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Scattering of Longitudinally Polarized Fermions

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Cross sections for scattering of longitudinally polarized Dirac particles are calculated. The results are presented in terms of the ratio of cross sections ϕ_p/ϕ_a. (The cross section is denoted by ϕ_p when the spins of the particles before scattering are parallel, by ϕ_a when antiparallel.) In the case of scattering of indistinguishable particles ϕ_p/ϕ_a is different from unity at all energies and scattering angles and can be as small as zero. For particle-antiparticle scattering ϕ_p/ϕ_a is close to unity at nonrelativistic energies, but at relativistic energies approaches the value of the corresponding ratio for indistinguishable-particle scattering. Thus, in both these cases, the scattering process provides a method for measuring longitudinal polarization. Thirdly, ϕ_p/ϕ_a is calculated for two particles of different masses m and μ. It is shown that also in this case, in principle, ϕ_p/ϕ_a could be used for polarization measurements.

I. INTRODUCTION

ONE of the implications of parity nonconservation in weak interactions is that the emerging fermions will be longitudinally polarized. This polarization may be detected in several ways. Mott scattering by a Coulomb potential may serve as an analyzer but then it is first necessary to change the longitudinal polarization to a transverse one.[1] One can observe the longitudinal polarization directly by measuring the circular polarization of the bremsstrahlung photons produced by the fermion.[2] In the case of a positron one may similarly establish its longitudinal polarization by measuring the circular polarization of the annihilation photons.[3] In the present paper we wish to report calculations on still another method; namely, scattering of longitudinally polarized fermions by longitudinally polarized fermions.

In Sec. II we present the results of the calculation when the two fermions are indistinguishable. The fermions are referred to as electrons, but of course they could be any two indistinguishable fermions. In Sec. III we present the results when the two fermions are each other's antiparticles; they are referred to in this case as positron and electron. In Sec. IV we present the results for different fermions, called now μ meson and electron. Throughout, the only interaction assumed is the electromagnetic interaction and the results are based on lowest order perturbation theory.

II. ELECTRON—ELECTRON SCATTERING

The matrix element M for scattering from an initial state of two electrons with four-momenta p_1 and p_2 into a final state with four-momenta p_1' and p_2' is given by[4]

$$-\frac{M}{4\pi e^2}=\frac{(\bar{u}_{\epsilon_1'}(p_1')\gamma_\mu u_{\epsilon_1}(p_1))(\bar{u}_{\epsilon_2'}(p_2')\gamma_\mu u_{\epsilon_2}(p_2))}{k\cdot k}-\frac{(\bar{u}_{\epsilon_2'}(p_2')\gamma_\mu u_{\epsilon_1}(p_1))(\bar{u}_{\epsilon_1'}(p_1')\gamma_\mu u_{\epsilon_2}(p_2))}{l\cdot l};$$
$$k\equiv p_1-p_1',\quad l\equiv p_1-p_2'. \quad (1)$$

The first term in Eq. (1) is the so-called direct term, the second is the so-called exchange term. We are using a system of units in which $\hbar=c=1$. All repeated Greek indices are to be summed over from 1 to 4 (thus $k\cdot k=k_\mu k_\mu=\mathbf{k}\cdot\mathbf{k}+k_4k_4$); $\gamma=i\alpha\beta$ and $\gamma_4=\beta$, are the usual Dirac matrices; $u_\epsilon(p)$ and $\bar{u}_\epsilon(p)=iu_\epsilon^*(p)\gamma_4$ are the electron spinors (u^* is the Hermitian conjugate of u). The subscript ϵ indicates that we are dealing with an eigenstate of the operator $\sigma_p\equiv\boldsymbol{\sigma}\cdot\mathbf{p}/|\mathbf{p}|$ to the eigenvalue ϵ, i.e.,

$$\sigma_p u_\epsilon(p)=\epsilon u_\epsilon(p), \quad (2)$$

where ϵ takes on the values $+1$ or -1. We define forward (backward) longitudinal polarization as $\epsilon=+1$ (-1). (That is, $\epsilon=+1$ means spin and momentum parallel.)

We are interested in the differential scattering cross section, $\phi(\epsilon_1,\epsilon_2)$, independent of the polarizations of electrons p_1' and p_2' but for a given longitudinal polarization of electrons p_1 and p_2; i.e., we want

$$\phi(\epsilon_1,\epsilon_2)=(2\pi)^{-2}E_1^2E_2^2(E_1+E_2)^{-2}d\Omega\sum_{\epsilon_1'}\sum_{\epsilon_2'}|M|^2. \quad (3)$$

We may formally sum over the polarizations of electrons p_1 and p_2 as well, if in M we replace $u_{\epsilon_1}(p_1)$ and $u_{\epsilon_2}(p_2)$ according to the identities

$$u_{\epsilon_1}(p_1)=\tfrac{1}{2}\sum_\epsilon(1+\epsilon_1\sigma_{p_1})u_\epsilon(p_1),$$
$$u_{\epsilon_2}(p_2)=\tfrac{1}{2}\sum_\epsilon(1+\epsilon_2\sigma_{p_2})u_\epsilon(p_2). \quad (4)$$

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[1] N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929), and **A135**, 429 (1932). Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **106**, 386 (1957).

[2] K. McVoy, Phys. Rev. **106**, 828 (1957); Goldhaber, Grodzins, and Sunyar, Phys. Rev. **106**, 826 (1957).

[3] Lorne A. Page, Phys. Rev. **106**, 394 (1957).

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), third edition, p. 238.

Finally we may extend the summation to include negative-energy states by introducing into M appropriate projection operators. This may be done by the use of the relation[5]

$$\sum_\epsilon(\bar{u}_\epsilon(p)Qu_\rho(r)) = (2|p_4|)^{-1}\sum_{\mathrm{p}}(u_\epsilon^*(p)(\gamma\cdot p+im)Qu_\rho(r)), \quad (5)$$

where $\sum_{\mathrm{p}}$ denotes a sum over all four solutions corresponding to a given three-momentum $\mathbf{p}$. Using Eqs. (4) and (5) and the completeness theorem

$$\sum_{\mathrm{p}}u_\epsilon^*(p)u_\eta(p)=\delta_{\epsilon\eta}, \quad (6)$$

we obtain

$$\phi(\epsilon_1,\epsilon_2)=\frac{e^4E_1E_2d\Omega}{E_1'E_2'(E_1+E_2)^2} \times\left\{\frac{A}{(k\cdot k)^2}+\frac{B}{(l\cdot l)^2}-\frac{C+D}{k\cdot k\, l\cdot l}\right\}, \quad (7)$$

where

$$A=\tfrac{1}{4}\,\mathrm{Tr}[(p_1'+im)\gamma_\mu(1+\epsilon_1\sigma_{p1})(p_1+im)\gamma_\nu] \times\tfrac{1}{4}\,\mathrm{Tr}[(p_2'+im)\gamma_\mu(1+\epsilon_2\sigma_{p2})(p_2+im)\gamma_\nu], \quad (8)$$

$$C=\tfrac{1}{16}\,\mathrm{Tr}[(p_1'+im)\gamma_\mu(1+\epsilon_1\sigma_{p1})(p_1+im) \times\gamma_\nu(p_2'+im)\gamma_\mu(1+\epsilon_2\sigma_{p2})(p_2+im)\gamma_\nu], \quad (9)$$

and B and D are obtained, respectively, from A and C by interchanging p_1' and p_2'. The symbol p denotes $\gamma\cdot p$.

There are essentially two frames of reference in which the calculation is of interest; the center-of-mass frame and the frame in which one of the electrons is at rest (this being usually the laboratory frame). These two frames can be transformed into each other by a Lorentz transformation, the velocity of the transformation being along the line defined by the spins of the electrons.[6] We therefore calculate A, B, C, and D in terms of quantities that are invariant under such a Lorentz transformation. They are

$$\begin{aligned} a&\equiv p_1\cdot p_2=p_1'\cdot p_2',\\ b&\equiv p_1\cdot p_1'=p_2\cdot p_2',\\ c&\equiv p_1\cdot p_2'=p_2\cdot p_1',\\ d&\equiv(\mathbf{p}_1')_\perp\cdot(\mathbf{p}_2')_\perp, \end{aligned} \quad (10)$$

where $(\mathbf{p}')_\perp$ denotes the part of p' transverse to the line defined by the spins. The quantities a, b, and c are invariant under any Lorentz transformation and the equality of $p_1\cdot p_2$ and $p_1'\cdot p_2'$, etc., follows from the energy and momentum conservation laws.

Evaluation of the traces in Eqs. (8) and (9) yields

$$A=2\{a^2+2m^2(m^2+b)+c^2+\epsilon_1\epsilon_2[ad+(m^2+b)^2]\}, \quad (11)$$

$$C=-2\{a^2+m^2(a+b+c)+m^4 +\epsilon_1\epsilon_2[a^2+m^2(-a+b+c+d)+m^4]\}, \quad (12)$$

and therefore

$$B=2\{a^2+2m^2(m^2+c)+b^2+\epsilon_1\epsilon_2[ad+(m^2+c)^2]\}, \quad (11')$$

$$D=C. \quad (12')$$

We notice the absence of terms containing either ϵ_1 or ϵ_2 alone. This means that if one of the electrons, for instance p_2, were unpolarized then the polarization of p_1 could not be measured by this method. One could anticipate this result by the following argument: in order to obtain a term containing only ϵ_1 it must be possible to form a scalar containing $\mathbf{s}_1$, the spin vector of electron p_1, but not $\mathbf{s}_2$. Such a scalar would have to be of the form $\mathbf{s}_1\cdot(\mathbf{p}_1\times\mathbf{p}_1')$ and this vanishes for longitudinal polarization.

We use as a measure of polarization the ratio of the scattering cross section ϕ_p (spins initially parallel) to the cross section ϕ_a (spins initially antiparallel):

$$\phi_p\equiv\phi(\epsilon_1,\epsilon_2)\quad\text{if}\quad\epsilon_1=-\epsilon_2, \quad (13)$$

$$\phi_a\equiv\phi(\epsilon_1,\epsilon_2)\quad\text{if}\quad\epsilon_1=+\epsilon_2. \quad (14)$$

In the center-of-mass frame of reference this ratio may be conveniently expressed in terms of the angle θ between $\mathbf{p}_1$ and $\mathbf{p}_1'$, and the velocity β of any one of the electrons. One has

$$\begin{aligned} a&=-E^2(1+\beta^2),\\ b&=-E^2(1-\beta^2\cos\theta),\\ c&=-E^2(1+\beta^2\cos\theta),\\ d&=-E^2\beta^2\sin^2\theta,\\ k\cdot k&=-2(m^2+b)=2E^2\beta^2(1-\cos\theta),\\ l\cdot l&=-2(m^2+c)=2E^2\beta^2(1+\cos\theta), \end{aligned} \quad (15)$$

where E is the energy of any one of the electrons. Therefore

$$\frac{\phi_p}{\phi_a}=\frac{2\cos^2\theta+\beta^2(3\cos^2\theta+\cos^4\theta)+\beta^4(1+\cos^2\theta)}{1+\cos^2\theta+\beta^2(2+3\cos^2\theta-\cos^4\theta)+\beta^4(5-4\cos^2\theta+\cos^4\theta)}. \quad (16)$$

We need only investigate Eq. (16) for θ between 0 and $\frac{1}{2}\pi$ since it is invariant under the substitution $\theta\rightarrow\pi-\theta$ as a consequence of indistinguishability of the two electrons. For no scattering, i.e., $\theta=0$, the ratio takes on its maximum value of unity. Otherwise it is always less than unity, the minimum value being reached at $\theta=\frac{1}{2}\pi$, when it becomes

$$\left.\frac{\phi_p}{\phi_a}\right|_{\text{minimum}}=\frac{\beta^4}{1+2\beta^2+5\beta^4}\begin{cases}\rightarrow 0 \text{ as } \beta\rightarrow 0\\ \rightarrow \frac{1}{8} \text{ as } \beta\rightarrow 1.\end{cases} \quad (17)$$

[5] Reference 4, p. 109.

[6] If we let electron p_2 be at rest in the laboratory frame, we must modify Eq. (2) to read: $\sigma_{p1}u_{\epsilon1}(p_1)=\epsilon_1u_{\epsilon1}(p_1)$; $-\sigma_{p1}u_{\epsilon2}(p_2)=\epsilon_2u_{\epsilon2}(p_2)$. This definition remains meaningful even if $\mathbf{p}_2=0$, and reduces to Eq. (2) in the center-of-mass frame since there $\sigma_{p1}=-\sigma_{p2}$.

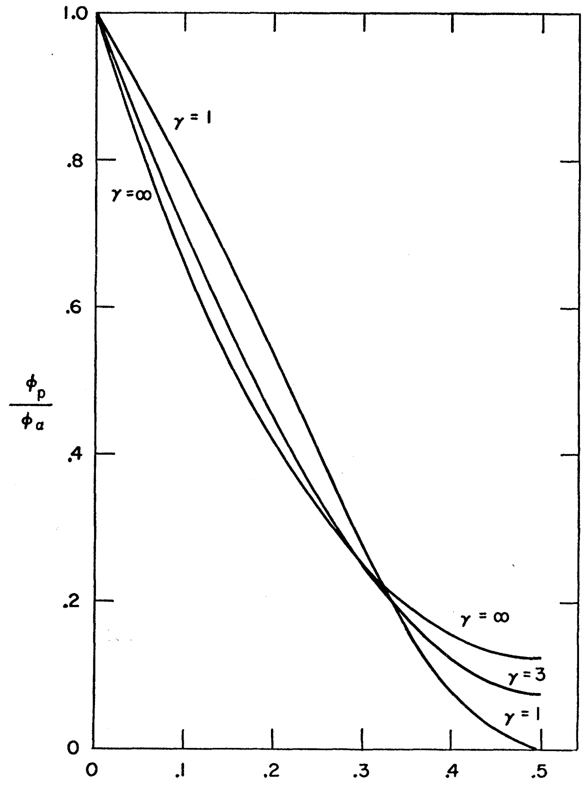

FIG. 1. The ratio ϕ_p/ϕ_a for electron-electron scattering. (γ equals E/m of the incident electron in the laboratory frame and w is its relative kinetic energy transfer.) The numbers on the abscissa can be interpreted as either w or $1-w$.

The reason why the minimum value occurs at $\cos\theta=0$ is to be found in the exclusion principle. For the case described by ϕ_p the spins of the two electrons in the initial state are parallel and hence the exclusion principle requires that the space part of the wave function be antisymmetric. If one assumes no spin flip in the interaction, the same requirement holds for the final-state wave function. If we expand this wave function in partial waves, only odd angular momenta can contribute. But all Legendre polynomials of odd order vanish at $\cos\theta=0$. Thus we obtain the value zero in the nonrelativistic limit when the assumption of no spin flip is valid. Equation (17) shows further that even in the relativistic limit the amount of spin flip is small.

As mentioned before, another frame of reference where the ratio ϕ_p/ϕ_a is of interest is the laboratory frame in which, say, electron p_2 is at rest. A convenient set of variables here is γ and w, where γ is the total energy of electron p_1 in units of m. The relative kinetic-energy transfer, w, is equal to W/T, where W is the kinetic energy lost in the collision by electron p_1 and $T=m(\gamma-1)$ is its kinetic energy before the collision. One has the following relation between the center-of-mass variables β and θ and the laboratory variables γ and w:

$$\beta^2=(\gamma-1)/(\gamma+1),\quad \cos\theta=1-2w. \tag{18}$$

Hence

$$\frac{\phi_p}{\phi_a}=\frac{\gamma^2(1+6x+x^2)-2\gamma(1-x)+1-x^2}{8\gamma^2-2\gamma(4-5x+x^2)+4-6x+2x^2},\quad x\equiv(1-2w)^2. \tag{19}$$

Equation (19) is invariant under the substitution $w\rightarrow 1-w$. The ratio ϕ_p/ϕ_a reaches its minimum value at $w=\frac{1}{2}$ when it becomes

$$\left.\frac{\phi_p}{\phi_a}\right|_{\text{minimum}}=\frac{(\gamma-1)^2}{4(2\gamma^2-2\gamma+1)}\begin{cases}\rightarrow 0 \text{ as } \gamma\rightarrow 1\\ \rightarrow \frac{1}{8} \text{ as } \gamma\rightarrow\infty.\end{cases} \tag{20}$$

We note that when $w=\frac{1}{2}$ the two electrons in the final state have equal energies, and come out on either side of the incident electron's direction at an angle θ^* to it, where

$$\sin^2\theta^*=2/(\gamma+3). \tag{21}$$

Figure 1 depicts the behavior of ϕ_p/ϕ_a as a function of w.

III. POSITRON-ELECTRON SCATTERING

Corresponding to Eq. (1), we have now

$$\frac{M}{4\pi e^2}=\frac{(\bar{u}_{\epsilon'}(p')\gamma_\mu u_\epsilon(p))(\bar{u}_\eta(-q)\gamma_\mu u_{\eta'}(-q'))}{k\cdot k}-\frac{(\bar{u}_\eta(-q)\gamma_\mu u_\epsilon(p))(\bar{u}_{\epsilon'}(p')\gamma_\mu u_{\eta'}(-q'))}{l\cdot l};$$

$$k\equiv p-p',\quad l\equiv p+q. \tag{22}$$

Equation (22) refers to scattering from an initial state of an electron with four-momentum p, polarization ϵ, and a positron with four-momentum q, polarization η,[7] into a final state where all the corresponding quantities are written with primes on them. The second term in Eq. (22) is the annihilation term corresponding to the exchange term in Eq. (1). Since the four-momenta q, q' have fourth components corresponding to positive energies $E_q, E_{q'}$, the spinors $u(-q), u(-q')$ are the negative-energy solutions. Hence, corresponding to Eq. (5) we now use[5]

$$\sum_\eta(\bar{u}_\eta(-q)Qu_\rho(r))=(2|q_4|)^{-1}\sum_\eta(u_\eta^*(q)(\gamma\cdot q-im)Qu_\rho(r)). \tag{23}$$

Therefore

$$\phi(\epsilon,\eta)=\frac{e^4EE_q d\Omega}{E'E_{q'}(E+E_q)^2}\left\{\frac{A}{(k\cdot k)^2}+\frac{B}{(l\cdot l)^2}-\frac{C+D}{k\cdot k\, l\cdot l}\right\}, \tag{24}$$

[7] We define longitudinal polarization for positrons in the same way as for electrons. Thus $\eta=+1$ means spin parallel to the positron's momentum.

where

$$A=\tfrac{1}{4}\,\mathrm{Tr}[(p'+im)\gamma_\mu(1+\epsilon\sigma_p)(p+im)\gamma_\nu]\times\tfrac{1}{4}\,\mathrm{Tr}[(-q+im)(1+\eta\sigma_{-q})\gamma_\mu(-q'+im)\gamma_\nu], \quad (25)$$

$$B=\tfrac{1}{4}\,\mathrm{Tr}[(-q+im)(1+\eta\sigma_{-q})\gamma_\mu(1+\epsilon\sigma_p)(p+im)\gamma_\nu]\times\tfrac{1}{4}\,\mathrm{Tr}[(p'+im)\gamma_\mu(-q'+im)\gamma_\nu], \quad (26)$$

$$C=\tfrac{1}{16}\,\mathrm{Tr}[(p'+im)\gamma_\mu(1+\epsilon\sigma_p)(p+im)\times\gamma_\nu(-q+im)(1+\eta\sigma_{-q})\gamma_\mu(-q'+im)\gamma_\nu], \quad (27)$$

$$D=\tfrac{1}{16}\,\mathrm{Tr}[(-q+im)(1+\eta\sigma_{-q})\gamma_\mu(1+\epsilon\sigma_p)\times(p+im)\gamma_\nu(p'+im)\gamma_\mu(-q'+im)\gamma_\nu]. \quad (28)$$

Evaluation of the above traces gives

$$A=2\{a^2+2m^2(m^2+b)+c^2+\epsilon\eta[ad+(m^2+b)^2]\}, \quad (29)$$

$$B=2\{b^2+2m^2(m^2+a)+c^2+\epsilon\eta[b^2+c^2-2(m^2+a)(a-d)]\}, \quad (30)$$

$$C=D=-2\{c^2+m^2(a+b+c)+m^4+\epsilon\eta[-a^2+ad+b^2+m^2(-a+b+c+d)]\}, \quad (31)$$

where

$$\begin{aligned} a&\equiv -p\cdot q=-p'\cdot q', \\ b&\equiv p\cdot p'=q\cdot q', \\ c&\equiv -p\cdot q'=-p'\cdot q, \\ d&\equiv -(\mathbf{p}')_\perp\cdot(\mathbf{q}')_\perp. \end{aligned} \quad (32)$$

We see that the polarization-independent parts of A, B, C, and D are obtainable from the corresponding expressions of Sec. II if use is made of the substitution law.[8] For the polarization-dependent parts the substitution law cannot be used since it relates outgoing positrons with incoming electrons (and vice versa) whereas we are specifying the polarization of the incoming particles always.

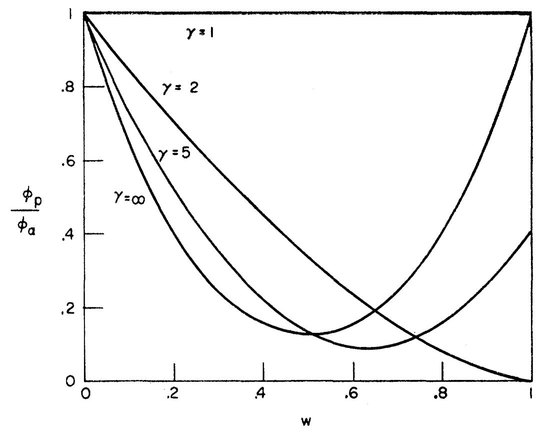

FIG. 2. The ratio ϕ_p/ϕ_a for positron-electron scattering. (γ and w refer to the positron in the laboratory frame and have the same meaning as in Fig. 1.)

In the center-of-mass frame, one has

$$\begin{aligned} a&=E^2(1+\beta^2), \\ b&=E^2(\beta^2\cos\theta-1), \\ c&=E^2(\beta^2\cos\theta+1), \\ d&=E^2\beta^2\sin^2\theta, \\ k\cdot k&=-2(m^2+b)=2E^2\beta^2(1-\cos\theta), \\ l\cdot l&=-2(m^2+a)=-4E^2. \end{aligned} \quad (33)$$

The ratio of cross sections for spins parallel ($\epsilon=-\eta$) and antiparallel ($\epsilon=\eta$) becomes now:

$$\frac{\phi_p}{\phi_a}=\frac{1+6\beta^4\cos^2\theta+\beta^6\cos^4\theta+(1-\beta^2)[1-4\beta^2+\beta^4+2\beta^2(4-\beta^2)\cos\theta+\beta^2\cos^2\theta+2\beta^4\cos^3\theta]}{8+(1-\beta^2)[-6-7\beta^2+\beta^4+6\beta^2(1-\beta^2)\cos\theta-\beta^2(1-7\beta^2+\beta^4)\cos^2\theta-2\beta^4(1-\beta^2)\cos^3\theta-\beta^6\cos^4\theta]}. \quad (34)$$

One translates Eq. (34) into the frame where the electron is initially at rest by use of Eq. (18), where γ is now the positron's E/m and w the relative kinetic energy transfer from the positron to the electron.

Equation (34), unlike Eq. (16), is not invariant under the substitution $\theta\to\pi-\theta$. This is an expression of the fact that electrons and positrons are distinguishable. In the relativistic limit, however, we obtain from Eq. (34):

$$\phi_p/\phi_a\to\tfrac{1}{8}(1+6\cos^2\theta+\cos^4\theta)\quad\text{as}\quad\beta\to 1. \quad (35)$$

This expression is identical with the relativistic limit of Eq. (16) and has the same minimum value of $\frac{1}{8}$ at $\cos\theta=0$ ($w=\frac{1}{2}$). In the nonrelativistic limit we have, on the other hand, independent of θ,

$$\phi_p/\phi_a\to 1\quad\text{as}\quad\beta\to 0. \quad (36)$$

The general behavior of ϕ_p/ϕ_a as a function of w is depicted in Fig. 2.

IV. μ-MESON–ELECTRON SCATTERING

We are now dealing with entirely different fermions; hence neither the exchange nor the annihilation term, but only the direct term contributes. Thus it is necessary to calculate only the quantity A of Secs. II and III. We obtain

$$A=2\{a^2+(m^2+\mu^2)(m^2+b)+c^2+\epsilon\eta[ad+(m^2+b)^2]\}, \quad (37)$$

[8] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Press, Inc., Cambridge, 1955), p. 161.

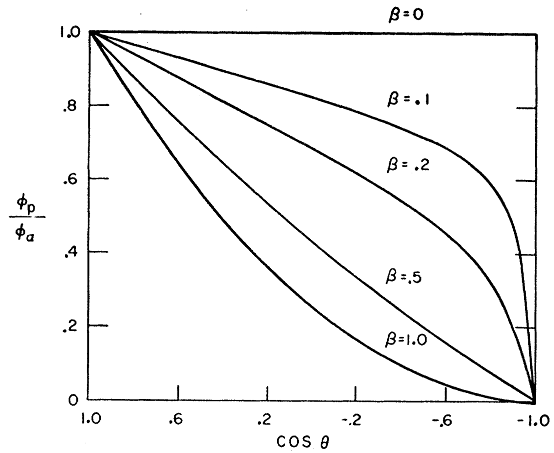

FIG. 3. The ratio ϕ_p/ϕ_a for μ-meson-electron scattering when the electron's rest mass is neglected. (β equals p/E of the μ-meson and θ is the scattering angle.) All quantities are in the center-of-mass frame of reference.

where

$$\begin{aligned} a &\equiv p_e\cdot p = p_e'\cdot p', \\ b &\equiv p_e\cdot p_e' = p\cdot p' + \mu^2 - m^2, \\ c &\equiv p_e\cdot p' = p_e'\cdot p, \\ d &\equiv (\mathbf{p}_e')_\perp\cdot(\mathbf{p}')_\perp, \end{aligned} \tag{38}$$

with p (p') and p_e (p_e') the μ-meson and electron in the initial (final) state. The quantities μ and m are the rest masses of the μ meson and electron. The polarizations of the initial state are specified by ϵ and η.

In the center-of-mass frame, we have

$$\begin{aligned} a &= -p^2 - E_eE, \\ b &= p^2\cos\theta - E_e^2, \\ c &= -p^2\cos\theta - E_eE, \\ d &= -p^2\sin^2\theta, \end{aligned} \tag{39}$$

with

$$p^2 = E_e^2 - m^2 = E^2 - \mu^2.$$

Thus the ratio ϕ_p/ϕ_a is given by

$$\frac{\phi_p}{\phi_a} = \frac{(p^2+E_eE)^2 + (p^2\cos\theta + E_eE)^2 - p^2(m^2+\mu^2)(1-\cos\theta) - p^2(p^2+E_eE)\sin^2\theta - p^4(1-\cos\theta)^2}{(p^2+E_eE)^2 + (p^2\cos\theta + E_eE)^2 - p^2(m^2+\mu^2)(1-\cos\theta) + p^2(p^2+E_eE)\sin^2\theta + p^4(1-\cos\theta)^2}. \tag{40}$$

Here, as in the positron-electron case, the effect is a relativistic one; that is

$$\phi_p/\phi_a \to 1 \quad \text{as} \quad p^2 \to 0. \tag{41}$$

If the electron can be taken as relativistic, we can rewrite Eq. (40) as follows:

$$\frac{\phi_p}{\phi_a} = \frac{(1+\cos\theta)[1+\beta(1+\cos\theta)+\beta^2\cos\theta]}{1+\cos\theta+\beta(1+\cos\theta)(3-\cos\theta)+\beta^2(4-3\cos\theta+\cos^2\theta)}, \tag{42}$$

where β stands for p/E. Equation (42) is plotted in Fig. 3 and it is seen that, in principle, ϕ_p/ϕ_a provides a method for measuring longitudinal polarization. One encounters, however, difficulties due to the kinematics of the problem. We see from Fig. 3 that, for instance, at $\cos\theta = 0$ we need $\beta \gtrsim 0.1$ for the effect to be appreciable. This can be accomplished if in the laboratory frame the μ meson is at rest and the electron has a momentum of the order 0.1 μ. However, the experimental situation is much more likely to be such that the electron is at rest in the laboratory frame. In this case we require a μ meson in the laboratory frame of momentum of the order 0.1 $\mu^2/m \simeq 2$ Bev.

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