\psi}{c^2}\right)F - \frac{\partial}{\partial p}pu\left(\frac{1-3\mu^2}{2}\,\frac{\sin^2\psi}{r}-\frac{1-\mu^2}{r}\right)F,\end{aligned} \tag{1}$$

where $F(t,\mu,r,p)$	$\equiv d^3N/(dpd\mu dr)$ is the density of particles in a given magnetic flux tube,
r	is the radius,
μ	is the pitch angle cosine in the fluid frame,
p	is the particle momentum in the fluid frame,
t	is the time in the fixed frame,
v	is the particle speed in the fluid frame,
$L(r)$	$\equiv -B/(dB/dz)$ is the focusing length,
$\psi(r)$	is the "garden-hose" angle between $\vec{B}$ and $\hat{r}$, and
φ	is the pitch angle scattering coefficient.

The distribution function, $F(t,\mu,r,p)$, is related to the phase space density, $f(t,\vec{x},\vec{p})$, by $F = 2\pi r^2 p^2 f$. We use F (following Ng & Wong 1979) because we can easily design the numerical finite difference method to strictly conserve this quantity (corresponding to conservation of particles) during streaming and convection.

3 Numerical Method:

Our simulations deal with solving equation (1) by means of a finite difference method over our rectangular simulation domain. The numerical method is a substantially modified version of that of Ruffolo (1995). In practice, we are unable solve the whole equation at once, but we simplify the method based on the "operator splitting" concept. That is, in a small enough time step, we group the right hand side of the transport equation into 3 groups, involving derivatives with respect to r, μ, and p, and then update $F\,(t,\mu,r,p)$ for each part consecutively.

In practice, the sequence of steps we used is as follows:

1. Update F for μ-changing processes over a time $\Delta t/2$.
2. Update F for p-changing processes (deceleration) over a time Δt.
3. Update F for r-changing processes (streaming and convection) over a time Δt. Crossing of a shock is also treated in this step.
4. Update F for μ-changing processes over another $\Delta t/2$.

Note that μ-changing processes are treated twice for $\Delta t/2$ each at the beginning and end. The reason why their treatment is split into two parts is because their symmetric treatment in time improves the convergence of the method to second order in Δt. (Every second term disappears in the Taylor series for the error, which is computed with respect to $t + \Delta t/2$.) Steps 2 and 3 do not need to be split because these operations commute to a reasonable approximation.

In step 3 we implement TVD differencing (Sweby 1984), tested and modified for a general Courant number (Nutaro, Riyavong, & Ruffolo 1999) and treat particles crossing the shock, allowing us to properly deal with a gradually varying $u \cos\psi$ or a discontinuous u. Other steps remain the same as in our previous work. Away from a shock, step 3 for updating F for r-changing processes involves solving

$$\frac{\partial}{\partial t} F\left(t, \mu, r, p\right) = -\frac{\partial}{\partial r}\left(\mu v \cos\psi + u - \frac{\mu^2 v^2 u \cos^2\psi}{c^2}\right) F\left(t, \mu, r, p\right). \tag{2}$$

Now consider the case of particles encountering an oblique, spherical shock. We assume that the shock curvature and spherical geometry only affect the particle transport via the diverging wind (Jokipii 1968) and the spiral field geometry; we assume that the shock's radius of curvature is much greater than the particle gyroradius and treat the shock crossing as if the shock were planar. In the framework of operator splitting, one step corresponds to spatial motions (streaming + convection) so the treatment of particles crossing the shock is naturally included in this step. We treat the transport of particles from a given cell by considering whether the particles encounter the shock during a time increment Δt. If not, then we use TVD scheme.

If particles encounter the shock, we first perform a Lorentz transformation of p and μ into the shock frame. In general, for a static magnetic field, $\vec{F} = q\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$, so the rate of doing work on the particle, $\vec{F} \cdot \vec{v}$, is zero; thus the momentum in the shock frame is conserved throughout the encounter. We also make the common approximation that the magnetic moment $p^2(1-\mu^2)/(2meB)$ is conserved as particles cross or are reflected by the shock (Decker 1983). Since the magnetic field strength differs on the two sides of an oblique shock, the pitch angle cosine μ must also change. Particles encounting the shock from downstream are transmitted upstream with higher μ in the range $(\mu_a, 1]$. The particles from upstream with a pitch angle cosine in the range (μ_a, μ_b) are reflected back into this range due to the magnetic mirroring effect, while only the particles which have μ less than μ_b can transport to the downstream region. Finally, we perform a Lorentz transformation of p and μ back into the local wind frame.

Note that the TVD algorithm effectively splits a cell into fractions of particles destined to move to two different spatial locations. Here apply a similar method, since when particles cross the shock, some particles might be transported to quite different μ values as well. Note that for the nonrelativistic fluid speeds and energetic cosmic ray particle speeds considered here, the fractional change in momentum for an individual shock encounter is not large. In principle the numerical method can be applied for a grid of p points. So far we have treated one p value and simply assumed $F \propto p^{-\gamma}$, through there are problems with this assumption when u/v is not small (Ruffolo 1999).

In practice, for a given p value we define a four-dimensional array (a "transfer matrix") that gives the fraction of each r-μ cell near the shock that is transported to every other r-μ cell near the shock; the values of this array are set at the start of the program. The splitting into different r cells still makes use of the modified TVD algorithm. Splitting in terms of μ is performed by mapping final μ-cell boundaries back into the initial μ-cell and splitting F according to the width of each segment of the initial μ-cell that is destined to arrive within a given final μ-cell. This technique could be refined, e.g., by linear interpolation of $F(\mu)$ between μ-grid points (which is effectively what is done in second-order differencing for the Lax-Wendroff method); however, we decided that this level of detail is not necessary given the approximate nature of the assumption of magnetic moment conservation.

Results will be presented at the Conference.

Acknowledgments:

This work was partially supported by a Basic Research Grant from the Thailand Research Fund.

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