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Note on Polarization Effects in Compton Scattering

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A derivation of the probability of Compton scattering for an arbitrary initial and final photon polarization is sketched. The result is applied to the case of two successive Compton scatterings, and some possibilities of experimental verification are discussed. A remark is made on a simplification of the calculations of Compton scattering when it is used to analyze the polarization of annihilation quanta.

THE aim of this note is to treat the Compton effect for polarized light and make two applications of the result. Although Nishina has already dealt with this problem[1] it is convenient to sketch in Section I another derivation which puts the result in a form more suitable for the present applications.[2] In Section II the result of Section I is used to obtain the azimuthal variation in intensity of initially unpolarized light which has undergone two Compton scatterings. The result has already been obtained by Nishina for the case of right angle scattering,[3] but to get the maximum variation intensity, as is desirable experimentally, one must go to angles other than ninety degrees. In Section III the result of Section I is used to show how a suitable choice of coordinates permits one to deduce directly from the Klein-Nishina formula the result of Pryce and Ward[4] for the coincidence rate in an experiment on the polarization of annihilation quanta.

I. DERIVATION OF THE TRANSFORMATION MATRIX

We wish to calculate the differential cross section for scattering of a photon of momentum $\mathbf{k}_0$ with an arbitrary initial polarization state to a final momentum $\mathbf{k}$ and an arbitrary final polarization state. To specify the initial and final polarization states we use statistical matrices U and W respectively.[5] U and W have two rows and columns labeled by two independent polarization states. The differential cross section is of the form

$$\sum_{lmst} U_{lm} S_{lm,\,st}(\mathbf{k}_0, \mathbf{k}) W_{st}. \tag{1}$$

To identify the matrix elements of S, write Eq. (18) p. 149 of QTR for the scattering of an initial photon specified by the polarization vector $\sum_{l=1,2} c_l \boldsymbol{\varepsilon}_l$ to a final state specified by the polarization vector $\sum_{s=1,2} d_s \boldsymbol{\zeta}_s$

$$\frac{d\sigma}{d\Omega} = e^4\left(\frac{k}{k_0}\right)^2 \frac{E}{\mu} \sum_u Av_{u_0} \times \left| \sum_{u'u''} \frac{(u_0, \sum c_l \boldsymbol{\varepsilon}_l\cdot\boldsymbol{\alpha} u')(u', \sum d_s^*\boldsymbol{\zeta}_s^*\cdot\boldsymbol{\alpha} u)}{\mu+k_0-E'} + \frac{(u_0, \sum d_s^*\boldsymbol{\zeta}_s^*\cdot\boldsymbol{\alpha} u'')(u'', \sum c_l\boldsymbol{\varepsilon}_l\cdot\boldsymbol{\alpha} u)}{\mu-k-E''}\right|^2. \tag{2}$$

(Equation (18) has been averaged over the initial and summed over the final spin states of the electron, the possibility of complex polarization vectors is not excluded.) This expression for the differential scattering cross sections may be rewritten in the form

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{k}{k_0}\right)^2 \sum_{lmst} c_m^* c_l d_t^* d_s S_{lm,\,st}, \tag{3}$$

where the matrix S is defined by the equation

$$S_{lm,\,st} = \frac{4\mu}{E}\sum_u Av_{u_0} \times\left(\sum_{u'u''}\frac{(u_0, \boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}_m u')(u', \boldsymbol{\alpha}\cdot\boldsymbol{\zeta}_s^* u)}{\mu+k_0-E'} + \frac{(u_0, \boldsymbol{\alpha}\cdot\boldsymbol{\zeta}_s^* u'')(u'', \boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}_m u)}{\mu-k-E''}\right)^* \times\left(\sum_{u'u''}\frac{(u_0, \boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}_l u')(u', \boldsymbol{\alpha}\cdot\boldsymbol{\zeta}_t^* u)}{\mu+k_0-E'} + \frac{(u_0, \boldsymbol{\alpha}\cdot\boldsymbol{\zeta}_t^* u'')(u'', \boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}_l u)}{\mu-k-E''}\right), \tag{4}$$

[1] Y. Nishina, Zeits. f. Physik. **52**, 869 (1929).
[2] The present derivation runs parallel to that of W. Heitler *Quantum Theory of Radiation* (Oxford University Press, London, 1936), pp. 146–60 for the Klein-Nishina formula. Heitler's book will be referred to hereafter as QTR.
[3] Reference (1) page 876.
[4] M. H. Pryce and J. C. Ward, Nature **160**, 435 (1947).
[5] For the use of a statistical matrix to describe a state see, for example, Kemble *Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), p. 434.

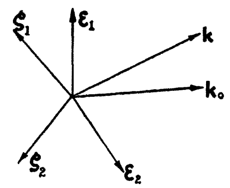

FIG. 1. Polarization vectors $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ for incoming photon $\mathbf{k}_0$, and $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ for scattered photon $\mathbf{k}$. $\boldsymbol{\zeta}_1$, $\boldsymbol{\varepsilon}_1$ and $\mathbf{k}$ lie in a plane perpendicular to the plane which contains $\boldsymbol{\zeta}_2$, $\boldsymbol{\varepsilon}_2$ and $\mathbf{k}_0$.

The case considered here is that in which the initial and final states are pure states. They have the statistical matrices

$$U_{lm}=c_m^*c_l,\quad W_{st}=d_t^*d_s,$$

respectively. U and W are normalized to unit trace:

$$\textstyle\sum_m U_{mm}=1,\quad \sum_s W_{ss}=1.$$

In the more general case of mixed states it will be impossible to decompose the elements of the statistical matrices U and W into simple products. However, the principle of superposition insures that the matrix S itself will remain unaltered in this situation.

The evaluation of $S_{lm,\,st}$ can be carried out by the methods described in QTR. The details will not be reproduced here. The result is

$$S_{lm,\,st}=\boldsymbol{\varepsilon}_l\cdot\boldsymbol{\zeta}_t^*\boldsymbol{\varepsilon}_m^*\cdot\boldsymbol{\zeta}_s\left[3+\frac{E}{\mu}+\frac{k-k_0}{\mu}\frac{\mathbf{k}_0\cdot\mathbf{k}}{k_0k}\right]+\left[\boldsymbol{\varepsilon}_l\cdot\boldsymbol{\varepsilon}_m^*\,\boldsymbol{\zeta}_s\cdot\boldsymbol{\zeta}_t^*-\boldsymbol{\varepsilon}_l\cdot\boldsymbol{\zeta}_s\,\boldsymbol{\varepsilon}_m^*\boldsymbol{\zeta}_t^*\right]\times\frac{E-\mu}{\mu}\left(1-\frac{\mathbf{k}_0\cdot\mathbf{k}}{k_0k}\right).\quad(5)$$

A useful basis for the polarization states of initial and scattered photon is that of Fig. 1. The unit vectors are chosen so that

$$\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_2=\boldsymbol{\zeta}_1\cdot\boldsymbol{\zeta}_2=0,\quad \boldsymbol{\varepsilon}_1\times\boldsymbol{\varepsilon}_2=\frac{\mathbf{k}_0}{k_0},\quad \boldsymbol{\zeta}_1\times\boldsymbol{\zeta}_2=\frac{\mathbf{k}}{k},$$

and so that $\boldsymbol{\zeta}_1$ lies in the plane of $\boldsymbol{\varepsilon}_1$ and $\mathbf{k}$, while $\boldsymbol{\zeta}_2$ lies in the plane of $\mathbf{k}_0$ and $\boldsymbol{\varepsilon}_2$. As a result $\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_2=0$, but $\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1$ will be zero only when $\mathbf{k}$, $\mathbf{k}_0$ and $\boldsymbol{\varepsilon}_1$ lie in the same plane. If the symbol $\|$ be used as equivalent to the subscript 1 and the symbol $\perp$ as equivalent to the subscript 2, then S takes the following form

$$S_{lm,\,st}=\left\{\begin{array}{c|cccc} & \|\,\| & \perp\,\perp & \|\,\perp & \perp\,\| \\ \|\,\| & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+4(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)^2 & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2 & 0 & 0 \\ \perp\,\perp & \dfrac{k}{k_0}+\dfrac{k}{k}-2+4(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1)^2 & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+4(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)^2 & 4(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) & 4(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) \\ \|\,\perp & 4(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) & 0 & -(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)\times\left(\dfrac{k}{k_0}+\dfrac{k_0}{k}-2\right) & (\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)\times\left(3+\dfrac{E}{\mu}+\dfrac{k-k_0}{\mu}\dfrac{\mathbf{k}_0\cdot\mathbf{k}}{k_0k}\right) \\ \perp\,\| & 4(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) & 0 & (\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)\times\left(3+\dfrac{E}{\mu}+\dfrac{k-k_0}{\mu}\dfrac{\mathbf{k}_0\cdot\mathbf{k}}{kk_0}\right) & -(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)\times\left(\dfrac{k}{k_0}+\dfrac{k_0}{k}-2\right) \end{array}\right\}.\quad(6)$$

It will be noted that the four matrix elements in the upper lefthand corner are just four cases of the Klein-Nishina formula, apart from a proportionality factor.

When the incident photon is unpolarized the scattering probability is determined by the sum

$$S_{,\,st}=\tfrac{1}{2}\textstyle\sum_m S_{mm,\,st}.$$

When the polarization of the scattered photon is to be averaged over, the scattering probability is determined by the sum

$$S_{lm,}=\tfrac{1}{2}\sum_{s} S_{lm,ss}.$$

The two sums just defined have the form

$$S_{,st}=\begin{array}{c|cc} & \parallel & \perp \\ \hline \parallel & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+2[(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1)^2+(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)^2] & 2(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) \\ \perp & 2(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+2(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)^2 \end{array},\tag{7}$$

and

$$S_{lm,}=\begin{array}{c|cc} \parallel & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+2(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)^2 & 2(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) \\ \perp & 2(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1) & \dfrac{k}{k_0}+\dfrac{k_0}{k}-2+2[(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)^2+(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1)^2] \end{array}.\tag{8}$$

When axes are chosen so that $\mathbf{k}_0$, $\mathbf{k}$ and $\boldsymbol{\varepsilon}_1$ lie in the same plane, $\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1=0$. Then all three matrices (6), (7), and (8) take on an especially simple form. In particular $S_{,st}$ and $S_{lm,}$ are diagonal. This circumstance finds application in Section III. It could be proved directly from the symmetry of the problem under reflection in the plane of $\mathbf{k}_0$, $\mathbf{k}$ and $\boldsymbol{\varepsilon}_1$.

II. THEORY OF DOUBLE COMPTON SCATTERING

In the process to be considered, a photon of momentum $\mathbf{k}_0$ undergoes Compton scattering; the resultant scattered photon of momentum $\mathbf{k}_1$ is again scattered to produce a final scattered photon of momentum $\mathbf{k}_2$, which is counted. The relative number of counts is to be determined as a function of the azimuth of $\mathbf{k}_2$ around the axis of $\mathbf{k}_1$.

If the statistical matrix of polarization of $\mathbf{k}_0$ is $U_{ij}{}^{(0)}$ before the first scattering, the statistical matrix of $\mathbf{k}_1$ will be proportional to

$$U_{kl}{}^{(1)}=\sum_{ij} U_{ij}{}^{(0)}S^{(1)}{}_{ij,kl}.\tag{9}$$

The second scattering will produce a statistical matrix for $\mathbf{k}_2$ proportional to

$$\sum_{kl} U_{kl}{}^{(1)}S^{(2)}{}_{kl,mn},$$

(see Fig. 2).

We consider the case that the original photons have random polarization. Thus the original statistical matrix $U^{(0)}$ is one-half the unit matrix. We take $\boldsymbol{\varepsilon}_1$ in the plane of $\mathbf{k}_0$ and $\mathbf{k}_1$. The matrix S takes a very simple form, for in this coordinate system,

$$1-(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1)^2-(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\zeta}_1)^2=\sin^2\theta_1\cos^2\phi_1=\sin^2\theta_1,$$

$$1-(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_2)^2=1-\frac{\cos^2\theta_1}{1-\sin^2\theta_1\cos^2\phi_1}=0,$$

$$\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\zeta}_1=0,$$

so that we have from section one,

$$\tfrac{1}{2}\sum_{iklm} S^{(1)}{}_{ii,kl}S^{(2)}{}_{kl,mm}=4[\gamma_{01}\gamma_{12}-\gamma_{01}\sin^2\theta_2-\gamma_{12}\sin^2\theta_1+2\sin^2\theta_1\sin^2\theta_2\cos^2\phi_2],$$

where

$$\gamma_{01}=\frac{k_1}{k_0}+\frac{k_0}{k_1},\quad\text{and}\quad \gamma_{12}=\frac{k_2}{k_1}+\frac{k_1}{k_2}.$$

For scatterers consisting of single electrons separated by the distance R, the number of photons $\mathbf{k}_2$ going into the solid angle $d\Omega_2$ per

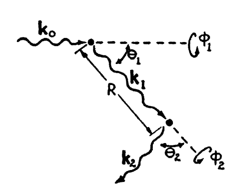

FIG. 2. Momentum vectors for the two successive Compton scatterings.

TABLE I: Optimum ratio of maximum to minimum number of counts for photons initially unpolarized but twice Compton scattered. The azimuth of the second counter is varied while the angles of scattering θ_1 and θ_2 are held fixed at values chosen to give the largest N max/N min.

K_0(Mev)	θ_1	θ_2	(N max/N min) max
0.5	$\sim 83°$	$\sim 85°$	3.85
1.0	$\sim 80°$	$\sim 83°$	2.14
5.0	$\sim 73°$	$\sim 80°$	1.17

second is[6]

$$dN=\frac{I}{16}\left(\frac{e^2}{mc^2}\right)^4\left(\frac{1}{4\pi R^2}\right)\left(\frac{k_1}{k_0}\right)^2\left(\frac{k_2}{k_1}\right)^2 \times 4[\gamma_{01}\gamma_{12}-\gamma_{01}\sin^2\theta_2-\gamma_{12}\sin^2\theta_1 +2\sin^2\theta_1\sin^2\theta_2\cos^2\phi_2]d\Omega_2, \quad (10)$$

where I is the current of incident photons (number per cm² per sec.). For fixed angles of scattering, θ_1 and θ_2, N varies from a maximum at $\phi_2=0$ (then $\mathbf{k}_2$ is in the plane of $\mathbf{k}_0$ and $\mathbf{k}_1$) to a minimum at $\phi_2=\pi/2$. The ratio N max/N min is

$$\frac{\gamma_{01}\gamma_{12}-\gamma_{01}\sin^2\theta_2-\gamma_{12}\sin^2\theta_1+2\sin^2\theta_1\sin^2\theta_2}{\gamma_{01}\gamma_{12}-\gamma_{01}\sin^2\theta_2-\gamma_{12}\sin^2\theta_1}. \quad (11)$$

The maximum of this ratio as a function of θ_1 and θ_2 is a monotonically decreasing function of energy which is only about 1.04 at $k_0=17$ Mev. Although an experiment to test (10) at 17 Mev appears uninteresting because of the smallness of the effect, the data of Table I have been computed in the hope that a test may be possible at lower energies.

Unfortunately, the small absolute number of counts is expected to make the experimental test quite difficult e.g., at $k_0=1$ Mev with $R=10$ cm and scatterers 1 cm on a side and containing 10^{24} electrons, only ~ 1 in 10^7 of the photons striking the first scatterer will be twice scattered into unit solid angle about the direction which gives the maximum number of counts. Possibly, some geometrical arrangement or focussing device could be used to increase the absolute intensity.

[6] This result can be obtained directly from the Klein-Nishina formula, once it is known that after the first scattering photons polarized parallel and perpendicular to the plane of scattering are incoherent.

III. REMARKS ON AN EXPERIMENT ON THE POLARIZATION OF ANNIHILATION QUANTA

The use of Compton scattering as an analyser of the polarization of annihilation photons was originally suggested by Wheeler[7] and has been investigated by numerous physicists.[8] Here we only wish to show under what conditions it is possible to neglect interference between the probability amplitudes of the photons.[9]

In the suggested experiment slow positons are allowed to annihilate, acting as a source of pairs of annihilation quanta traveling in (nearly) opposite directions. (See Fig. 3.) After having gone through a scattering block, each photon (or a negaton which it has knocked on) is given a chance to trip a counter. Coincidences are measured as a function of relative azimuth $\phi_1-\phi_2$.

The following simple argument shows that the interference of the photon amplitudes can be quite essential. For, neglecting interference we might reason (incorrectly) as follows: Calculate the probabilities that photons (1) and (2) are right and right, left and left, right and left, and left and left circularly polarized respectively. Then, multiply each of these probabilities by the probability that, given that state, a coincidence should result when the counters are at a given relative azimuth $\phi_1-\phi_2$. Then, sum over the four states. Now, for circularly polarized photons the probability of Compton scattering is independent of azimuth. Consequently the sum (which is proportional to the number of coincidences) should be independent of azimuth. This result is known to be wrong.[4]

A virtue of the general formalism developed in Section I is that one can see when the neglect of interference is justified.[7] The probability of a coincidence is proportional to

$$w=\sum_{1,1',2,2',3=3',4=4'}U_{12,1'2'}S^{(1)}{}_{11',33'}S^{(2)}{}_{22',44'},$$

[7] J. A. Wheeler, Annals N. Y. Acad. Sci. **48**, 219 (1946).

[8] See reference 4. Snyder, Pasternack and Hornbostel, Phys Rev. **73**, 440 (1948). E. Bleuler and D. Ter Haar, Science **108**, 10 (1948). R. P. Feynman, unpublished. I am indebted to Professor Feynman for the opportunity to read his elegant treatment of two quantum annihilation. His method suggested the use of the statistical matrix in the present work.

[9] In paragraph 2 of their paper, Snyder, Hornbostel and Pasternack give a very simple justification of the neglect of interference for the particular bases of polarization vectors below.

where $U_{12,1'2'}$ is the statistical matrix of photons (1) and (2), and $S^{(1)}{}_{11',33'}$ is the transformation matrix from the statistical matrix of (1) to (3), $S^{(2)}{}_{22',44'}$ the transformation matrix from the statistical matrix of (2) to that of (4).

For simplicity, consider the case in which the position and negaton annihilate at zero velocity. Then to conserve angular momentum the resultant photons must be in a state of zero angular momentum, for the following reason: From the polarization states of the two photons one can make up four polarization states of the combined system of which three have angular momentum 2 and the fourth angular momentum 0. Now a negaton and positon at rest have at most total angular momentum 1. Consequently, the resultant annihilation photons must be a pure state of angular momentum 0. (In passing we note that this is in agreement with the standard result that annihilation is forbidden in first approximation for a slow negaton-positon pair in a triplet spin state.) As a result the matrix U takes a simple form. Using the basis for polarization vectors of figure one, we find

$$U=\tfrac{1}{2}\begin{array}{c} \\ \parallel\parallel \\ \perp\perp \\ \parallel\perp \\ \perp\parallel \end{array}\begin{array}{cccc} \parallel\parallel & \perp\perp & \parallel\perp & \perp\parallel \\ \end{array}\left\{\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{array}\right\}.$$

Thus

$$\begin{aligned} w=4\{&U_{\parallel\perp,\parallel\perp}S_{\parallel\parallel,}{}^{(1)}S_{\perp\perp,}{}^{(2)}+U_{\perp\parallel,\perp\parallel}S_{\perp\perp,}{}^{(1)}S_{\parallel\parallel,}{}^{(2)} \\ &+U_{\parallel\perp,\perp\parallel}S_{\parallel\perp,}{}^{(1)}S_{\perp\parallel,}{}^{(2)}+U_{\perp\parallel,\parallel\perp}S_{\perp\parallel,}{}^{(1)}S_{\parallel\perp,}{}^{(2)}\} \\ =2(&S_{\parallel\parallel,}{}^{(1)}S_{\perp\perp,}{}^{(2)}+S_{\perp\perp,}{}^{(1)}S_{\parallel\parallel,}{}^{(2)} \\ &-S_{\parallel\perp,}{}^{(1)}S_{\perp\parallel,}{}^{(2)}-S_{\perp\parallel,}{}^{(1)}S_{\parallel\perp,}{}^{(2)}). \end{aligned}$$

The coincidence rate is given by the expression:

$$\begin{aligned} &NN_1t_1N_2t_2wd\Omega_1d\Omega_2 \\ &=\frac{1}{4}\left(\frac{e^2}{mc^2}\right)^4NN_1t_1N_2t_2\left(\frac{k}{k_0}\right)_1^2\left(\frac{k}{k_0}\right)_2^2 \\ &\quad\times[\gamma_1\gamma_2-2\gamma_1\sin^2\theta_2-2\gamma_2\sin^2\theta_1 \\ &\qquad+2\sin^2\theta_1\sin^2\theta_2\sin^2\phi_1-\phi_2]d\Omega_1d\Omega_2, \end{aligned}$$

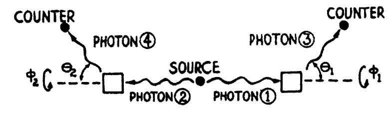

FIG. 3. Geometry of experiment on the polarization of annihilation quanta.

where $N\equiv$number of annihilations per second which result in photons striking scatterers, N_i $\equiv$number of electrons per unit volume in scatterer i, $t_i\equiv$thickness of scatterer i, $d\Omega_i\equiv$solid angle of counter at i,

$$\gamma_i\equiv\left(\frac{k}{k_0}+\frac{k_0}{k}\right)_i,$$

Moreover, when the coordinate system is chosen so that one of the counters is in the plane determined by $\boldsymbol{\varepsilon}_1$ and the momentum of an annihilation quantum, then the off-diagonal elements of either $S^{(1)}$ or $S^{(2)}$ vanish so that

$$w=2(S_{\parallel\parallel,}{}^{(1)}S_{\perp\perp,}{}^{(2)}+S_{\perp\perp,}{}^{(1)}S_{\parallel\parallel,}{}^{(2)}).$$

Thus, the coincidence rate can be written down immediately from the diagonal elements of $S_{qr,}$ or, equivalently, from the Klein-Nishina formula. (This simplification also extends to the case in which the negaton-position pair annihilates with velocity greater than zero, provided the annihilation is considered in the frame in which the pair has zero total momentum.[10] In the general case of arbitrary $U_{ij,kl}$ and arbitrary basis for the polarization vectors one can determine from Eq. (12), whether the neglect of interference of the photon amplitudes is permissible in any given case.) Apart from effects of finite geometry, the expression agrees in its angular dependence with the result of the above authors.

I wish to thank Dr. A. Pais, Professor J. A. Wheeler, and Professor E. P. Wigner for helpful discussion.

[10] The U for this case has been calculated by Feynman in the unpublished manuscript mentioned in reference 4.