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Angular Correlation of Compton-Scattered Annihilation Photons and Hidden Variables (*) (**).

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Summary. — The relative linear polarization of protons from two-quantum annihilation of positrons in copper was measured by Compton scattering. Measurements of the angular distribution of Compton-scattered photons arriving in coincidence were carried out over a wide range of scattering angles, both polar and azimuthal. The results agree with standard quantum-mechanical calculations assuming opposite parity of the electron and the positron. This result has implications regarding hidden-variable theories in quantum mechanics. A theorem by Bell restricts the values that any local hidden-variable theory can predict for certain relations between measurements made on correlated systems such as the photon pair from positron annihilation. It is shown that the distributions we observed could not give results allowed by Bell's theorem if the photons were measured by ideal polarization analyzers, assuming the correctness of the usual quantum-mechanical Compton-scattering formulae. Our results are thus evidence against local hidden-variable theories.

1. – Introduction.

A measurement has been made of the relative linear polarization of the photons emitted when a positron annihilates at rest. The results of this exper-

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iment have implications regarding the belief of EINSTEIN and others that it is possible to find a theory which provides more than the statistical predictions of quantum mechanics.

When a positron annihilates at rest, conservation of linear momentum requires the creation of more than one photon, usually two photons with equal and opposite momenta. The polarization states of the two photons are also related, as was first show by WHEELER [1]. YANG [2] showed that this was a consequence of invariance under rotation and parity transformations. Suppose that the two annihilation photons are moving in opposite directions along the z-axis, and assume that the electron and positron have opposite parities. The allowed linear polarization state is

$$\psi = \frac{|XY\rangle - |YX\rangle}{\sqrt{2}}, \tag{1}$$

where the symbol $|XY\rangle$ denotes a state with photon No. 1 polarized in the x-direction and photon No. 2 polarized in the y-direction, and $|YX\rangle$ denotes a state with photon No. 1 polarized in the y-direction and photon No. 2 polarized in the x-direction. The linear polarizations of the two photons are sometimes said to be « at right angles to each other ».

It would be very convenient to demonstrate this with standard optical tools like polaroids or birefringent crystals. However, such ideal polarization analyzers do not exist for the high-energy gamma-rays emitted in positron annihilation. Therefore, one must use Compton scattering to measure the relative polarization of the photons. To see why Compton scattering acts like a linear-polarization analyzer, consider the classical analogue of Compton scattering, which is Thompson scattering. When a linearly polarized wave hits an electron, the electron vibrates in the direction of the electric vector and radiates like a dipole, so that scattered rays tend to be perpendicular to the electric vector. Returning to the quantum-mechanical description, one might guess that finding a scattered photon at a certain angle corresponds to finding the linear polarization at the perpendicular angle. Thus, in the case of the two photons, which are « polarized at right angles », one might guess that the scattered photons would tend to scatter in perpendicular directions.

The experimental arrangement used to investigate this is shown schematically in Fig. 1. The positrons annihilate between two scatters. The emerging annihilation photons are scattered by them into two detectors. If the two photons are polarized at right angles to each other, then one expects a maximum coincidence counting rate when the difference in azimuthal angle $(\varphi_2 - \varphi_1)$ between the detectors is 90°.

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(1) A. WHEELER: *Ann. New York Academy of Sciences*, **48**, 219 (1946).
(2) C. N. YANG: *Phys. Rev.*, **77**, 242 (1950).

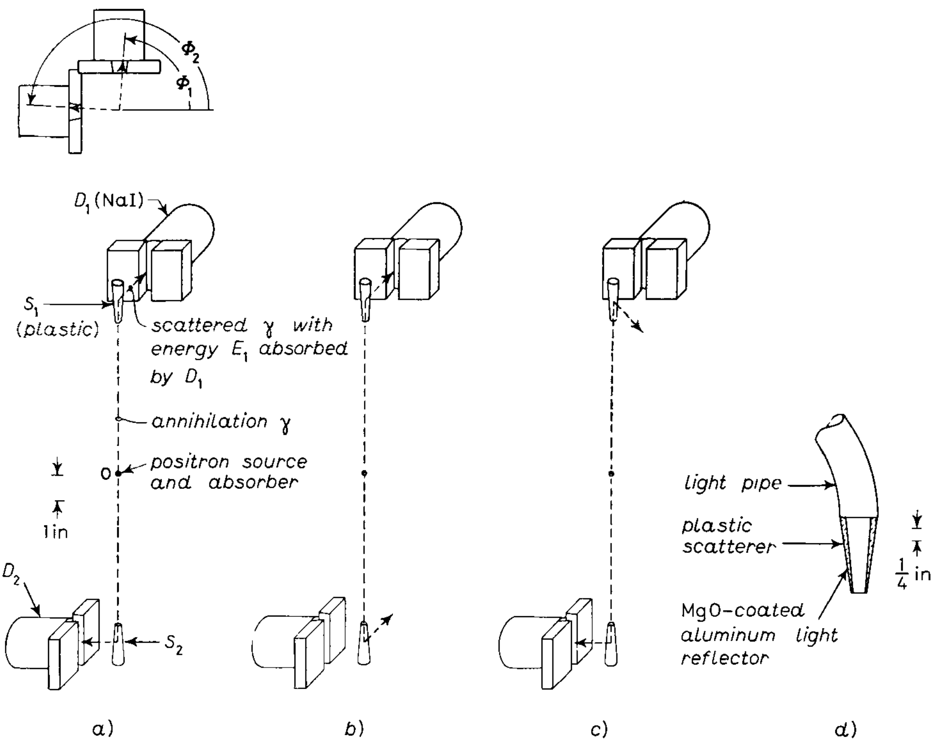

Fig. 1. – Schematic view, to scale, of the experimental arrangement. The lead collimator is omitted. *a*) Fourfold coincidence event (N-event); *b*), *c*) threefold coincidence events (n_1- and n_2-event respectively); *d*) detail of scatterers S_1, S_2.

The explicit expression for the probability of detecting a pair of scattered photons in this geometry given polarization states before scattering as in eq. (1) was first worked out by PRYCE and WARD [3].

Let the line connecting the source and the two scatterers be the z-axis. When a pair of annihilation photons moving in opposite directions along the z-axis scatter off electrons in the scatterers S_1 and S_2, let the scattering angles with respect to the z-axis be θ_1 and θ_2 and let the azimuthal scattering angles be Φ_1 and Φ_2, as shown in Fig. 1. Since the kinematics of Compton scattering give a definite relation between the scattering angles θ_1 and θ_2 and the energies of the two photons after scattering E_1 and E_2, one can write the probability of finding the two scattered photons as a function of E_1, E_2, Φ_1 and Φ_2:

$$P(E_1E_2\Phi_1\Phi_2) = \frac{1}{4\pi^2}F(E_1)F(E_2)[1 - m(E_1)m(E_2)\cos 2(\Phi_2 - \Phi_1)]\,, \tag{2}$$

[3] M. H. L. PRYCE and J. C. WARD: *Nature*, **160**, 435 (1947); see also H. S. SNYDER, S. PASTERNAK and J. HORNBOSTEL: *Phys. Rev.*, **73**, 440 (1948).

where $F(E)$ is the usual Klein-Nishina cross-section for Compton scattering of a photon with initial energy equal to one electron rest mass and final energy E, and $m(E)$ is a function plotted in Fig. 6 whose explicit form is given in the Appendix. For photons from two-quantum annihilation, $m(E)$ has a maximum value when E is just above half an electron rest mass ($\theta = 82°$). The Φ-dependence of the countingg rate is therefore of the form $[1 + A\cos 2(\Phi_2 - \Phi_1)]$, with the coefficient A of the $\cos 2\Phi$ term depending on the energy (or angle θ) of the scattered photons.

Using aluminum scatterers and anthracene detectors, WU and SHAKNOV (4) measured this Φ-dependence is early as 1950 and found good agreement with theory, showing that the electron and positron did have opposite parity. However, the inefficient detectors available at the time required collection of events over a wide range of scattering angles and thus large corrections for geometrical effects. Later, in 1960, LANGHOFF (5) did a thorough measurement with good geometry at many azimuthal angles, but it still might be argued that the agreement he found at one particular value of e (polar angle $\theta = 82°$) was fortuitous. Our aim was to test the predictions of quantum mechanics for this distribution over a range of scattering angles with as few uncertainties about normalization or geometrical corrections as possible.

The reason why it is worth-while to lavish so much attention on this measurement is because it is often referred to in discussions of « hidden variable » theories in quantum mechanics. The implications of this experiment for such theories have been discussed by one of us elsewhere (6,7), so we will only outline them briefly here.

In a well-known paper published in 1936 EINSTEIN, PODOLSKY and ROSEN (8) critically examined the usual quantum-mechanical treatment of measurement of two noncommuting variables in a system of two particles which had interacted in such a way that measurements on them were correlated even though the particles had separated before the measurement. They conclude that the « real, factual situation » which they assumed must exist independent of our observations could not possibly be completely described by quantum mechanics. They were answered by many critics but the argument remained mostly on a philosophical level, without any experimental tests being proposed. Then in 1957 BOHM and AHARONOV (9) pointed out that the relative polariza-

(4) C. S. WU and I. SHAKNOV: *Phys. Rev.*, **77**, 136 (1950). Earlier measurements were made by: E. BLEULER and H. L. BRADT: *Phys. Rev.*, **73**, 1398 (1948); R. C. HANNA: *Nature*, **162**, 332 (1948).

(5) H. LANGHOFF: *Zeits. Phys.*, **160**, 186 (1960).

(6) L. KASDAY: *Rendiconti S.I.F.*, Course IL (New York, N. Y., and London, 1971).

(7) L. KASDAY: Thesis, Columbia University (1972).

(8) A. EINSTEIN, N. ROSEN and B. PODOLSKY: *Phys. Rev.*, **47**, 777 (1935).

(9) D. BOHM and Y. AHARONOV: *Phys. Rev.*, **108**, 1070 (1957); *Nuovo Cimento*, **17**, 964 (1960).

tion of annihilation photons was an example of the kind of situation discussed by EINSTEIN, PODOLSKY and ROSEN. They were able to show that the measurements of WU and SHAKNOV were sufficient to rule out certain hypothetical modifications of quantum mechanics motivated by Einstein's ideas. In 1964 BELL ([10]) showed that a whole class of such theories, known as local hidden-variable theories, could be tested by experiment. Bell's theorem placed limits on the values that any such theory could predict for certain correlations among measurements that might be made on system of the Einstein-Podolsky-Rosen type. We will show in Sect. **3** that this experiment is not ideal for testing Bell's theorem, but it does make any theory that satisfies Bell's theorem and reproduces our results look quite artificial.

2. – Methods and results.

2·1. *Experimental method.* – The experimental arrangement is shown in Fig. 1 and 2. Positrons were emitted by a radioactive source, stopped and

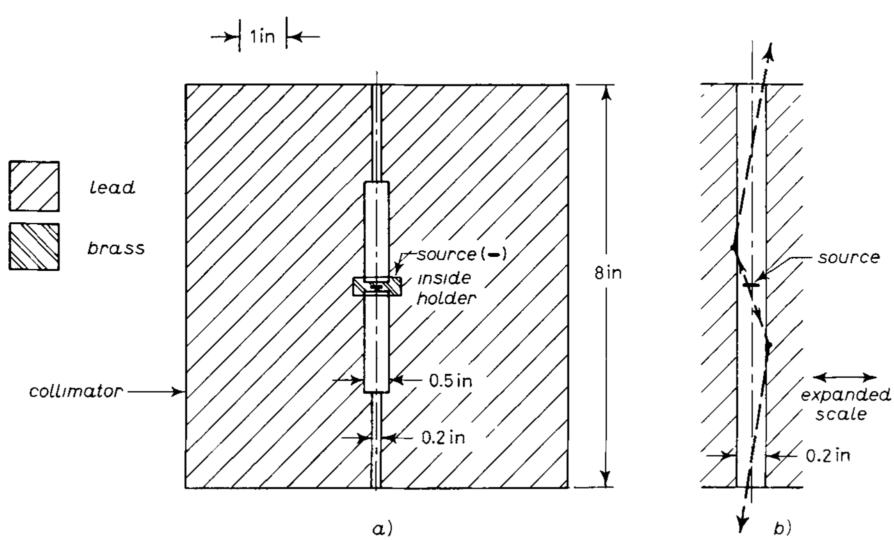

Fig. 2. – *a*) Collimator, source holder and source. The 0.5 in diameter cavity prevents events of the type shown in *b*). Note the expanded horizontal scale in *b*).

annihilated (in copper) at 0. The annihilation gamma-rays were emitted in all directions; the vertical direction was selected by a lead collimator which is omitted in Fig. 1 but is drawn in Fig. 2. Events were sought in which the

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([10]) J. S. BELL: *Physics*, **1**, 195 (1964).

annihilation photons Compton-scattered off electrons in S_1 and S_2 and entered detectors D_1 and D_2, which measured their energies. Lead slits selected the range of azimuthal angles Φ_1 and Φ_2 which were accepted. The top slit-detector assembly was rotated to vary the relative azimuthal angle.

False background events were virtually eliminated by making the scatterers out of plastic scintillators [11]: we required a 4-fold time coincidence among the two scatterers and the two detectors, and also imposed a « sum energy requirement » that the total energy deposited in each scatterer plus detector equal the energy of the annihilation photon.

Instead of simply measuring the coincidence rate as a function of azimuthal angle $(\Phi_2 - \Phi_1)$, we measured the quantity R defined by

$$R(\varphi_1\varphi_2 e_1 e_2) = \frac{N/N_{ss}}{(n_1/N_{ss})(n_2/N_{ss})}\,, \tag{3}$$

where

N_{ss} = number of times the two photons Compton-scatter,

N = number of times the two photons Compton-scatter and both photons are dectected,

n_1 = number of times the two photons Compton-scatter *and* only photon 1 is detected,

n_2 = number of times the two photons Compton-scatter *and* only photon 2 is detected,

φ_1, φ_2 = the azimuthal angles at which the lead slits are positioned (to be distinguished from Φ_1, Φ_2 which refer to the *photons*);

e_1, e_2 = the outputs of the energy detectors D_1 and D_2 (to be distinguished from E_1 and E_2, the « real » photon energies).

If it is assumed that the source, scatterers and detectors are very small, the polarizations of the photons are as in eq. (1) and each photon Compton-scatters once in each scatterer, calculating R with the appropriate Compton-scattering cross-section gives

$$R(\varphi_1\varphi_2) = 1 - m(e_1)m(e_1)\cos 2(\varphi_2 - \varphi_1)\,. \tag{4}$$

This is just the Φ-dependent term of eq. (2). For comparison of our results with theory, the quantity R has a number of useful properties:

1) If the momenta of the scattered photons were uncorrelated, R would equal 1. Deviations of R from 1 correspond to correlations between the mo-

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(11) This arrangement had been used earlier by LANGHOFF (see note (5)).

menta. Assuming constant source strength and geometry, R is proportional to the coincidence counting rate, as measured in earlier experiments.

2) A number of instrumental effects that would disturb a simple measurement of the coincidence counting rate cancel out of the expression for R, for example, variations in source strength and slit width (to first order).

For our experiment with real slits and detectors the expression for R must be modified by a number of geometric corrections of the order of a few percent in size. These will be discussed in Subsect. **2**'3.

2'2. *Detailed description of the experiment.*

2'2.1. Radioactive sources. ^{64}Cu positron sources were used for the data-taking runs. Their β^+-activity at the beginning of the runs was 10 mCi. The main features of the ^{64}Cu decay scheme, Fig. 3, are a 12.8 h half-life, a

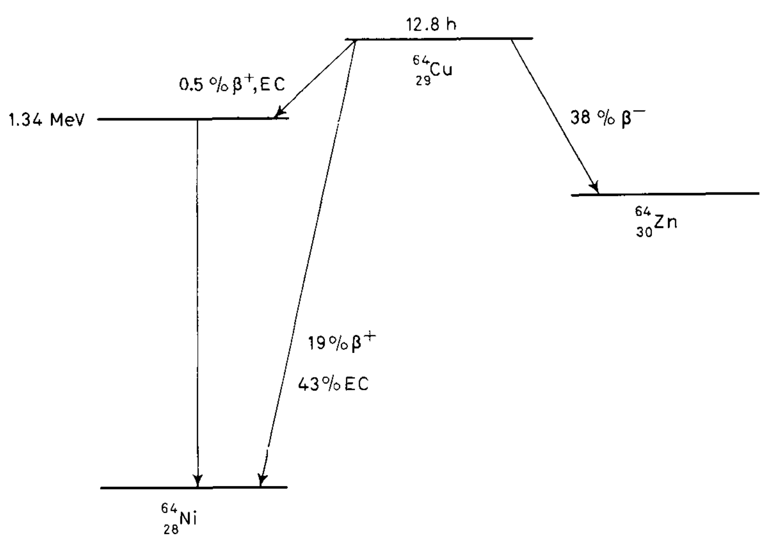

Fig. 3. – Decay scheme of ^{64}Cu.

19 % β^+ branching ratio, and $\sim$1 MeV gamma-rays accompanying 0.5 % of the positron emissions. The sources were made of $\frac{1}{8}$ in. diameter, $\frac{1}{16}$ in. thick natural-copper discs, which were neutron irradiated. Natural copper could be used since it contains 69 % ^{63}Cu. (The irradiation was performed at the Industrial Reactor Laboratories.)

For energy calibration we used the 122 keV ^{57}Co line and the 511 keV ^{22}Na line. Two pairs of sources were made, one for each counter. These pairs were held at standard positions with respect to the counters during calibration runs.

2.2.2. Source holder and collimator. The source was supported by a brass holder which slid into a rectangular hole in the lead collimator, Fig. 2. (This rectangular hole is perpendicular to the plane of the paper.) The positrons were stopped and annihilated in the source and in a thin layer of the surrounding holder material. Holes in the lead of 0.2 in. diameter collimated the annihilation photons; these holes were enlarged to 0.5 in. diameter near the source to avoid the events shown in part *b*) of the Figure.

If one of the annihilation photons underwent large-angle Compton scattering inside the collimator, its momentum would no longer be opposite the other photon's momentum, so both photons could not escape the collimator. This event would not be counted and was of no concern.

A photon could scatter through a small angle in the collimator, emerge and reach the scatterer. To set a limit on how many did so, we examined the energy spectrum of the emerging photons, using a lithium-drifted germanium detector. We required a coincidence between the Ge(Li) detector and a plastic scintillator placed below the collimator. The spectrum was compared to the spectrum taken without the collimator. Scattered photons comprised at most a few percent of all those reaching the scatterer position. Furthermore, photons scattering through such small angles lose only a few percent of their polarization. Hence, the net effect of small-angle Compton scattering in the collimator is only (a few per cent)$^2 \sim 10^{-3}$, which is negligible in this experiment.

2.2.3. Scatterers. The length of each scatterer was large enough (1.5 in.) for 33 % of the entering photons to Compton-scatter, but it was necessary to keep the diameter small to minimize the chance of the photons scattering a second time.

We used a conical scatterer surrounded by a slightly larger conical light reflector, coated on the inside with MgO (for efficient, diffuse reflection), Fig. 1. Total internal reflection in the scintillator tends to send light toward the light pipe, and the MgO reflects most of the remaining light. The resolution for 90° scattered photons was 30 % full width at half the maximum of the peak.

2.2.4. Azimuthal angle defining slits. The slits were made of lead and were 0.48 in. thick. The inside edges were « aimed » at the axis of the collimator to minimize scattering. The top slit and the detector behind it were mounted so they could rotate about the axis of the collimator. The slits subtended an angle of about 20° at the source.

2.2.5. Detectors and electronics. The detectors were 2 in. diameter by 2 in. long NaI crystals made by Harshaw, used with Radio Corporation of America (R.C.A.) type 8575 bi-alkalai 12 stage phototubes. The function of the electronics was to collect the numbers required to calculate R (defined in eq. (3)) for a given set of values of e_1 and e_2, the measured energies

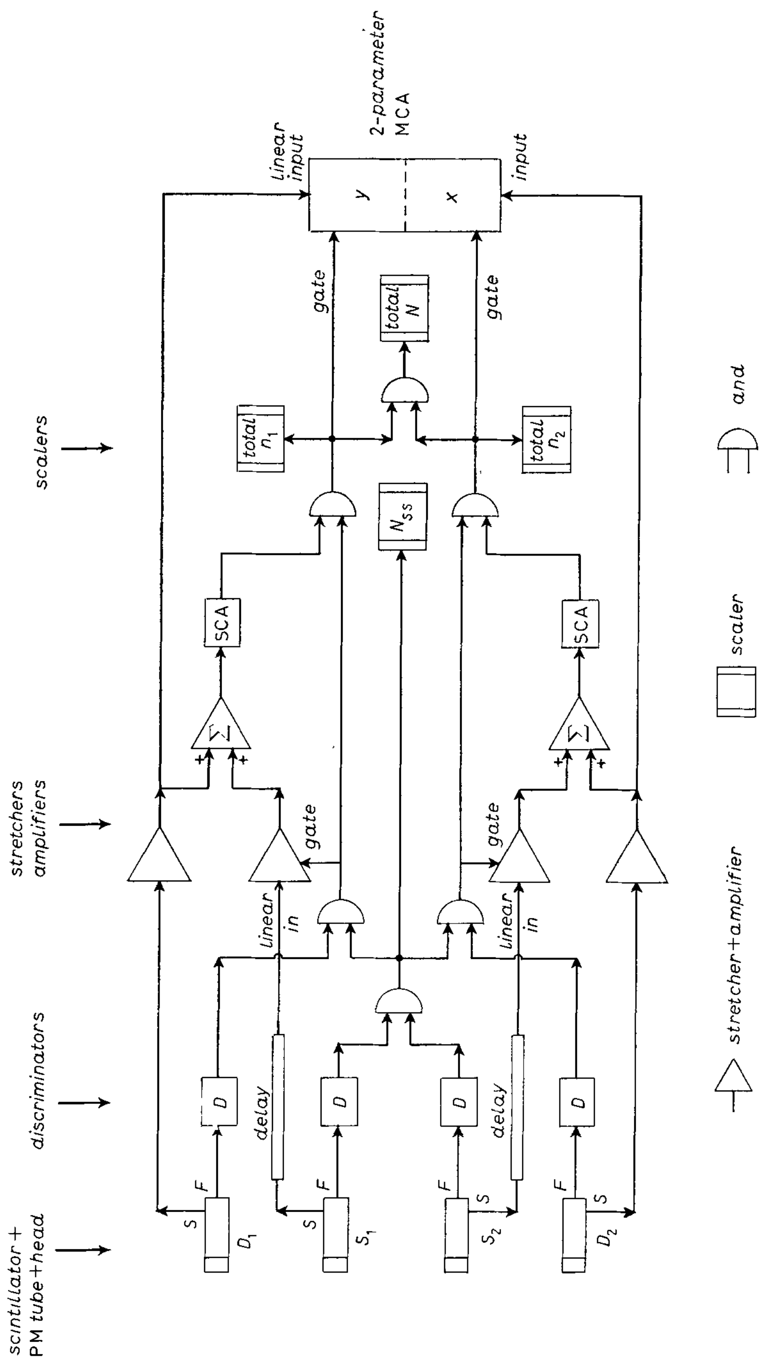

Fig. 4. – Block diagram of the electronic logic. MCA: multichannel analyzer, S: slow output, F: fast output, D: discriminator, Σ: sum amplifier, SCA: single-channel analyzer.

of the two scattered photons after scattering. These include a two-parameter e_1 *vs.* e_2 spectrum of the 4-fold coincidence events, e_1 and e_2 spectra for the 3-fold coincidence events and the total number of 2-fold coincidences between the scatterers only.

A simplified block diagram of the electronics is shown in Fig. 4. Discriminators connected to the fast outputs of the photomultipliers generated the fast logic pulses S_1 and S_2 from the two scatterers, and D_1 and D_2 from the two NaI detectors. The fast (S_1S_2) logic pulses were generated from the S_1 and S_2 pulses by a fast AND (21 ns resolving time) and counted by a scaler. The fast logic pulses $(S_1S_2D_1)$ and $(S_1S_2D_2)$ were also generated.

The slow outputs of the photomultipliers were stretched and amplified to form the slow analogue pulses s_1 and s_2 from the scatters, and d_1 and d_2 from the NaI detectors. Because of the high singles rate in the scatterers, the s_1 and s_2 stretchers were gated by the $(S_1S_2D_1)$ and $(S_1S_2D_2)$ coincidence pulses respectively. This made it necessary to run the inputs of these two stretchers through delay lines.

The s_1 and d_1 analogue pulses were then summed. This gave the total energy left in the scatterer and detector by a scattered photon, which should add up to e_m = one electron mass (0.511 MeV). An observed spectrum of these sum pulses is shown in Fig. 5. The sum was fed to a single-channel analyzer (SCA).

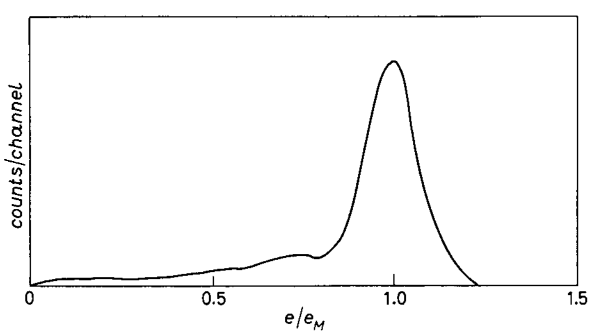

Fig. 5. – Typical sum energy spectrum. e=energy deposited in scatterer and energy deposited in detector; e_M=energy of annihilation gamma-ray.

When the sum pulse was between $0.83e_m$ and $1.17e_m$ the logic pulse Σ_1 was generated. Then the slow logic pulse $(S_1S_2D_1\Sigma_1)$ was generated, and sent to a scaler and the gate of the Y ADC (analogue to digital converter) of the MCA (multichannel analyzer). Similar, the slow logic pulse $(S_1S_2D_2\Sigma_2)$ was generated and sent to a scaler and the X ADC gate.

The analogue pulses d_1 and d_2 were fed to the analogue inputs of the Y and X ADC's respectively. The ADC's digitized d_1 and d_2 to form e_1 and e_2 whenever a logic pulse appeared at their respective gates. If one and only one of the ADC gates was opened, the corresponding e_1 or e_2 pulse would be added to the appropriate 1-parameter spectrum. If both ADC gates were opened in coincidence (within 1.5 μs), the (e_1, e_2) pulse pair would be added to the 2-parameter spectrum. Thus, the logic requirement on the pulses in the 2-parameter spectrum was

$$[(S_1S_2D_1\Sigma_1)\cdot(S_1S_2D_2\Sigma_2)] = (S_1S_2D_1D_2)\Sigma_1\Sigma_2\,,$$

the desired 4-fold coincidence requirement. The 1-parameter spectra did not actually contain all the pulses which satisfied the 3-fold coincidence requirements $[(S_1S_2D_1\Sigma_1), (S_1S_2D_2\Sigma_2)]$; the 4-fold coincidence events were missing. The missing events were added later using the computer in the MCA.

The scalers were gated with the « busy » output of the MCA so that they would only count when the MCA was accepting pulses.

2·3. *Corrections to R.*

2·3.1. Types of corrections considered. With point scatterers and detectors and perfect collimators, the measured ratio of coincidence counting rates R as defined in eq. (3) would be given by eq. (4), which may be rewritten as

$$R = A + B\cos 2(\varphi_2 - \varphi_1)$$

with $A = 1$ and $B = m(e_1)m(e_2)$.

Our nonideal geometry introduced two classes of corrections to this expression. First there are effects which simply change the value of A and B. These include the effective angular widths of the slits which select photon azimuthal angle, the combined effect of the finite-energy resolution of the detectors and the energy intervals selected for analysis, the possibility of photons scattering more than once in the scatterer and the « axial » correlation between the points where the two photons scatter. Then there are corrections for factors which change the form of the angular dependence of R. These factors include the « radial » correlation between the points in the two scatterers where the photons scatter, caused by the 180° angular correlation between the photons, chance coincidence rates which apparently vary with angle because the average source strengths happened to be different when measurements were made at different angles, and apparent variation with angle of the energy intervals selected by the multichannel analyzer because of electronic drift.

All of these effects were carefully considered and are discussed elsewhere [7]. Most of them gave corrections at the 1% level or lower. The most important of them will be discussed below.

2·3.2. Effect of angular widths of the slits. The effective widths of the slits which selected photon azimuthal angles were measured two ways.

First the slit dimensions were measured with a ruler to obtain « geometric widths ». Then the « bottom » slit and detector were moved to the top of the collimator, keeping the distance between the slit and the collimator axis constant. They were rotated to a position approximately opposite the « top » slit and detector, and a ^{22}Na source was placed on the collimator axis. Since the annihilation photons emerge at 180° from the ^{22}Na source, « empirical » slit widths could be obtained by measuring the coincidence rate in the two detectors as the angle between the detectors was varied. Simple calculations showed that the geometric and empirical widths led to 6 % and 5% reductions in B respectively; a value of (5 ± 0.5) % was used.

2˙3.3. Effect of multiple scattering in the scatterers. It is difficult to compute the effect on R of photons that scatter more than once in the scatterer, and this was responsible for the major part of the uncertainty in B. The number of photons which scatter twice and emerge with an energy of $e_m/2$ (the same energy as photons which scatter once through 90°: $e_m =$ one electron mass) was computed, assuming that the cross-section for each of the two scatterings was independent of the azimuthal scattering angle, in other words, polarization information was assumed lost.

The value of B was accordingly multiplied by {1— (no. double scattered)/(no. single scattered)} to give an upper limit on the reduction in B. The limit obtained was 7%: accordingly, we took the reduction in B to be (3.5 ± 3.5)%.

2˙3.4. Effect of chance coincidences. These were calculated for all of the AND gates and for each run. Necessary corrections were made; the maximum correction was 1.3 %.

2˙4. *Data reduction and results.*

2˙4.1. Energy spectrum of triple-coincidence events. A typical energy spectrum of the triple-coincidence events is shown in Fig. 6, with schematic drawings of events associated with different parts of the spectrum. The factor $m(e)$ of the angular dependence term of eq. (2) is also shown in the Figure. For single scattered photons the spectrum should have the shape of $m(e)$ multiplied by the effect of solid angle and efficiency of the detectors for scattered photons of energy e. The spectrum should go to zero at e_b and e_d where the photons will miss the counters. A typical true event, with scattering angle $\theta = 90°$, has an energy of $0.5e_m$ (e_m is one electron mass) and is shown as c) in the Figure. However, there is a bump at $e = 0.25e_m$ which cannot be caused by true events.

There are two major contributions to this bump. First, there are events in which the photon which has scattered in the scatterer proceeds to Compton-scatter in the detector and escape, thereby leaving only part of its energy in the detector as shown in *a*) in the Figure. These events have a spectrum which would normally extend from 0 to about $0.25e_m$, except that the lower-energy events are vetoed by the sum energy requirement. Of the order of 15% of the events with energy $0.25e_m$ leak through because of the finite resolution of the detectors. The common event shown, in which a photon scatters through 90° and then backscatters out of the detector, has an energy of exactly $0.25e_m$, the energy of the bump.

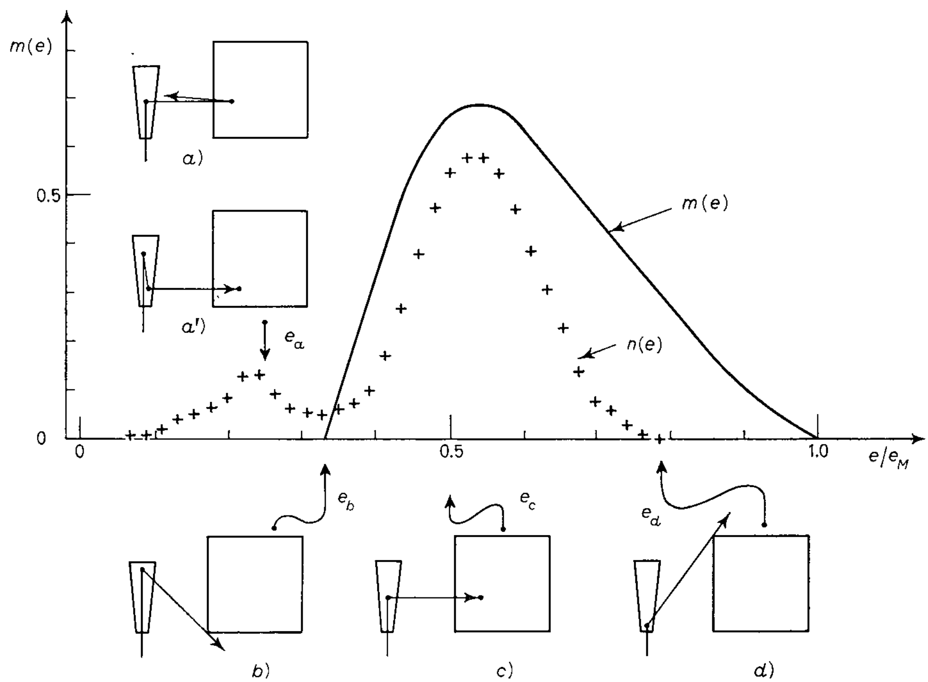

Fig. 6. – Spectrum of threefold « n_1 »-coincidence events $n(e)$ and typical events contributing to various parts of the spectrum. The amplitude of the ($\cos 2\varphi$)-dependence of R is proportional to the theoretical function $m(e)$ shown as a solid line.

Then there are events in which the photon scatters twice in the scatterer before being totally absorbed by the detector as shown in *a'*). These have an energy spectrum which extends from 0.2 to $1.0e_m$. The energy spectrum of these events rises at low energies, with peaks expected near $0.25e_m$ and $0.5e_m$. Thus these events, especially the event shown as *a'*), contribute to the $0.25e_m$ bump. In Subsect. **2**˙3.3 and upper limit of 7% for the contribution of the false events near $0.5e_m$ was obtained.

2˙4.2. Measured angular correlations of scattered photons (R). First the angular-correlation function R was computed using the total numbers of threefold and fourfold coincidence events in eq. (3). That is, included in N,

n_1 and n_2 were all events which satisfied the appropriate time coincidence and sum energy requirements. The numbers were obtained from the corresponding scalers.

There were corrections which changed the form of the theoretical R *vs.* φ curve (see Subsect. **3·3**.1) due to correlation between the points where the two photons scatter, and accidental coincidences. Each experimental value of R was moved by an amount equal in magnitude but opposite in sign to the correction to the corresponding theoretical value of R. These corrections were small, ~ 0.01, but comparable to the statistical accuracy. After these corrections are made, R should exhibit a $(\cos 2\varphi)$-dependence; therefore, the experimental values of R were plotted against φ. As expected, the points could be fitted by

$$R = A - B\cos 2\varphi\,, \qquad A = 1.0071 \pm 0.0036 \text{ (corrected data)}\,,$$

$$B = 0.3419 \pm 0.0051$$

$$\left(\chi^2/\text{degrees of freedom} = 0.84 \text{ (10 degrees of freedom: } p = 0.6)\right)\,.$$

The observed R *vs.* φ is plotted in Fig. 7. Agreement with the expected cosine behavior is excellent: indeed, better than we would have expected, since deviations due to misalignments were estimated to be a few percent. We therefore neglected any error in B due to misalignment.

It was also our intention to measure the angular correlation as a function of the energy e (or scattering angle θ) of the scattered photons so R was also calculated for restricted regions of the energies e_1 and e_2 of the two scattered

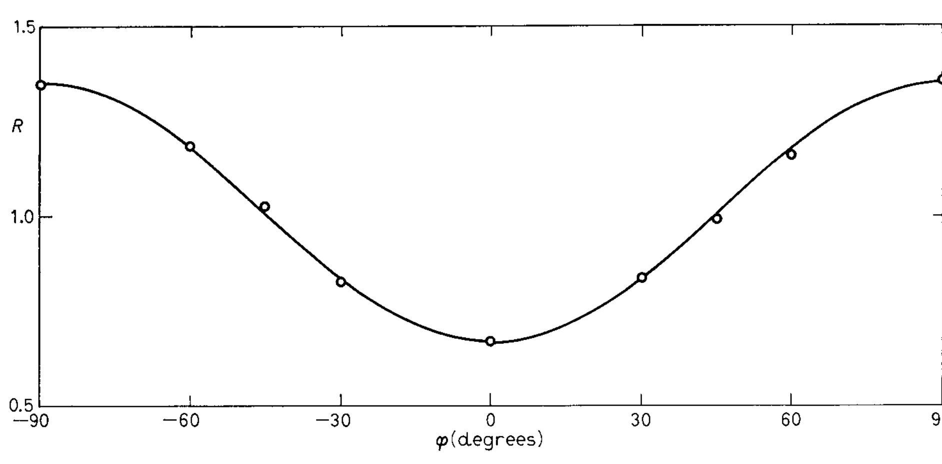

Fig. 7. – Plot of experimental values of R *vs.* relative azimuthal angle. R was computed from the total numbers of fourfold and threefold coincidence events. These data verify the prediction of quantum mechanics that R *vs.* φ can be fitted by $A + B\cos 2\varphi$, with A, B adjustable. The best fit is shown as a solid line (χ^2/degrees of freedom $= 0.84$). $\varphi = \varphi_2 - \varphi_1$, ○ typical $\pm 1\sigma$ error.

photons. The coefficient B in the angular-correlation function R is larger for these restricted regions than for the average over the whole distribution, but the statistics are naturally not as good.

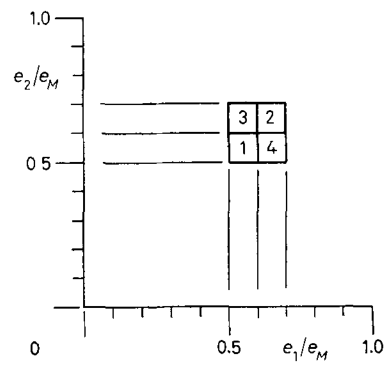

Fig. 8. – The four energy regions chosen to study the amplitude of the cosine dependence of R. The quantities e_1, e_2 are the energies of the scattered photons; $e_M = 1$ electron mass.

We chose four energy regions, as shown in Fig. 8. With the techniques just described R was computed for each region and corrections were applied. One additional correction was needed. The limits of the energy regions were fixed at certain channels in the MCA, and as the experiment progressed, the actual energies corresponding to these channels drifted by several percent. To compensate for this, calibration spectra were taken before and after each run; the change in R which was caused by the drift was calculated using the theoretical R *vs.* energy spectrum. The data points were then moved the same amount in the opposite direction. The theoretical and experimental values of the parameters A and B of the straight-line fits are displayed in Table I. Region 1 was chosen at the maximum of $m(e)$. Since regions 3 and 4 are symmetric when the energies of the two scattered photons are interchanged, the R's of these regions were added. Any systematic deviations from the cosine form are less than or equal to the statistical uncertainties, so they cannot be

TABLE I. – *Comparison of experimental A and B/A with theory.*

Region	Theory		Experiment		χ^2/n'
	A	B/A	A	B/A	
1	1.00 ± 0.05	0.415 ± 0.015	1.021 ± 0.010	0.409 ± 0.018	1.1
2	1.00 ± 0.05	0.372 ± 0.010	0.984 ± 0.019	0.392 ± 0.030	0.7
3+4	1.00 ± 0.05	0.395 ± 0.015	1.020 ± 0.010	0.390 ± 0.017	1.5

(n' = degrees of freedom = 2)

parametrized. Therefore, the errors quoted in the straight-line fits, which are purely statistical, would be doubled to account for the possible systematic errors.

2'4.3. Comparison with the predictions of quantum mechanics. The form of R given in eq. (4) must be modified by the corrections discussed in Subsect. **2'3.1** to compare our results with the predictions of quantum mechanics. When this is done, R takes the form

$$R = A - B\cos 2\varphi\,, \tag{5}$$

with $B/A = (\overline{m}_1 + \Delta m_1)(\overline{m}_2 + \Delta m_2)(1-\varepsilon_\varphi)(1-\varepsilon_m)(1-\varepsilon_s)$, and $A = 1 +$ corrections due to Z-correlations,

$\Delta\overline{m}_1, \Delta\overline{m}_2$ are the finite-energy resolution corrections,

ε_φ is the correction for finite angular resolution,

ε_m is the correction for photons scattering more than once in the scatterer and

ε_s is a correction due to correlation between the points where the two photons scatter.

The values of $\overline{m}_1, \overline{m}_2, \Delta\overline{m}_1, \Delta\overline{m}_2$ were found by numerical integration over the spectra of the triple-coincidence events. The integration used to find these quantities can be viewed as finding the weighted average of $m(e)$, using the triple-coincidence spectrum as the weighting function. For the evaluation of R over the entire spectrum we refer again to Fig. 6, containing a triple-coincidence spectrum and a plot of $m(e)$. The events in and near the bump at e_a, discussed above, had an unknown angular distribution. Therefore, we used the value of $\overline{m}_1$ obtained by integrating from $e = 0.33e_M$ to $E = e_M$: $\overline{m}_1 = 0.601$. Since $0 \leqslant m \leqslant 0.69$ and the events below $0.33e_M$ amounted to 15% of the total counts, the possible error caused by the pump is given by

$$0.51 \leqslant \overline{m}_1 \leqslant 0.60\,,$$

or

$$\overline{m}_1 = 0.56 \pm 0.04\,,$$

similarly

$$\overline{m}_2 = 0.58 \pm 0.03\,.$$

The values of Δm_1 and Δm_2 were found to both be 0.016. The finite-angle factor ε_φ was found to be

$$\varepsilon_\varphi = (4.5 \pm 0.5)\%\,.$$

Also, the reduction in the effect due to the annihilation photons scattering more than once in the scatterer was found to be

$$\varepsilon_m = (3.5 \pm 3.5)\% .$$

The correction to B due to correlation between the points where the two photons scatter was found to be

$$\varepsilon_s = 0.006 .$$

The net result for the theoretical B is

$$B = 0.32 \pm 0.05 ,$$

thus the theoretical prediction becomes

$$B/A = 0.32 \pm 0.05 ,$$

$$A = 1.00 \pm 0.05 .$$

There is agreement within the quoted uncertainties between the theoretical and experimental values of A and B/A.

Similar calculations of A and B where carried out for the restricted energy regions and the results are given in Table I. In these cases the theoretical uncertainties were smaller, though the experimental statistics are not as good. Here also our results are in good agreement with the predictions of quantum mechanics.

Detailed evidence for the theoretical R *vs.* $\cos 2\varphi$ dependence is best provided by the excellent fit to R for the total region Fig. 7, because of its good statistics and freedom from uncertainties due to energy *vs.* channel drift. Evidence that the magnitude of the cosine dependence is in accordance with the quantum-mechanical prediction is provided by the excellent agreement between the theoretical and experimental values of B/A for the energy regions, Table I.

3. – Conclusions.

3.1. *Bell's theorem.* – As was mentioned in the Introduction, our results are related through Bell's theorem to the possibility of constructing a physical theory that describes the « real factual situation » of EINSTEIN, PODOLSKY and ROSEN. We will very briefly outline the arguments leading up to Bell's theorem, state the theorem and define the terms used in it.

It is, of course, well known that the quantum-mechanical description of a given state of a physical system cannot specify with certainty the result of all possible measurements that can be made on the system. For example, if the position of a particle is specified with certainty, only a probability distribution is specified for the momentum. EINSTEIN, PODOLSKY and ROSEN argued, in effect, that associated with any physical system was a set of variables which determine with certainty the results of *all* possible measurements. Such variables are sometimes referred to as « hidden variables ».

It might appear that to disprove the existence of hidden variables is to perform the impossible task of disproving the null hypothesis. But in 1964 BELL ([10]) showed that a certain ideal experiment could rule out all « local » theories of hidden variables. A local theory satisfies the *locality postulate*: a measurement made on a physical system does not influence the values of the hidden variables that determine the results of measurements on another, « distant » physical system.

Bell's theorem may be stated as follows: consider two measuring instruments A and B. Instrument A performs measurements on one physical system, and the other B performs measurements on a « distant » physical system. Instruments A and B have « knobs » which are set to positions a and b, respectively. The locality postulate requires that the knob setting a has no effect on measurement B and *vice versa*. According to the version of his theorem that BELL proved in 1970 and discussed in his review of the hidden-variables question ([12]), when locality holds

$$|\overline{\alpha_4\beta_2}+\overline{\alpha_4\beta_3}|+|\overline{\alpha_1\beta_2}-\overline{\alpha_1\beta_3}|\leqslant 2\,, \tag{6}$$

where

α_i is an *output* of A and when its knob is at position i,

β_j is an *output* of B when its knob is at position j,

$\overline{\alpha_i\beta_j}$ is the mean over many trials of $\alpha_i\beta_j$.

BELL pointed out that his inequality would be violated if instruments A and B « perfectly » measured the spin components (selected by knob settings a and b) of two spin-$\frac{1}{2}$ particles in a state of zero total spin. (« Perfect » measurements are defined to produce the value $+1$ or -1 when either value of a two-value observable, such as the z-component of spin of a spin-$\frac{1}{2}$ particle, is encountered.) BOHM ([13]) had previously shown that such measurements demonstrated the Einstein-Podolsky-Rosen « paradox ». BOHM and AHARONOV ([9]) pointed out that « perfect » measurements of the linear polarization of photons produced in positron annihilation were essentially equivalent to such spin measurements. Suppose we place a source of annihilation photons between

([12]) J. S. BELL: *Rendiconti S.I.F.*, Course IL (New York, N. Y., and London, 1971).
([13]) D. BOHM: *Quantum Mechanics* (New York, N. Y., 1951).

two « perfect » detectors of linear polarization. Define the quantities α and β in eq. (6) as the outputs of the two detectors, and a and b as the angles the detectors' axes make with the horizontal plane. A simple quantum-mechanical calculation yelds

$$\overline{\alpha\beta} = -\cos 2(a-b);$$

if we substitute

$$2a = 0^\circ, \quad 2b = 135^\circ, \quad 2c = 45^\circ, \quad 2d = 90^\circ$$

into Bell's inequality, eq. (6), we obtain

$$2\sqrt{2} \leqslant 2 .$$

The inequality is violated. Therefore if the quantum predictions are correct, a hidden-variable theory would be ruled out.

Unfortunately, this experiment cannot be realized. No ideal polarization detectors ([14]) have yet been found for annihilation photons (or for the optical photons involved in an analogous experiment discussed by HORNE ([15]) and CLAUSER, HORNE, SHIMONY and HOLT ([16])). Consider, for example, Compton polarimeters. The output of a Compton-polarization measurement is either « a photon was scattered into the gamma detector » or « the photon was not scattered into the gamma detector ». In order to apply Bell's inequality *directly* to the polarimeter outputs, it is necessary to assign numerical values to the possible outputs. For example, the output A of one detector might be defined as $+1$ (-1) when the scattered photon hits (does not hit) the gamma detector; and the output B of the other detector can similarly denfied. Also, the quantities a, b in $P(a, b)$ can be taken as the angular placements of the gamma detectors. But when this is done, it turns out that the $P(ab)$ that results does not violate Bell's inequality ([15]). Hence, for these definitions of A and B, a *direct* application of Bell's inequality to the instrumental outputs cannot rule out local hidden-variable theories.

One might think that some other definitions of A and B, or some clever arrangement of many gamma detectors could circumvent this difficulty. But this is not the case, for it is possible to construct an *ad hoc* local hidden-variable theory that reproduces all the results of Compton scattering of annihi-

([14]) Actually an ideal analyzer need not exist. There could in principle exist an « almost ideal analyzer » which not be perfectly efficient but would produce outputs which would violate Bell's inequality. But no one has found such an « almost ideal analyzer » either.

([15]) M. A. HORNE: Thesis, Boston University (1969).

([16]) J. F. CLAUSER, M. A. HORNE, A. SHIMONY and R. A. HOLT: *Phys. Rev. Lett.*, **23**, 880 (1969).

lation photons. Therefore, no direct analysis of Compton scattering could possibly violate Bell's inequality. BELL [17] has produced a counter-example in which the correlation between the scattering events at the two detectors arises from their dependence on a single hidden variable. The model reproduces the quantum predictions for all momentum measurements that could be made on the two scattered photons. Clearly no function of momentum measurements, including any $P(ab)$, could ever violate Bell's inequality. Hence, no such Compton-scattering experiment can absolutely rule out a local hidden-variable theory. Bell's counter-example does not apply when the photons have energies somewhat lower than the masses of the particles which scatter them. For this reason BELL suggests that it might be useful to perform the experiment on photons of different energy. It should be noted, though, that another counter-example, simpler if perhaps more artificial than Bell's, is not subject to this restriction on the photon energy: it is given elsewhere [6,7].

Even though a Compton experiment cannot rule out hidden-variable theories, it can provide strong evidence against them. The following assumptions can be made:

1) it is possible in principle to construct an ideal linear-polarization analyzer,

2) the *results* obtained in an experiment using ideal analyzers and the *results* obtained in a Compton-scattering experiment are correctly related by quantum theory.

Assumption 2) may be clarified as follows. Suppose one or more photons Compton-scatter. It can be shown that, according to quantum theory, the angular distribution of the scattered photons can be computed from the results which would have been obtained in an ideal polarization analysis of the photons and *vice versa*. The computation involves only the Compton-scattering results and the ideal-polarization results. *No* specification of the photon state is necessary. The basis of this proof is given in the Appendix, and the details of the proof are given elsewhere [7]. Assumption 2) is that this relation between the ideal-measurement results and the Compton results is correct.

SNYDER *et al.* [4] showed that this relation is possible because when the photons' polarizations are resolved into components parallel and perpendicular to the scattering planes, interference effects between the components vanish when the Compton scattering is computed. The Appendix uses an alternative argument, involving parity and angular-momentum conservation. The experimental evidence for the validity of the theory of Compton scattering is discussed elsewhere [7].

With the aid of assumptions 1) and 2) Bell's inequality for ideal polarization

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[17] J. S. BELL: private communication. Bell's example has been outlined by one of us elsewhere: see notes [6,7].

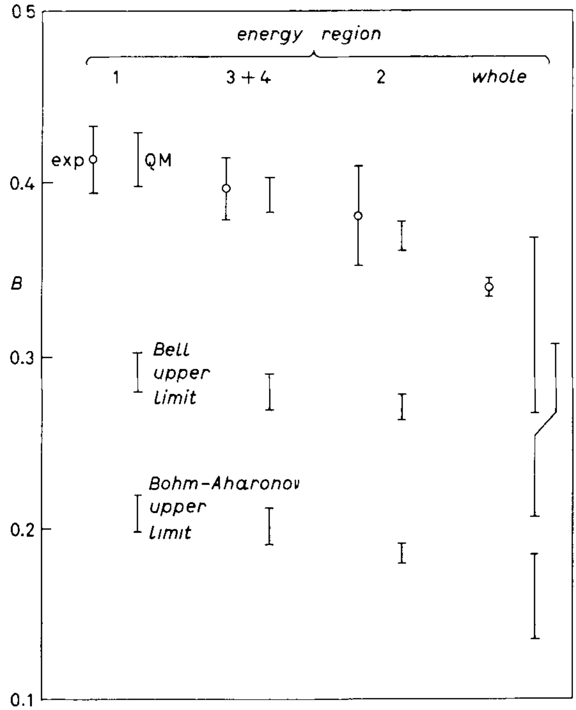

Fig. 9. – Comparison between experimental (exp) results and quantum (QM) predictions for B, and the upper limits on B derived from Bell's inequality and the Bohm-Aharonov hypothesis. This error bars on the experimental points indicate uncertainties in instrumental corrections of the various theoretical predictions.

analyzers was used to calculate corresponding restrictions on the angular distribution of Compton-scattered photons. The result was that the value of B in our expression for R was limited to no more than $1/\sqrt{2}$ of the value predicted by quantum mechanics. This is shown in Fig. 9.

3·2. *The Bohm-Aharonov hypothesis.* – Consideration of the Einstein-Podolsky-Rosen situation have led BOHM and AHARONOV [9] to consider the hypothesis that quantum theory breaks down in a particular way for widely separated particles. JAUCH [18] has shown how considerations involving the notion of a state in axiomatic quantum theory can also motivate the hypothesis.

BOHM and AHARONOV examined the following hypothesis: that quantum theory is valid for particles which are close together, but that after the photons are some « large distance » apart their state vector changes into a product of state vectors for the individual photons. Then a measurement on photon 1 would effect the state vector of 1 but not the state vector of 2. JAUCH [19]

([18]) J. M. JAUCH: *Rendiconti S.I.F.*, Course IL (New York, N. Y., and London, 1971).
([19]) J. M. JAUCH: private communication.

has remarked that in the case of positron annihilation the « large distance » involved might be much larger than the coherence length (~ 7 cm) of the annihilation process.

BOHM and AHARONOV showed that it is impossible in practice to rule out this hypothesis by means of position and momentum measurements on the annihilation photons (or on the particles involved in any scattering experiment). However, the hypothesis can be tested by measuring the linear polarizations of the annihilation photons. A direct calculation [6] shows that all mixtures obeying this hypothesis (with rotational and reflexive symmetry) lead to a value of B in our expression for R which is less than $\frac{1}{2}$ the value predicted by quantum mechanics, as shown in Fig. 7 [20].

3·3. *Conclusions.* – It would be pleasing to be able to say that the results of this experiment rule out local hidden-variable theories. We cannot say that, and in fact it appears that no experiment done with currently available techniques could lead to such a definite conclusion. There are two main difficulties, which will now be discussed with reference to other experiments as well as our own.

First, polarization measurements that are perfect, or sufficiently close to perfection to directly demonstrate Bell's inequality, cannot be made. One must make a measurement with an imperfect instrument and infer from the measurement what the output of a perfect instrument would have been. The reasons why this is true for a Compton polarimeter have been discussed here. One might hope to avoid this problem by doing an experiment at optical frequencies, where better polarimeters are available. This has been done. KOCHER and COMMINS [21] showed that linear-polarization correlation measurements on certain atomic cascades demonstrated the Einstein-Podolsky-Rosen « paradox ». CLAUSER *et al.* [16] related this directly to Bell's theorem and FREEDMAN and CLAUSER [22] carried out the optical experiment, getting results in agreement with quantum mechanics. However, as CLAUSER *et al.* point out, the efficiency of present-day detectors does not allow a direct violation of Bell's inequality when polarizing filters and optical photon detectors are used to measure polarization correlations. The assumption they must make to infer the response of ideal analyzers from the response of their real ones is that the probability of detection of a photon is independent of whether it has passed through a polarizer or reached the detector directly. It would seem that Nature would be very peculiar if this assumption were violated. However, the question raised

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[20] It has also been pointed out by J. F. CLAUSER: *Phys. Rev. A*, **6**, 49 (1972), that semi-classical radiation theory gives the same prediction as the Bohm-Aharonov hypothesis.

[21] C. A. KOCHER and E. D. COMMINS: *Phys. Rev. Lett.*, **18**, 575 (1967).

[22] S. J. FREEDMAN and J. F. CLAUSER: *Phys. Rev. Lett.*, **28**, 938 (1972).

by this assumption is not trivial. It has been pointed out by PEARL [23] that unless measurements are made without polarizers, and without making such an assumption, a hidden-variable theory could be constructed which reproduces the predictions of quantum mechanics for such experiments.

In our experiment we do not have to assume anything about the efficiency of our detectors for photons which do not scatter. However, we must make two different assumptions: first, that the Compton-scattering equation correctly relates the distribution of momenta of the scattered photons to the result that would have been obtained in an ideal experiment (see Appendix), and, second, that the probability of detecting a photon is independent of the direction in which either photon scatters. The first of these assumptions looks less intuitively axiomatic than the one assumption necessary for the Freedman and Clauser experiment. However, if it were not true, our experiment would be consistent with a local hidden-variable theory only if there were a large error in the predictions of quantum mechanics for Compton scattering (if nowhere else) that had not yet been noticed. We believe that the two experiments require different assumptions and complement each other as tests of local hidden-variable theories.

Another problem is: how far apart must the measuring instruments be to satisfy the locality postulate? This problem has been considered by McGUIRE and FRY [24], who show that it is difficult to construct even a nonlocal hidden-variable theory if this distance is much larger than the coherence length of the process that creates the photons. If the coherence length is taken as the length of the wave train produced during the mean life of the state whose decay produces the photon, this is about 7 cm for positrons annihilating in copper and more than 1 m for the photons in the cascade described by KOCHER and COMMINS [21] and used in the experiment of FREEDMAN and CLAUSER [22]. McGUIRE and FRY state that the source-detector distance in the Freedman and Clauser experiment is larger than the coherence length, but it would appear that this is not so if the polarization analyzer is included as part of the detector [23]. In our experiment the scatterers are distant from the source by about one coherence length, and the detectors from the scatterers by about another coherence length.

APPENDIX

The relation between ideal-polarimeter and Compton-polarimeter results.

Linear-polarization measurements can be made on a photon with either 1) an « ideal » polarization analyzer (in principle), or 2) a Compton polarimeter.

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[23] P. M. PEARL: *Phys. Rev. D*, **2**, 1418 (1970).
[24] J. H. McGUIRE and E. S. FRY: *Phys. Rev. D*, **7**, 555 (1972).

An ideal polarization analyzer is *defined* to produce a unique output, *viz.* $+1$ (-1), upon measuring a photon with linear polarization parallel (perpendicular) to the analyzer axis. In contrast, a Compton polarimeter does not give a unique output for any particular polarization state of the photons. Instead, such a polarimeter Compton-scatters the photon, and the polarization of the incoming photon determines the probability that the scattered photon will be found with various directions of momentum.

There exists a function Q relating ideal and Compton polarization measurements. The existence of Q for measurements on a single photon shall now be shown to follow from general principles of quantum mechanics, plus parity and angular-momentum conservation. This existence proof may be extended to measurements on systems of more than one photon and, if we use the Klein-Nishina formula, the explicit form of Q may be written for such systems. The details are given elsewhere ([7]).

A.1. *The existence of the relation.* – Consider a photon which Compton-scatters off an electron which is initially at rest. The initial state Ψ_1 of the electron-photon system is given by

$$\Psi_1 = |i\rangle[q|X\rangle + r|Y\rangle] = q|Xi\rangle + r|Yi\rangle \,, \tag{A.1}$$

where

$|i\rangle$ = an electron with zero linear momentum and spin state i,

$|X\rangle$, $|Y\rangle$ = a photon with momentum along the z-axis and linear polarization in the x, y-direction,

q, r = numbers, complex in general, normalized so that

$$qq^* + rr^* = 1 \,. \tag{A.2}$$

The final state Ψ_2 of the system is given by

$$\Psi_2 = |j\boldsymbol{k}\rangle \,, \tag{A.3}$$

where

$\boldsymbol{k}$ = the momentum of the scattered photon,

j = the polarizations of the recoil electron and scattered photon.

These are the variables which will be summed over to find the final result. Also let

E, θ, Φ = the energy, polar scattering angle and azimuthal scattering angle of the scattered photon.

The probability $\mathrm{d}F_{\boldsymbol{k}}$ for finding a scattered photon with momentum $\boldsymbol{k}$ is given by

$$\mathrm{d}F_{\boldsymbol{k}}(qr) = \varrho(E)\tfrac{1}{2}\sum_i\sum_j |\langle\Psi_2|S|\Psi_1\rangle|^2\,\mathrm{d}E\,\mathrm{d}\Phi\,\mathrm{d}\theta \,, \tag{A.4}$$

where

$\varrho(E)$ = the density of final states,

S = the scattering matrix,

$\frac{1}{2}\sum_i$ = the average over initial electron spin states.

Substituting for Ψ_1 and Ψ_2 from eqs. (A.1) and (A.3) we obtain

$$\text{(A.5)} \qquad \mathrm{d}F_{\boldsymbol{k}}(qr) = \varrho(E)\tfrac{1}{2}\sum_{ij}|\langle j\boldsymbol{k}|S|(q|X\rangle + r|Y\rangle)|i\rangle|^2\,\mathrm{d}E\,\mathrm{d}\Phi\,.$$

The differential $\mathrm{d}\theta$ does not appear in eq. (A.5) because energy and momentum conservation relates θ to E. Equation (A.5) may be expanded as

$$\text{(A.6)} \qquad \mathrm{d}F_{\boldsymbol{k}}(qr) = \tfrac{1}{2}\varrho(E)\sum_{ij}\{qq^*|\langle j\boldsymbol{k}|S|Xi\rangle|^2 + rr^*|\langle j\boldsymbol{k}|S|Yi\rangle|^2 + \\ + qr^*\langle j\boldsymbol{k}|S|Xi\rangle\langle j\boldsymbol{k}|S|Yi\rangle^* + rq^*\langle j\boldsymbol{k}|S|Yi\rangle\langle j\boldsymbol{k}|S|Xi\rangle^*\}\,\mathrm{d}E\,\mathrm{d}\Phi\,,$$

which is of the form

$$\text{(A.7)} \qquad \mathrm{d}F_{\boldsymbol{k}} = [\alpha|q|^2 + \beta|r|^2 + 2\,\mathrm{Re}\,(\gamma qr^*)]\,\mathrm{d}E\,\mathrm{d}\Phi\,,$$

with α and β real (and positive). γ would be complex in general. However, we shall now show that, because of conservation of parity and angular momentum, γ is real.

Now, the electromagnetic interaction is invariant under rotation and parity transformation. Therefore (since the electrons are not polarized) the scattering probability must be the same for right- and left-hand circularly polarized photons. Since

$q, r = 1,\ i$ for right circular polarization and

$q, r = 1,\ -i$ for left circular polarization,

we have

$$\text{(A.8)} \qquad \mathrm{d}F_{\boldsymbol{k}}(1, i) = \mathrm{d}F_{\boldsymbol{k}}(1, -i)\,.$$

Substituting eq. (A.8) into eq. (A.7) yields

$$\alpha + \beta + 2\,\mathrm{Re}\,(\gamma i) = \alpha + \beta + 2\,\mathrm{Re}\,(\gamma[-i])$$

or

$$2\,\mathrm{Re}\,(i\gamma) = 0\,.$$

Hence γ is real and may be taken outside of the « Re » in eq. (A.7) to yield

$$\text{(A.9)} \qquad \mathrm{d}F_{\boldsymbol{k}} = [\alpha|q|^2 + \beta|r|^2 + 2\gamma\,\mathrm{Re}\,(qr^*)]\,\mathrm{d}E\,\mathrm{d}\Phi$$

or

$$\frac{\mathrm{d}F_k}{\mathrm{d}E\,\mathrm{d}\Phi} = [q^* r^*]\begin{bmatrix}\alpha & \gamma\\ \gamma & \beta\end{bmatrix}\begin{bmatrix}q\\ r\end{bmatrix}. \tag{A.10}$$

Now rotate the x and y axes along which polarization is measured through an angle ξ about the z-axis. Call the new axes x' and y'. The quantities q' and r' are related to q and r by

$$\begin{bmatrix}q'\\ r'\end{bmatrix} = \begin{bmatrix}\cos\xi & -\sin\xi\\ \sin\xi & \cos\xi\end{bmatrix}\begin{bmatrix}q\\ r\end{bmatrix}.$$

Since the matrix in eq. (A.10) is real, it follows that there exists a ξ such that $\mathrm{d}F_k$ is *diagonal* in q' and r'. Calling the elements of the diagonal matrix α' and β', we have

$$\mathrm{d}F_k(qr) = \left(\alpha'|q'|^2 + \beta'|r'|^2\right)\mathrm{d}E\,\mathrm{d}\Phi \equiv \mathrm{d}F'_k(q'r'). \tag{A.11}$$

Note that we have introduced and defined the quantity $\mathrm{d}F'_k(q'r')$.

Since $\mathrm{d}F'_k$ depends only on $|q'|^2$ and $|r'|^2$, $\mathrm{d}F'_k$ can be related to measurements made with the ideal analyzer defined above. An « ideal analyzer » gives an output $L' = +1$ (-1) for photons polarized along (or perpendicular to) the analyzer axis. Now let the ideal-analyzer axis be oriented parallel to the x'-axis, which was defined just under eq. (A.10). Then clearly the mean value of L' for a photon with polarization components q' and r' is

$$\bar{L}'(q'r) = |q'|^2(+1) + |r'|^2(-1). \tag{A.12}$$

But eq. (A.11) may be written

$$\mathrm{d}F'_k(q'r') = \tfrac{1}{2}\left[(\alpha'+\beta')\left(|q'|^2 + |r'|^2\right) + (\alpha'-\beta')\left(|q'|^2 - |r'|^2\right)\right]\mathrm{d}E\,\mathrm{d}\Phi.$$

If we use eq. (A.12) this becomes

$$\mathrm{d}F'_k(q'r') = \tfrac{1}{2}[\alpha'+\beta'+(\alpha'-\beta')\bar{L}']\,\mathrm{d}E\,\mathrm{d}\Phi. \tag{A.13}$$

This proves what we set out to show that there exists a function relating F'_k (the probability of a photon Compton scattering in the direction $\boldsymbol{k}$) and $\bar{L}'$ (the average output of an ideal analyzer oriented along the x'-axis.)

The result of extending this existence proof to measurements made with two polarization analyses on two photons is given by [7]

$$\mathrm{d}F(\boldsymbol{k}'_1\boldsymbol{k}'_2) = f(E_1)f(E_2)[1 + m(E_1)m(E_2)P(\Phi_1\Phi_2)]\,\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\Phi_1\,\mathrm{d}\Phi_2/4\pi^2, \tag{A.14}$$

where

$\mathrm{d}F(\boldsymbol{k}'_1\boldsymbol{k}'_2)$ = the probability of finding the photons scattered in directions $\boldsymbol{k}'_1$ and $\boldsymbol{k}'_2$,

$P(\Phi_1\Phi_2) = \overline{L_1' L_2'}$ with L_1' and L_2' the outputs of the two analyzers,

$f(E) \quad = \dfrac{r_0^2}{2} \dfrac{m_e C^2}{E_0^2} \chi(E_0 E)$,

$m(E) \quad = -\sin^2\theta / \chi(E_0 E)$,

E_0 = the energy of the incident photon,

E = the energy of the scattered photon,

Φ = the angle between the incident-photon polarization and the scattering plane,

r_0 = the classical electron radius ($\sim 2.82 \cdot 10^{-13}$ cm),

$m_e C^2$ = the electron rest mass energy,

θ = the scattering angle related to E by

$$m_e C^2 (1/E + 1/E_0) = 1 - \cos\theta,$$

$\chi(E_0 E) \quad = E_0/E + E/E_0 - \sin^2\theta$.

For positron annihilation, $P(\Phi_1\Phi_2) = \cos 2(\Phi_2 - \Phi_1)$; when this is substituted into eq. (A.14), eq. (2) of the text follows (7).

Note added in proofs.

Since this paper was submitted, a report of another measurement of the same angular correlation has appeared in this journal (G. FARACI, D. GUTKOWSKI, S. NOTARRIGO and A. R. PENNISI: *Lett. Nuovo Cimento*, **9**, 607 (1974)). We would like to add an addendum to our paper on that work here.

It is stated in the paper of FARACI *et al.* that their results agree with our results, as reported in Varenna in 1970 (ref. (6)). This is not correct. Our results are in formal disagreement with theirs. The preliminary report of 1970 may not have explicitly stated that our experiment is different in a very important way from most other measurements of this correlation. The scattering angle θ is *not* defined by the size of the counter, requiring averaging over a wide range of angles. Instead, it is measured for both scatterings and, for each event, by recording the energy of the scattered photons. Therefore, our average over all scattering angles, which FARACI *et al.* plot on their Fig. 2 as a measurement taken at a scattering angle of 90°, should *not* be interpreted as a correlation at any particular angle θ. The values of θ_1 and θ_2 for each event had actually been measured, but results were summed together for all θ_1 and θ_2 only to better test the φ-dependence of the results. A proper measurement of the correlation *at a particular scattering angle* θ would be our data for particular energy regions, as shown in our Fig. 8 and 9. In region 1, corresponding to a scattering angle region centered on $\theta = 82°$, we have $\beta = 0.400 \pm 0.018$, corresponding in the notation of Fig. 2 of FARACI *et al.* to $R = 2.33 \pm 0.10$, *not* the value of approximately 2.05 at 90° which is shown. This result is nearly three standard deviations above their value as plotted at about the same angle, *without applying any geometrical corrections.* When geometrical corrections are included for both results, the discrepancy becomes greater.

In essence: With geometrical corrections, our results agree with the predictions of quantum mechanics. Without corrections, they still significantly disagree with the results of FARACI *et al.* One possible reason for this disagreement might be the effect of the length of the flight path of the annihilation photons, as discussed in both of our papers. However, the range of flight paths investigated by FARACI *et al.* included the values used in our work, so a disagreement remains. There appear to be two possible reasons for this:

A) because of some error, our correlation is too large;

B) because of some error, the correlation of FARACI *et al.* is too small.

If we assume *A*) is the case, there are very few possible explanations, since most experimental effects tend to *weaken* a correlation. One might suppose that perhaps there was some effect, such as misalignment of the apparatus or variations of counting rate from run to run due to some random displacements of the source, which happened to increase the counting rate near $\varphi = 90°$ or decrease it near $\varphi = 0°$. We were aware of this possibility during our experiment and took precautions to avoid it. With the normalization described in Subsect. **2**·1 of our paper, any effect influencing the annihilation rate or the rate at which photons struck the two scatterers would be removed from our results.

There are several reasons why *B*) could be the case. We will here discuss only the two that appear to us to be the most important. In our experiment the largest correction after that for geometrical effects was the correction for multiple scattering in the scatterer (see Subsect. **2**·3.3 of our paper), although our apparatus was designed to minimize this effect. FARACI *et al.* do not discuss this correction. We are not able to estimate how large it would be without more detailed information than is given in the brief paper of FARACI *et al.* If it was not considered in designing the apparatus, it could significantly weaken their correlation.

Furthermore, FARACI *et al.* do not mention any check on the total energy left in the detectors in their fourfold coincidence events. If in fact there was no sum energy requirement, events in which a photon has suffered an additional Compton scattering between scatterer and detector will be counted. These will also weaken the measured correlation. The number of these events would tend to increase as the distance between scatterer and detector was increased. This could explain the weakening of the correlation with increased scatterer-detector distance shown in Fig. 4 of FARACI *et al.*

● RIASSUNTO (*)

Si è misurata per mezzo dello scattering di Compton la polarizzazione lineare relativa dei fotoni provenienti dall'annientamento in due quanti di positoni nel rame. Si sono eseguite misure della distribuzione angolare di fotoni che arrivano in coincidenza dopo aver subito lo scattering di Compton in corrispondenza di un esteso intervallo di angoli di scattering, sia polari che azimutali. I risultati concordano con misure standard della meccanica quantistica supponendo che elettrone e positone abbiano parità opposte. Questo risultato ha dei risvolti che riguardano le teorie delle variabili nascoste della meccanica quantistica. Un teorema di Bell restringe il campo dei valori che una teoria locale delle variabili nascoste può prevedere per certe relazioni fra misure eseguite su sistemi correlati come le coppie di fotoni provenienti dall'annientamento di positoni. Se si suppongono corrette le abituali formule della meccanica quantistica per lo scattering di Compton, si mostra che le distribuzioni che si sono osservate non possono dare risultati compatibili col teorema di Bell se si misurano i fotoni con analizzatori ideali di polarizzazione. I nostri risultati quindi forniscono argomenti contro le teorie delle variabili nascoste.

(*) *Traduzione a cura della Redazione.*

Угловая корреляция аннигиляционных фотонов, испытавших комптоновское рассеяние, и скрытые переменные.

Резюме (*). — Используя комптоновское рассеяние, измеряется относительная линейная поляризация фотонов, образованных в результате двух-фотонной аннигиляции позитронов в меди. Измерения углового распределения фотонов, испытавших комптоновские рассеяния, в схеме совпадений проводятся в широком интервале углов рассеяния, полярного и азимутального. Полученные результаты согласуются с обычными квантовомеханическими вычислениями, предполагая противоположную четность электрона и позитрона. Этот результат имеет следствия для теории скрытых переменных в квантовой механике. Теорема Белла ограничивает значения, которые любая локальная теория скрытых переменных может предсказать для некоторых соотношений между измерениями, произведенными с коррелированными системами, такими как фотонная пара, образованная при аннигиляции позитрона. Показывается, что наблюденные нами распределения не могут дать результатов, которые соответствовали бы теореме Белла, если фотоны регистрируются с помощью идеальных поляризационных анализаторов, предполагая правильность обычных формул квантовой механики для комптоновского рассеяния. Таким образом, наши результаты свидетельствуют против локальных теорий скрытых переменных.

(*) *Переведено редакцией.*