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Challenges for an 'ab initio' theory of cosmic ray modulation

S. Parhi[1], **R. A. Burger**[2], **J. W. Bieber**[1], **and W. H. Matthaeus**[1]

[1]Bartol Research Institute, University of Delaware, Newark, DE 19716
[2]School of Physics, Potchefstroom University for CHE, Potchefstroom, South Africa

Abstract. Recent efforts to improve and complete modulation theory are described, emphasizing factors that arise in attempts to understand the diffusion tensor based upon turbulence theory and the theory of charged particle scattering. Direct numerical solutions of the transport equations are used to study a new perpendicular diffusion formalism that can smoothly vary between the field line random walk limit and a recent treatment based upon the Taylor–Green–Kubo equation. Using this new formalism we show that modulation is strongly influenced by the radial variation of the correlation length. This variation is still poorly understood, in part owing to the uncertain impact of pickup ion driven turbulence in the outer heliosphere.

1 Introduction

Cosmic ray modulation is a complex subject relating galactic particle populations to heliospheric structure, while involving at a detailed level incompletely understood theoretical features of charged particle scattering and plasma turbulence. Scattering theory involves turbulence parameters, and thus one needs to understand how plasma turbulence evolves throughout the heliosphere. Even in the simplest formulations of scattering and turbulence theories this would involve specification of a turbulence energy density and a similarity length scale everywhere in the three dimensional heliosphere. Boundary data in the inner heliosphere can stand in place of a fuller understanding of the origins of solar wind turbulence, but it is also necessary to understand how turbulence is driven within the heliosphere, for example, by shear and excitation of fluctuations by scattering of interstellar pickup ions. In more elaborate models of turbulence the fluctuations may be represented as consisting of two or more components, each with a known symmetry (Matthaeus et al., 1990; Bieber et al., 1994). A useful example is the two-component model which includes one dimensional "slab" and two dimensional (2D) ingredients. Scattering theory itself also involves many issues and is incompletely understood. Perpendicular diffusion, especially at lower energies (Giacalone and Jokipii, 1999; Mace et al., 2000) is not well accounted for theoretically. For these reasons it is obvious why there is at present no accepted *ab initio* modulation theory, that is, one in which the diffusion coefficients are determined on the basis of scattering theory and the underlying magnetic fluctuation parameters are computed from plasma theory and known features of heliospheric structure. Recently we have been attempting to further develop an *ab initio* approach to modulation, building on recent efforts along these lines (Zank et al, 1998; Burger et al., 2000).

¿From the start it becomes apparent that modulation is relatively sensitive to features of turbulence and scattering which themselves are active areas of research. Thus it appears appropriate to view these efforts as investigations of scattering and turbulence on the same footing as they are investigations of modulation. One such parameter that sensitively affects modulation is the parallel correlation scale of magnetic fluctuations, and this is the subject of emphasis in the present short paper. The main point of the present paper is to show the type of effects that can be expected in modulation when the assumptions concerning the radial variation of the correlation scale are varied in a modulation model.

2 Modulation model

The modulation of galactic cosmic rays is described by Parker's equation. In our model we describe the radial coefficient by $\kappa_{rr} = \kappa_{\|} cos^2\psi + \kappa_{\perp}^{r\phi} \sin^2\psi$. Here $\kappa_{\|}$ is the diffusion coefficient parallel to the mean magnetic field, $\kappa_{\perp}^{r\phi}$ the diffusion coefficient perpendicular to the field in the radial/azimuthal direction and ψ the spiral angle. In our two-dimensional model $\kappa_{\perp}^{r\phi}$ acts only in the radial direction. $\kappa_{\theta\theta}$ is the diffusion coefficient perpendicular to the mean magnetic field in the polar direction which we assume to be larger than $\kappa_{\perp}^{r\phi}$ by a constant factor. We use a steady state two-

Correspondence to: S. Parhi (sparhi@bartol.udel.edu)

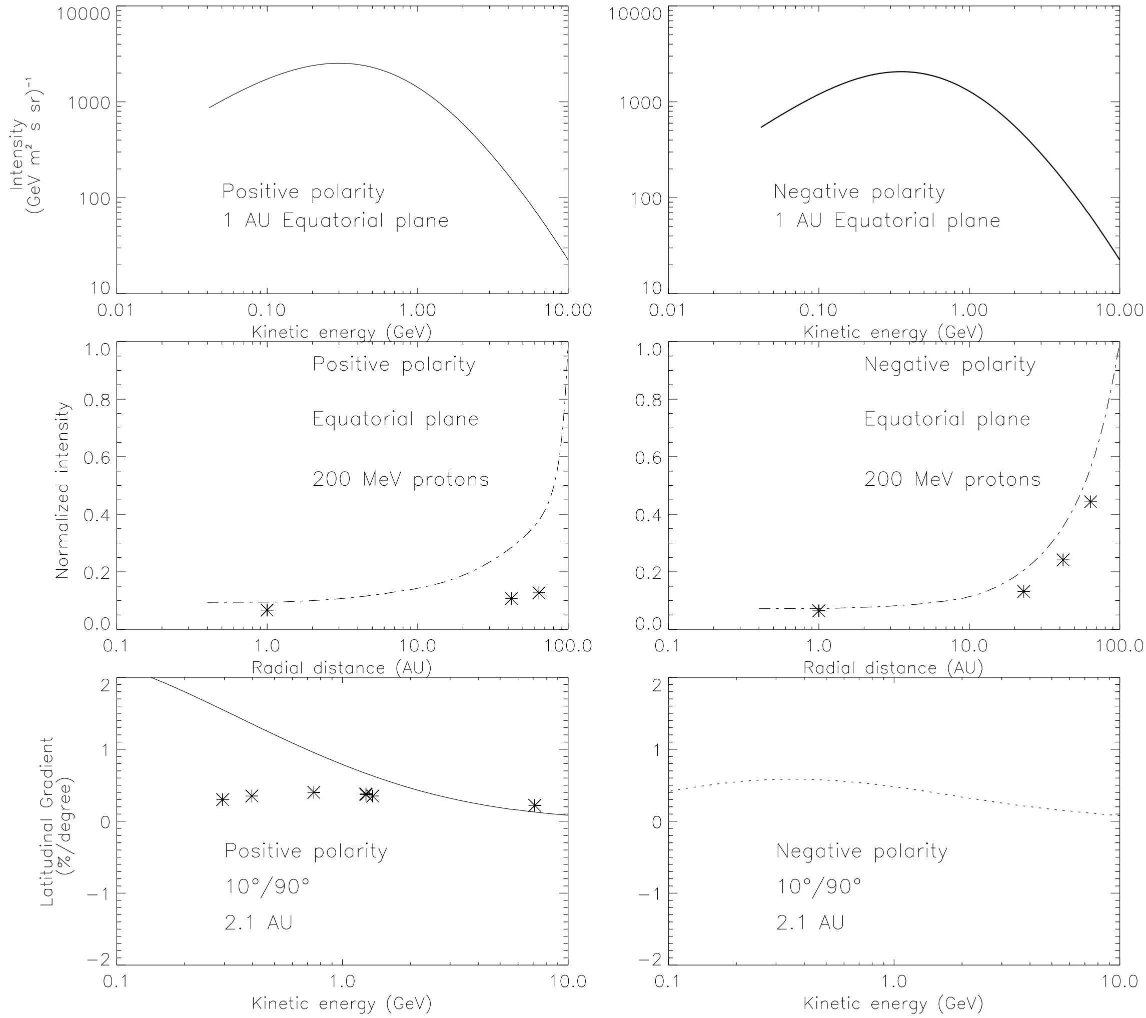

Fig. 1. 1 AU spectrum (top) in the equatorial plane, radial gradient (middle) for 200 MeV particles in the equatorial plane and latitudinal gradient (bottom) at 2.1 AU for $l_{slab} \propto r^{-0.3}$, $\zeta = 0.15$, and $\tilde{l}/l_{slab} = 100$. Radial gradient data come from Voyager and IMP and the latitudinal gradient data come from Ulysses.

dimensional model that simulates the effect of a wavy current sheet (Burger and Hattingh, 1995) by using an averaged drift field with only an r and θ component. We consider $\kappa_\perp^{r\phi}$ as a combination of limits established by Green-Kubo-Taylor equation (Bieber And Matthaeus 1997; hereafter "BAM") and by field line random walk. Thus we define $\kappa_\perp^{r\phi}$ as

$$\kappa_\perp^{r\phi} = \frac{VR_L}{3}\frac{1}{\Omega\tau}\left(\frac{\Omega^2\tau^2}{1+\Omega^2\tau^2}\right)^{1-\zeta}, \quad (1)$$

where $\Omega\tau = 2R_L/3D_\perp$. Here $2D_\perp = D_{slab} + (D_{slab}^2 + 4D_{2D}^2)^{1/2}$, $D_{slab} = \delta B_{slab}^2 l_{slab}/2B^2$, $D_{2D} = \delta B_{2D}\tilde{l}/B$, where l_{slab} is the correlation length for slab-component fluctuations, $\tilde{l}$ is the turbulence ultrascale (Matthaeus et al., 1995), R_L is the particle Larmor radius, V is the particle speed, and $\Omega = V/R_L$ is the angular gyrofrequency. We also have introduced the timescale τ to describe the decorrelation of the particle trajectories. Here ζ is the parameter such that when $\zeta = 0$ it corresponds to BAM limit and when $\zeta = 1$ it corresponds to field line random walk limit (Jokipii and Parker, 1969). Values of ζ between 0 and 1 correspond to a rigidity variation of $\kappa_\perp$ intermediate between BAM and field line random walk limits, as reported in some numerical simulations (Giacalone and Jokipii, 1999). The heliospheric boundary is assumed to be at 100 AU. The solar wind speed is 400 km/sec in the equatorial plane and increases to 800 km/sec in the polar regions. The tilt angle α of the wavy current sheet is set at 20°, a value appropriate for low solar activity. The interstellar proton spectrum as function of rigidity is as given in Burger et al (2000). We consider $\lambda_\parallel$ as defined in Zank et al (1998):

$$\lambda_\parallel = 3.1371\frac{B^{5/3}}{\delta B_{x,slab}^2}\left(\frac{R}{c}\right)^{1/3} l_{slab}^{2/3}\left\{1+\frac{7/9A}{(q+\frac{1}{3})(q+\frac{7}{3})}\right\}, \quad (2)$$

where $A = (1+s^2)^{5/6} - 1$, $s = 0.746834R_L/l_{slab}$, and $q = (5s^2/3)/(1+s^2-(1+s^2)^{1/6})$, r is the radial heliocentric distance, $\delta B_{x,slab}^2$ the variance of the x component of slab geometry fluctuations, and $R = pc/Ze$ the particle rigidity (p momentum, c the speed of light, and Ze particle

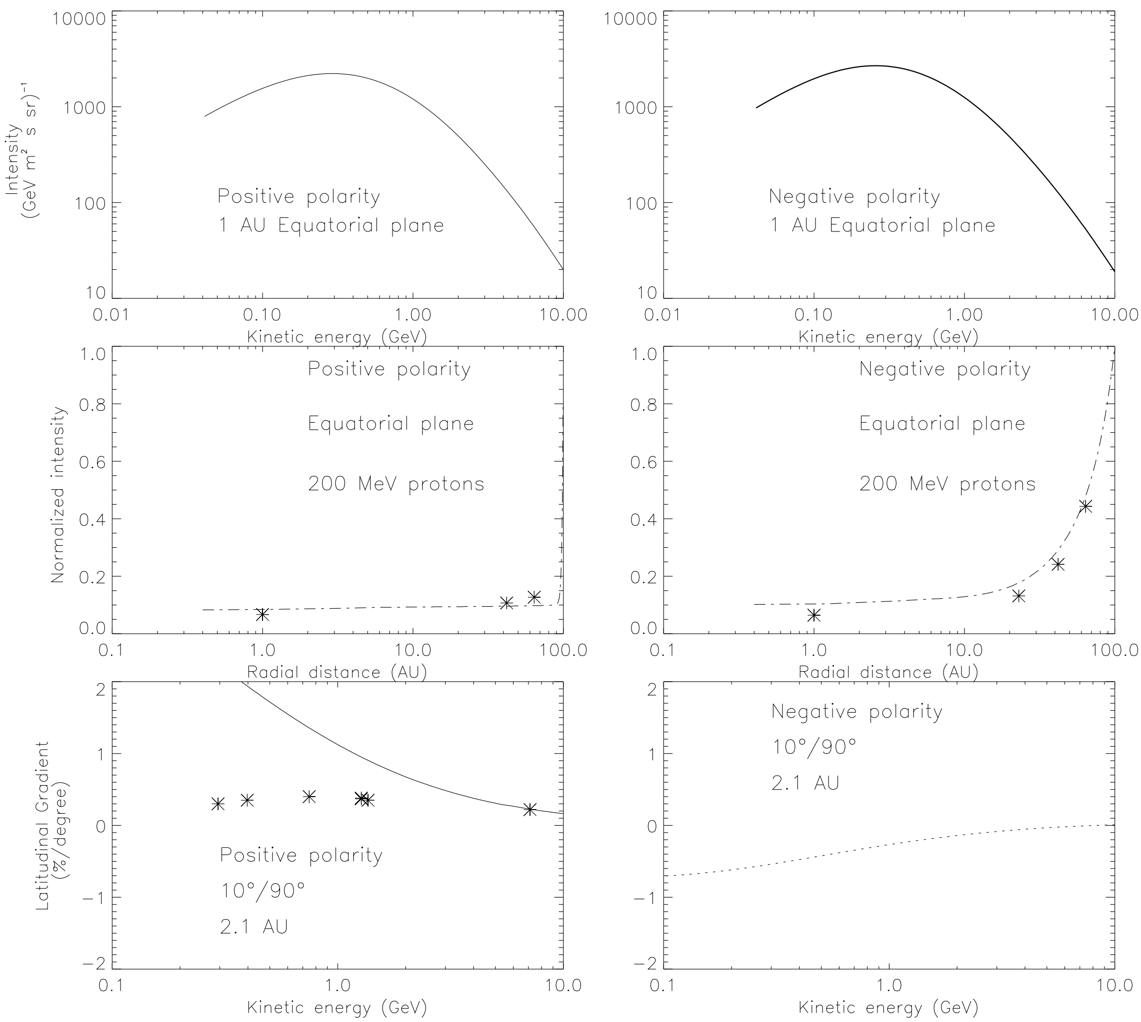

Fig. 2. Same as in Fig. 1 but for $l_{slab} \propto r$.

charge). The Eq(2) is in very close accord with the exact Fokker-Planck result [Zank et al., 1998]. The fractional term in braces in Equation (2) is important when $R_L > l_{slab}$. In this regime, the particles are resonant with fluctuations in the turbulence energy range, and $\lambda_\parallel$ scales with rigidity as R^2. In contrast, when $R_L < l_{slab}$, the particles are resonant with the inertial range, and $\lambda_\parallel$ scales as $R^{1/3}$. This is a key reason that modulation is sensitive to the radial variation of l_{slab}, because particles can shift from one regime to the other depending upon how l_{slab} changes.

Note that the radial dependence of δB^2 follows from the models of Zank et al [1996]. In the equatorial plane its radial dependence changes from $\sim r^{-3.6}$ to $r^{-2.2}$ at about 7 AU. Since the radial dependence of the magnitude of the average magnetic field changes from r^{-2} close to the Sun to r^{-1} in the rest of the heliosphere that we consider, the ratio $\delta B^2/B_0^2$ has only a weak radial dependence in most of the heliosphere. This is also true in polar regions. For drift we use the form $\kappa_T = vR_L/3$. The motivation for this weak scattering form is given in Burger et al [2000].

3 Numerical results

Our base line quantities are $\zeta = 0.15$, $\tilde{l}/l_{slab} = 100$. Keeping them constant we vary the radial dependence of the correlation length from $r^{-0.3}$ in Fig. 1 to r^{+1} in Fig. 2. Fig. 1 describes the intensity of particles (top left) at 1 AU in the equatorial plane for positive polarity as kinetic energy varies. The middle left plot describes radial gradient for 200 MeV particles in the equatorial plane and the bottom left describes the latitudinal gradient at 1 AU for positive polarity. The right panel describes the same for negative polarity. Drift effects in Fig. 1 are quite moderate, as is evident in the small difference between the radial profiles for the different polarity states.

For the case that $l_{slab} \propto r$ (Fig. 2), the increase in correlation length leads to an increase in λ_{rr} in the inner heliosphere but then a decrease in the outer heliosphere. More pronounced is the decrease in $\lambda_{\theta\theta}$ resulting in increased drift effects. The radial profiles are now very different in the two polarity cycles and in fact agree quite well with the data.

We define latitudinal gradient as follows:

$$G_\theta(r) = \ln \frac{I(r,\theta_2)}{I(r,\theta_1)} \frac{100}{(\theta_2 - \theta_1)}$$

with $I(r,\theta_i)$ the intensity at position (r,θ_i). We have the observational Ulysses data only for positive polarity. The latitudinal gradient for the negative cycle is negative and for the positive cycle it is larger than for the previous case. It appears some new physics, e. g., the Fisk field (Fisk 1996) is required to explain this behavior.

4 Conclusions

Efforts to develop an ab initio model of solar modulation face numerous challenges, including the lack of a satisfactory theory of perpendicular transport, poor understanding of the turbulence ultrascale, and the undetermined radial variation of the magnetic correlation length (for slab-like fluctuations). This work has focused upon the latter effect by comparing a model in which the correlation length scales as $l_{slab} \propto r^{-0.3}$ (motivated by pickup ion-driven turbulence) with one in which the correlation length scales as $l_{slab} \propto r^{+1}$.

For the functional form given by equation (2), the correlation length delimits two regimes with different rigidity (R) scalings of the parallel mean free path $\lambda_\parallel$. The scalings are as $R^{1/3}$ or R^2 depending upon whether the particle Larmor radius is respectively smaller than or larger than the correlation length. For representative solar wind conditions at 1 AU, the boundary between the two regimes is at ~ 10 GV, and hence the $\lambda_\parallel \propto R^2$ regime is not particularly relevant for most cosmic rays at the orbit of Earth.

The question is what happens in the outer heliosphere, as we trace the cosmic ray particle back in time. For a field magnitude scaling as r^{-1}, the Larmor radius scales proportionally to r. If $l_{slab} \propto r^{-0.3}$, then there is a rapid migration of particles from the $\lambda_\parallel \propto R^{1/3}$ regime to the $\lambda_\parallel \propto R^2$ regime as we trace them backwards into the outer heliosphere. With larger mean free paths, diffusion becomes relatively more important, leading to diffusion-dominated modulation. In contrast, if $l_{slab} \propto r^{+1}$, then < 10 GV particles remain in the $\lambda_\parallel \propto R^{1/3}$ regime throughout the modulation process. With small mean free paths, modulation becomes drift-dominated.

Our modeling results are consistent with the considerations discussed above. In Fig. 1 (diffusion-dominated) radial gradients are only mildly sensitive to solar polarity, and latitude gradients are positive in both polarity epochs. In Fig. 2 (drift-dominated), radial gradients differ strongly in the two polarity epochs, and the latitude gradient reverses sign. For the rather limited range of parameters considered here, drift-dominated modulation provides a better description of radial gradients, but neither model fits the latitude gradients. This suggests a possible need for new features, e.g., implementation of the Fisk (1996) field, or development of more realistic models of turbulence transport at high latitudes. For preliminary results on the effect of the Fisk field using a three-dimensional model, see Burger and Hattingh (2001).

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