Multiple Magnetic Field-Shock Crossings and Particle Acceleration at Quasi-perpendicular Shocks

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Abstract

The importance of accounting for diffusion perpendicular to the mean magnetic field during quasi-perpendicular shock acceleration is well documented. Here we note that perpendicular diffusion is typically envisioned as due to the random walk of field lines, with particle guiding centers closely tied to and diffusing back and forth along the field. A turbulent magnetic field line can cross and recross the shock, like a sawtooth edge. In this "sawtooth mechanism," if there are N magnetic field-shock crossings that are separated by distances $L > \lambda_{||}$, the scattering mean free path, a particle diffusing along the field line will cross the shock an average of N^2 times before escaping. This could increase the total shock-drift distance and energization of particles. We have verified that multiple field-shock crossings do occur for reasonable values of $(\delta B/B_0)^2$ near the shock, and have measured the distribution of N, θ_{Bn}, and L for simulated random magnetic fields. For the special case of the solar wind termination shock, this mechanism may help explain the observationally inferred drift of anomalous cosmic rays (ACR) over much of the distance from the solar equator to the poles or vice-versa.

1 Introduction:

When particles are largely tied to a given magnetic field line, for a single field-shock crossing the acceleration rate can greatly increase as the field-shock normal angle, θ_{Bn}, approaches 90°, i.e., for a nearly perpendicular shock (Jokipii 1987). The energization of particles can be viewed (in the fixed frame) as mainly due to the shock-drift mechanism (Schatzman 1963), in which particles drift along the electric field while encountering the shock. Particle diffusion perpendicular to the mean magnetic field direction has been shown to play an important role (e.g., Jokipii 1987; Jokipii, Kóta, & Giacalone 1993; Jones, Jokipii, & Baring 1998). Giacalone, Jokipii, and Kóta (1994) and Ellison, Baring, and Jones (1995) have performed Monte Carlo (MC) simulations for this situation that include ad hoc diffusion perpendicular to the magnetic field, and the latter authors found that while the acceleration rate rose with θ_{Bn}, the injection rate declined.

In theoretical models of particle transport in turbulent magnetic fields such as those found in the solar system (e.g., Bieber & Matthaeus 1997), particle diffusion perpendicular to the mean magnetic field direction is mainly ascribed to a random walk of the magnetic field, i.e., particle guiding centers are basically tied to field lines, which themselves wander perpendicular to the mean field. Note that a turbulent magnetic field line can cross and recross the shock, like a sawtooth edge (Figure 1). In this work, we consider the implications of this concept of perpendicular diffusion, and we identify a mechanism, which we term the "sawtooth mechanism," that can greatly enhance the particles' total energization and shock drift distance, which could give an improved physical explanation of the observationally inferred drifts (Cummings, Stone, & Webber 1985) and associated energy cutoffs (Mewaldt et al. 1996) of ACR.

Our proposed mechanism involves similar basic physics to that contained in MC calculations with ad hoc perpendicular diffusion (e.g., Giacalone, Jokipii, & Kóta 1994; Ellison, Jones, & Baring 1999). Our goal is to elucidate the key physical mechanisms of energetic particle acceleration at nearly perpendicular shocks, which would also underly the MC results. Here we do not address the issue of injection. The mechanism we consider is appropriate when the particle speed is fast relative to the convection speed; for slow particles or a fast convection speed, a more important mechanism might be that involving multiple reflection in collapsing magnetic traps (Decker 1990, 1993).

2 Multiple Magnetic Field-Shock Crossings:

In order to determine the characteristics of magnetic field-shock crossings, we computationally generated random magnetic fields for a specified power spectrum matrix using inverse Fourier transforms and a random phase. Figure 1a shows an example of such a random magnetic field line for slab turbulence with $P_{xx}(k_z) = P_{yy}(k_z) = C(1 + k_z^2/k_0^2)^{-\nu}$, $\langle\delta B^2\rangle/B_0^2 = 0.05$, and a correlation length of 16Δ, where Δ is the grid size for the inverse Fourier transform. The mean magnetic field is in the horizontal direction, and the shock plane (solid line) is slightly tilted relative to the mean magnetic field; note the greatly expanded vertical axis. Actually, the magnetic field line should be vertically compressed downstream of the shock, which for clarity is not shown in Figure 1a; that would not affect the number or characteristics of field-shock crossings studied in the present work. This is an example of how one does find multiple field-shock crossings for reasonable values of the turbulent energy. In fact, stronger turbulence, with $\langle\delta B^2\rangle/B_0^2 \sim 0.1$, is expected to be generated by a quasi-perpendicular shock, according to the hybrid simulations of Liewer, Rath, & Goldstein (1995).

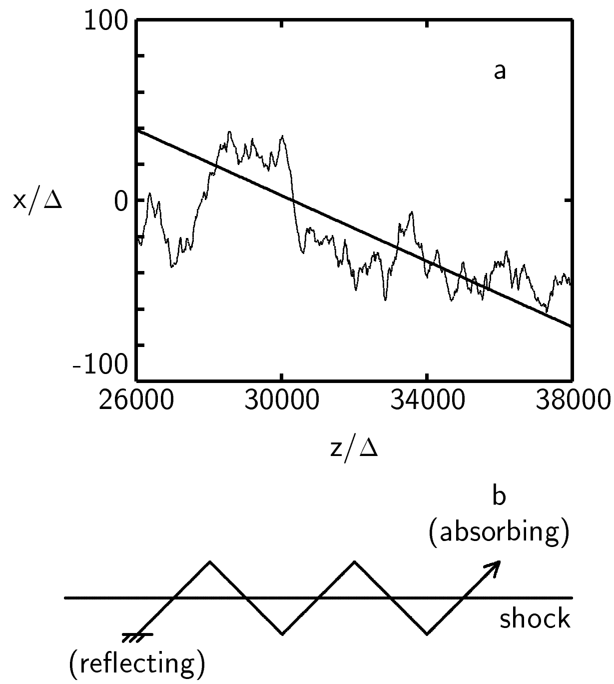

Figure 1: a) A magnetic sample field line that crosses a shock (diagonal line) multiple times. Note the greatly expanded vertical scale. b) Schematic of the above “sawtooth” magnetic field, and boundary conditions for the random walk of particles along $\vec{B}$.

Figure 2 shows examples of the statistics we can collect regarding field-shock crossings, for the same type of turbulence. The calculation of the upstream field-shock angle, θ_{Bn}, takes into account the orientation of $\vec{B}$ in three dimensions. It is noteworthy that even when taking magnetic field irregularities into account, the distribution of field-shock angles has peaks near 90°, indicating that particle-shock encounters can potentially yield a large amount of shock-drift acceleration. As shown by Jokipii (1982), the ratio of energy gain to $q\Delta\Phi$, the change in potential energy when drifting along the electric field, is approximately unity for such high angles. The distribution of L, the distance between consecutive crossings, helps determine the number of crossings which are sufficiently far apart for the sawtooth mechanism to take effect (see §3). These calculations will be extended to consider 2D and 2D + slab (three-dimensional) turbulence models as well.

3 Sawtooth Mechanism:

Let us first consider the random walk of particles along the random magnetic field by ignoring the possibility of reflection when approaching the shock from upstream. Referring to Figure 1, in this framework we view the acceleration process in terms of discrete episodes of diffusive shock acceleration (which includes shock drift acceleration) when the particle encounters a field-shock crossing. In addition to the correlation lengths, other relevant length scales for a given particle species and energy include the gyroradius r_g and the scattering mean free path $\lambda_{||}$. Since the particle motion follows a sort of average of $\vec{B}$ over a gyroradius, field-shock crossings closer together than r_g should be grouped together so that the particle interaction in that region is considered to constitute a single particle-shock encounter.

Next, field-shock crossings spaced farther than r_g but closer than $\lambda_{||}$ will generally be traversed in sequence; N such crossings can then yield an N-fold enhancement in shock acceleration. We refer to this as a linear enhancement.

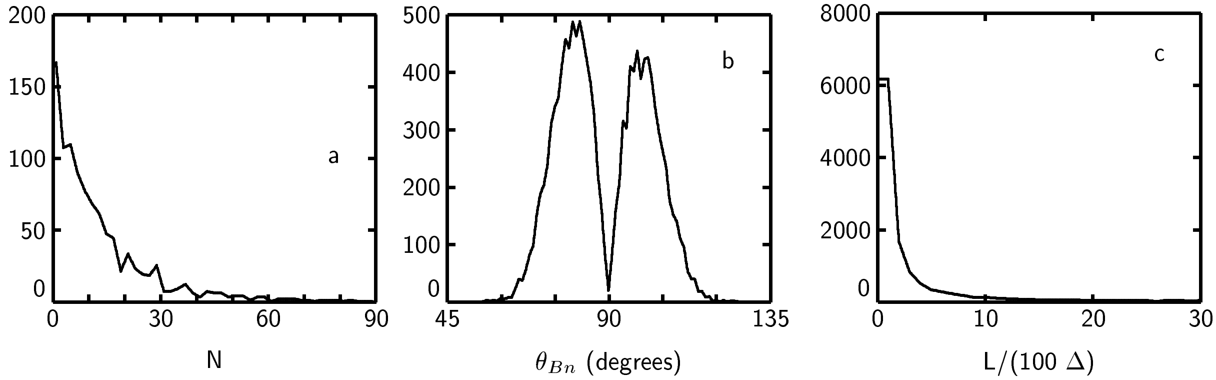

Figure 2: Histograms of a) the number of field-shock crossings, b) the upstream angle between the field and the shock normal, and c) the distance between crossings for 1000 simulated turbulent magnetic fields.

Now consider only field-shock crossings or groups of field-shock crossings that are spaced farther apart than $\lambda_{\|}$. For this purpose, the magnetic field in Figure 1a can be conceptually simplified as in Figure 1b. (Again, for simplicity we have not indicated the refraction of magnetic field lines.) Between two crossings, a particle's motion is randomized, and there is an equal probability of either moving forward to the next crossing or returning to the previous crossing.

This then becomes a classic random walk problem and is amenable to mathematical analysis (Chandrasekhar 1943). Starting from upstream of the shock on the left hand side, let n be the number of times a particle encounters the shock, and let m indicate the regions between field-shock crossings from left ($m = 0$) to right ($m = N$). To represent the ultimate return of particles upstream of the first field-shock crossing, due to convection, we place a reflecting barrier at $m = 0$, and to represent escape downstream, we use the conservative assumption of absorption at $m = N$. The probability of escape after n shock encounters can be shown to be

$$P(n) = \sum_{j=0,m\leq n} \frac{(-1)^j}{2^{n-1}} \binom{n-1}{(n+m-2)/2} - \sum_{j=0,m\leq n-2} \frac{(-1)^j}{2^{n-1}} \binom{n-1}{(n+m)/2} \tag{1}$$

where $n + N$ is even and $m = (2j + 1)N$. This probability sums to 1 (when summing over all $n \geq N$ such that $n + N$ is even), and the mean value of n is N^2. The mean number of shock encounters before escape should actually be even larger if we consider that a large fraction of particles approaching a shock from upstream (87% for a strong shock with a compression ratio of 4) should be reflected backup upstream, which gives some probability of trapping between two adjacent field-shock crossings.

Therefore, even with conservative assumptions the sawtooth model predicts a quadratic enhancement by a factor of $\sim N^2$ in the shock drift and total energization of particles, where N is the number of field-shock crossings spaced farther than λ. Presumably this enhancement is occurring in MC simulations of particle acceleration at nearly perpendicular shocks. In practice the total energy gain of particles will also be limited by the lateral extent of the shock, and the convection of field lines past the shock. In fact, for the case of the solar wind termination shock, observations of anomalous cosmic rays can be understood in terms of particle drift at the shock over a large fraction of the distance from the heliospheric equator to the poles or vice-versa (Cummings, Stone, & Webber 1985), with even an indication of a spectral break corresponding to particles that traverse that entire distance (Mewaldt et al. 1996). Such observations point to successful shock-drift acceleration up to the limit of the size of the termination shock.

Further work will aim to clarify the effect of the level of turbulence on the acceleration of energetic charged particles at nearly perpendicular shocks. We hope that this framework will also prove useful for assessing a possible species dependence of the acceleration efficiency.

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