

# Implications of Increased Perpendicular Diffusion on the Tilt Angle Dependence of Electron Modulation in the Heliosphere

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## Abstract

For numerical solutions of Parker's cosmic ray transport equation to be compatible with the small latitudinal gradients observed for protons by Ulysses, enhanced perpendicular diffusion seems needed in the polar regions of the heliosphere. The role of enhanced perpendicular diffusion was further investigated by examining electron modulation as a function of the "tilt angle"  $\alpha$  of the wavy current sheet, using a comprehensive modulation model including convection, diffusion, gradient, curvature and neutral sheet drifts. We found that by increasing perpendicular diffusion in the polar direction, a general reduction occurs between the modulation differences caused by drifts effects for galactic cosmic ray electrons as a function of  $\alpha$  for the  $A > 0$  (e.g. ~1990 to ~2000) and  $A < 0$  (e.g. ~1980 to ~1990) solar magnetic polarity cycles.

## 1 Introduction

It is well-known that the wavy heliospheric current sheet (HCS) is a very important modulation parameter as were predicted by drift models (Jokipii & Thomas, 1981, le Roux & Potgieter, 1990). The computed effects of the HCS "tilt angle"  $\alpha$  which represents the extend to which it is warped is however dependent of other modulation parameters, in particular the parallel ( $K_{\parallel}$ ) and perpendicular diffusion coefficients ( $K_{\perp}$ ). Concerning  $K_{\perp}$  it was argued by Kóta & Jokipii (1995) that it is not isotropic but seems enhanced in the polar directions. This enhancement has been studied intensively in modulation models (e.g. Potgieter 1997) and it was illustrated that the enhancement is necessary to make these models compatible with the small latitude effects observed for protons onboard the Ulysses spacecraft. For the present work, the effect of enhancing  $K_{\perp}$  in the polar direction on the "tilt angle" dependence of electron modulation was studied. This aspect is also pursued in Ferreira & Potgieter (SH3.1.07), where the effects are illustrated for spectra and differential intensities as a function of radial distance and polar angle. For a detailed description of the importance of the various parameters in electron modulation, the reader is referred to Potgieter (1996).

## 2 The Modulation Model

The model for this study is based on the numerical solution of the Parker's (1965) transport equation:

$$\frac{\partial f}{\partial t} = -(\mathbf{V} + \langle v_D \rangle) \cdot \nabla f + \nabla \cdot (K_s \cdot \nabla f) + \frac{1}{3} (\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln R}, \quad (1)$$

where  $f(\mathbf{r}, R, t)$  is the cosmic ray (CR) distribution function,  $R$  is rigidity,  $\mathbf{r}$  is position, and  $t$  is time. Terms on the right-hand side represent convection, gradient and curvature drifts, diffusion and adiabatic energy changes respectively, with  $\mathbf{V}$  the solar wind velocity. The symmetric part of the tensor  $K_s$  consists of a parallel diffusion coefficient ( $K_{\parallel}$ ) and a perpendicular diffusion coefficient ( $K_{\perp}$ ). The anti-symmetric element  $K_A$  describes gradient and curvature drifts in the large scale heliospheric magnetic field (HMF) with the pitch angle averaged guiding center drift velocity for a near isotropic CR distribution is given by  $\langle v_D \rangle = \nabla \times K_A \mathbf{e}_B$ , where  $\mathbf{e}_B = \mathbf{B}/B$ , with  $B$  the magnitude of the background HMF. Eq. 1 was solved in a spherical coordinate system assuming azimuthal symmetry, and for a steady-state, that is  $\partial f / \partial t = 0$ .

The HMF was modified according to Jokipii & Kóta (1989). Qualitatively, this modification is supported by measurements made of the HMF in the polar regions of the heliosphere by Ulysses (Balogh et al. 1995). The solar wind speed  $V$  was assumed to change from  $450 \text{ km.s}^{-1}$  in the equatorial plane ( $\theta = 90^\circ$ ) to a maximum of  $850 \text{ km.s}^{-1}$  when  $\theta \leq 60^\circ$ , with  $\theta$  the polar angle. The outer boundary of the simulated heliosphere was assumed at 100 AU which is a reasonable consensus value. The galactic electron spectrum published from the COMPTEL results (Strong et al., 1994) was used as the local interstellar spectrum; see also Potgieter (1996). Solutions for  $\alpha$  up to  $70^\circ$  were computed for both  $A > 0$  and  $A < 0$  polarity epochs.

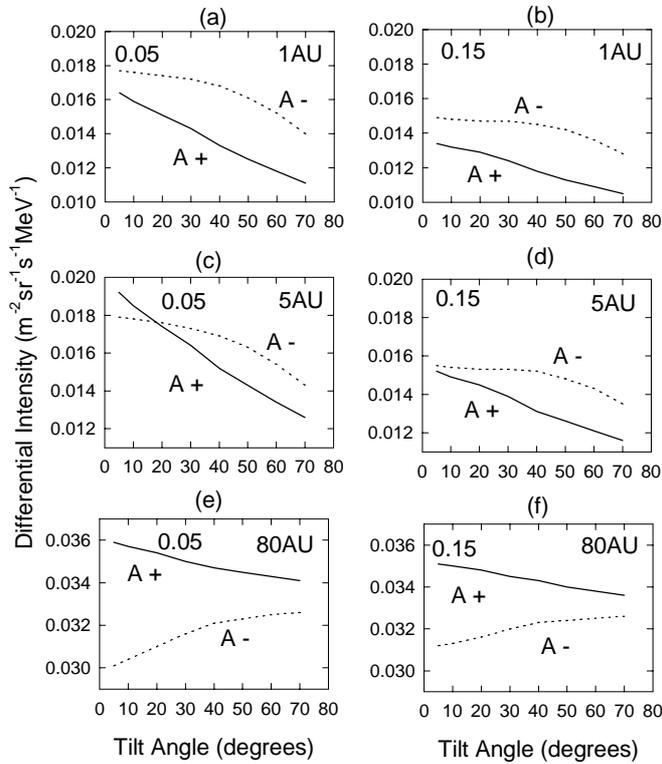
For the parallel and perpendicular diffusion coefficients, and the “drift” coefficient, the following general forms were assumed respectively:

$$K_{\parallel} = K_0 \beta f_1(R) f_2(\theta, r); K_{\perp r} = a K_{\parallel}; K_{\perp \theta} = b K_{\parallel}; K_A = (K_A)_0 \frac{\beta R}{3B_m}. \quad (2)$$

Here  $\beta$  is the ratio of the speed of the CR particles to the speed of light;  $f_1(R)$  gives the rigidity dependence in GV;  $K_0$  is a constant in units of  $6.0 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$  with  $K_0 = 25$ ;  $a = 0.05$  is a constant which determines the value of  $K_{\perp r}$  which contributes to perpendicular diffusion in the radial direction, and  $b$  is a constant determining the value of  $K_{\perp \theta}$  which contributes to perpendicular diffusion in the polar direction. Diffusion perpendicular to the HMF was therefore enhanced in the polar direction by assuming  $K_{\theta\theta} = K_{\perp \theta} = b K_{\parallel}$  with  $b = 0.05$  and  $0.15$  respectively. (See also Kóta & Jokipii, 1995; Potgieter, 1996).  $(K_A)_0$  specifies the amount of drifts allowed, with  $(K_A)_0 = 1.0$  a maximum. The effective radial diffusion coefficient is given

by  $K_{rr} = K_{\parallel} \cos^2 \psi + K_{\perp r} \sin^2 \psi$ , with  $\psi$  the angle between the radial direction and the averaged HMF direction. Note that  $\psi \rightarrow 90^\circ$  when  $r > \sim 10 \text{ AU}$  with the polar angle  $\theta \rightarrow 90^\circ$ , and  $\psi \rightarrow 0^\circ$  when  $\theta \rightarrow 0^\circ$ , which means that  $K_{\parallel}$  dominates  $K_{rr}$  in the inner and polar regions and  $K_{\perp r}$  dominates in the outer equatorial regions of the heliosphere. Differential intensities,  $j \propto R^2 f$ , are calculated as particles  $\text{m}^{-2} \text{sr}^{-1} \text{MeV}^{-1}$ .

Solutions were computed with a simple rigidity dependence for  $K_{\parallel}$  and  $K_{\perp}$  (meaning both  $K_{\perp r}$  and  $K_{\perp \theta}$ ) given by  $f_1(R) = \beta R/R_0$  with  $R > 0.4 \text{ GV}$ , and  $f_1(R) = \beta (0.4 \text{ GV})/R_0$ , with  $R \leq 0.4 \text{ GV}$  and  $R_0 = 1 \text{ GV}$ . This simple approach has proven to be most useful (Potgieter 1996). For the spatial dependence,  $f_2(\theta, r) = 1 + r/r_0$  was assumed, with  $r_0 = 1 \text{ AU}$ . Note that  $K_{\parallel}$  and  $K_{\perp}$  have the same rigidity dependence that becomes flat and constant below  $0.4 \text{ GV}$ . This feature causes the electron modulation at a given position in the heliosphere to become almost constant at energies  $< \sim 50 \text{ MeV}$ . At energies  $< \sim 10 \text{ MeV}$  Jovian electron may contribute to the computed spectra (Haasbroek et al., 1996) but is



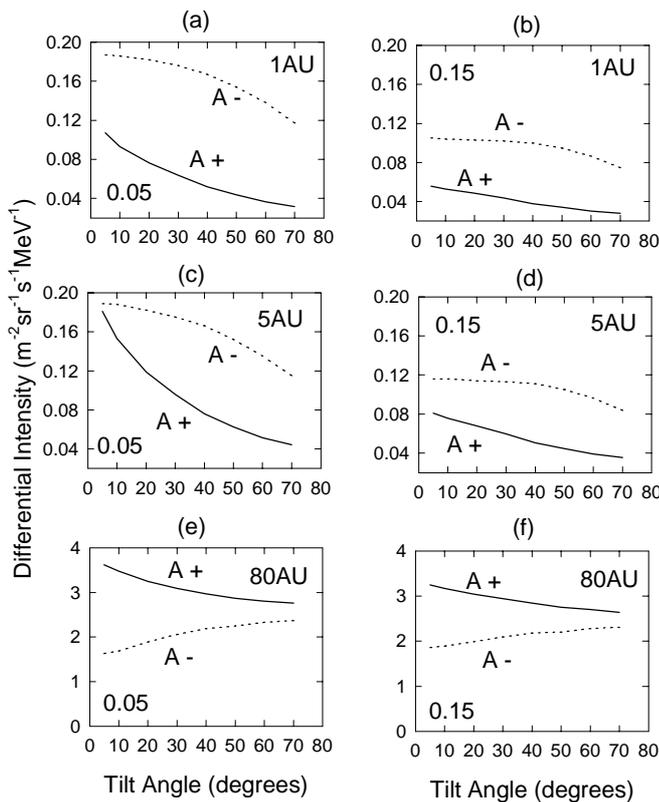
**Figure 1:** Electron differential intensity at 1.94 GeV as a function of tilt angle  $\alpha$ , shown in the equatorial plane ( $\theta = 90^\circ$ ) for  $A > 0$  and  $A < 0$  polarity epochs. Solutions are shown at 1 AU, 5 AU and 80 AU for different values of  $K_{\perp \theta}$ : Panels (a), (c) and (e) with  $b = 0.05$  and panels (b), (d) and (f) with  $b = 0.15$  in Eq. (2).

neglected for this study. Different assumptions for  $f_i(R)$  may change the slope of the spectra at low energies as was illustrated in detail by Potgieter (1996) but this is not important for the results and conclusions of this study.

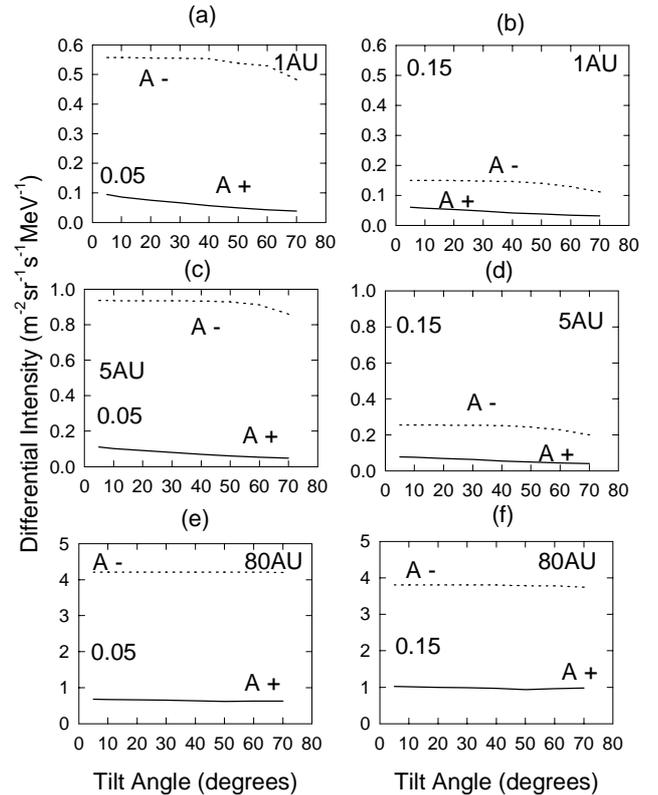
Modeling the modulation of electrons in the heliosphere, a two-dimensional (2D) model with an emulated wavy current sheet was used as developed by Hattingh & Burger (1995a). Obviously, the 2D model differs from a 3D model in the way the HCS is handled. However, using a 2D model is well justified and for a comparison between this 2D model and the 3D model developed by Hattingh (1998) - see also Hattingh & Burger, 1995b) - the reader is referred to Hattingh (1998) and to Ferreira, Potgieter & Burger (SH3.1.20). For an additional description of the “tilt angle” dependence of the model, see Burger & Potgieter (SH3.1.04).

### 3 Results and Discussion

The electron differential intensities as a function of  $\alpha$  are shown in Figure 1 for 1.94 GeV electrons at  $\theta = 90^\circ$  (equatorial plane) for both polarity cycles. Solutions are shown at three radial distances and for two different values of  $K_{\perp\theta}$ . Panels (a) and (b) show solutions at 1 AU; panels (c) and (d) for 5 AU and panels (e) and (f) for 80 AU for  $b = 0.05$  and  $b = 0.15$  respectively. From this figure follows that at 1 AU the intensity for the  $A < 0$  polarity cycle is higher than for the  $A > 0$  cycle. As  $K_{\perp\theta}$  was enhanced by increasing  $b$  from 0.05 to 0.15, a reduction occurs in the differences between the two epochs. The intensities for both epochs are lower for the increased value of  $K_{\perp\theta}$  and do not have such a strong  $\alpha$  dependence as for a smaller  $K_{\perp\theta}$ . For  $A > 0$  this diminished dependence on  $\alpha$  is especially evident for  $\alpha < 40^\circ$ . At 5 AU the intensities for the  $A > 0$  and  $A < 0$  cycles cross at  $\alpha = \sim 15^\circ$  with the  $A < 0$  intensities lower than those for the  $A > 0$  for  $\alpha < 15^\circ$ . As for 1 AU, the increase in



**Figure 2:** As in Figure 1, but for 0.30 GeV electrons.



**Figure 3:** As in Figure 2, but at a polar angle of  $\theta = 5^\circ$ .

$K_{\perp\theta}$  led to a decrease of the  $\alpha$  dependence, especially with  $\alpha < 40^\circ$ . The spectra shown no longer cross, but it still occurs at a slightly larger radial distance. For 80 AU, the intensities for the  $A > 0$  are consistently higher than for the  $A < 0$  epoch and the increase in  $K_{\perp\theta}$  had little or no effect on the differential intensities as a function of  $\alpha$ . This indicates that the increase in  $K_{\perp\theta}$  is more important in the inner and middle heliosphere (compare also Ferreira & Potgieter, SH3.1.07).

The process was repeated for 0.30 GeV electrons and the results are shown in Figure 2. Qualitatively, Figure 2 shows a similar response to changes in  $K_{\perp\theta}$  and no significant deviations occur with the changing  $\alpha$  compared to Figure 1. The cross-over shown in Figure 1 now occurs at  $r > 5$  AU. Quantitatively, the  $\alpha$  dependence of the  $A > 0$  intensities is less linear and the differences between the two epoch solutions are evidently larger than for the higher electron energies shown in Figure 1.

Figure 3 shows solutions as in Figure 2 except that they were obtained at  $\theta = 5^\circ$  to illustrate what happens to the  $\alpha$  dependence of the solutions in the polar regions of the heliosphere. The  $\alpha$  dependence is clearly negligible compared to the equatorial regions as follows from comparing panels (a) to (f) in Figure 3 with those in Figure 2. The only significant  $\alpha$  dependence is when  $\alpha > 60^\circ$ . This is understandable, because when the tilt angles become very large the modulation of intensities must respond to the presence of the wavy HCS in these high latitude regions of the simulated heliosphere. The increase in  $K_{\perp\theta}$  from 5% to 15% of  $K_{\parallel}$  gives the largest effect on the level of modulation in the polar regions as a comparison of panels (b), (d) and (f) in Figure 3 with those in Figure 2 illustrates. This is due to the fact that the enhancement of  $K_{\perp\theta}$  in the polar directions becomes more effective with decreasing polar angles. The significant reduction of the latitude dependence of the electron intensities due to the enhancement of  $K_{\perp\theta}$  also follows clearly from comparing panels (b),(d) and (f) in Figure 3 with those in Figures 2.

## 4 Summary and Conclusions

Studying the effects on electron modulation of enhancing  $K_{\perp\theta}$  in the polar directions, from 5% to 15% of  $K_{\parallel}$ , it was found that this increase reduced the differences between the modulated intensities as a function of “tilt angle”  $\alpha$  for the two magnetic polarity cycles. This is especially strong for the inner heliosphere in the equatorial regions and most of the heliosphere in the polar regions. The increase in  $K_{\perp\theta}$  also led to a decrease in the  $\alpha$  dependence of the differential intensities for  $\alpha < 40^\circ$  for the inner heliosphere in the equatorial regions as shown in Figures 1 and 2. For the polar regions, shown in Figure 3, the increase in  $K_{\perp\theta}$  had little or no change in the  $\alpha$  dependence of the intensities for  $\alpha < 60^\circ$ , but it caused a significant reduction in the global latitude dependence of electron modulation. Obviously, the enhancement of  $K_{\perp\theta}$  plays an important role in the modulation of cosmic rays and need further study, especially of a more fundamental nature.

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