Département de physique nucléaire et corpusculaire
Section de Physique
Université de Genève

Semestre de printemps 2023

ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - 2^{nd} Assignment

Distributed: February 28, 2023
To be returned: March 9, 2023

Isospin

1. Show that $e^{i\pi\sigma_2/2} = i\sigma_2$, where σ_2 is one of the Pauli matrices.

2. Using isospin symmetry, show that the rates for the following strong interaction decays occur in the ratios

$$\Gamma(\Delta^- \to \pi^- n) : \Gamma(\Delta^0 \to \pi^- p) : \Gamma(\Delta^0 \to \pi^0 n) : \Gamma(\Delta^+ \to \pi^+ n) : \Gamma(\Delta^+ \to \pi^0 p) : \Gamma(\Delta^{++} \to \pi^+ p) =$$

$$= 3 : 1 : 2 : 1 : 2 : 3$$

3. The G-parity is defined as a rotation of 180° around the y-axis in isospin space followed by a charge conjugation:

$$G = Ce^{i\pi I_2} \ .$$

   The G-parity is defined such that charged particles can also be eigenstates of G-parity. Since strong interaction is invariant under isospin rotations and charge conjugation, G-parity is conserved in strong interactions. Show that for a system of n pions

$$G(n\pi) = (-1)^n \ .$$

   What is the G-parity of the ρ^0 and ω mesons (study their decays)?

Meson decays

4. Show how the intrinsic parity of the π^0 meson can be determined from the decay $\pi^0 \to \gamma\gamma$ by measurements of the photon polarization. By similar measurements one can check that the electron-positron pair has odd relative parity, since the annihilation process $e^+e^- \to 2\gamma$ can occur in the $^1S_0$ positronium state.

5. How does the ρ^0 meson decay? Why the ρ^0 meson cannot decay to two π^0's ? Can it decay to two γ's?

6. The leptonic decay of neutral vector mesons ($J^{PC} = 1^{--}$) can be pictured as proceeding via a virtual photon γ^*:
$$V(q\bar{q}) \to \gamma^* \to e^+e^- \ .$$
Neglecting a possible dependence on the vector meson mass, show that the leptonic decay widths are in the ratio:
$$\rho : \omega : \phi : \psi = 9 : 1 : 2 : 8 \ .$$
Hint: To calculate these ratios it suffices to notice that the $V - \gamma$ coupling is proportional to the charges of the quarks (the matrix element for this decay is proportional to $\langle\psi|\hat{Q}_q|\psi\rangle$, there is no need to calculate explicitly the amplitudes).

Baryon wavefunctions

7. Write down the proton's wave function (flavor and spin). Using isospin symmetry write down the neutron's wave function.

8. Derive the magnetic moment of the proton, neutron, and Λ.

9. If the color did not exist, baryon wavefunctions would be constructed from
$$\psi = \phi_{flavor}\chi_{spin}\eta_{space} \ .$$
Taking $L = 0$ and using the flavor and spin wavefunctions derived above
i) show that it is still possible to construct a wavefunction for a spin-up proton for which $\phi_{flavor}\chi_{spin}$ is totally antisymmetric;
ii) predict the baryon multiplet structure for this model;
iii) show that μ_p is negative and that $\mu_n/\mu_p = -2$ in this model (which rules it out).

10. The Ω^- baryon was discovered in the strong interaction process $K^- + p \to \Omega^- + X$. What is the minimal composition of X in terms of quarks compatible with the strong interaction? Show the quark composition of all interacting hadrons. Show in detail the decay chain of Ω^- and the strangeness *flow*.