Département de physique nucléaire et corpusculaire
Section de Physique
Université de Genève

Semestre de printemps 2023

ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - 3rd Assignment

Distributed: March 7, 2023
To be returned: March 16, 2023

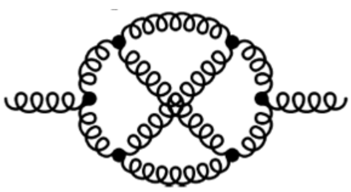
Miscellanea

1. When comparing e^+e^- to $q\bar{q}$ cross sections, we introduce the ratio

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

   Why do we use $\sigma(e^+e^- \to \mu^+\mu^-)$ in the denominator instead of $\sigma(e^+e^- \to e^+e^-)$?

2. What is the amplitude of the following diagram?



   Hint: assign a specific color to the gluon on the left and follow the color flow.

3. From the expression of the running coupling constant $\alpha_S(Q^2)$ for $N_f = 3$ determine the value of Q^2 at which α_S appears to become infinite. Comment on this result.

4. Show that, if a particle (for instance, the neutron) has a non vanishing electric dipole moment d_E, both parity and time reversal invariance are violated (assuming CPT invariance, violation of the time reversal invariance implies CP violation).

Color Factors

5. Color factors involve always expressions of the Gell-Mann matrices in the form $\lambda^a_{ij}\lambda^a_{kl}$ (sum over the color index a). Using the expression

$$\sum_{a=1}^{8} \lambda^a_{ij}\lambda^a_{kl} = 2\delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}$$

   calculate $\lambda^a_{11}\lambda^a_{11}$, $\lambda^a_{11}\lambda^a_{22}$, $\lambda^a_{12}\lambda^a_{21}$ and $\lambda^a_{ij}\lambda^a_{ij}$. These color factors will be used in the following exercises.

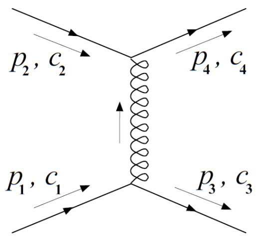
6. Assume that the $q\bar{q}$ potential is Coulomb-like

$$V_{q\bar{q}}(r) = -f_C \frac{\alpha_s \hbar c}{r} \ .$$

where f_C ia the color factor. Show that quarks and antiquarks attract each other most strongly when they are in the color singlet configuration ($3 \otimes \bar{3} = 1 \oplus 8$).

Hint: Since the color symmetry holds, it is enough to consider one octet state. Take for instance $(r\bar{r} - b\bar{b})/\sqrt{2}$. The singlet state is $(r\bar{r} + g\bar{g} + b\bar{b})/\sqrt{3}$.

7. Do the same calculation for the interaction of two quarks (shown in the Figure below). The decomposition of a qq system is $3 \otimes 3 = \bar{3} \oplus 6$. Find for which configuration the force is attractive and for which it is repulsive.



Hint: Take for instance the state $(rb + br)/\sqrt{2}$ for the sextet and $(rb - br)/\sqrt{2}$ for the triplet configuration.

8. Find the overall 'color factor' for $qq \to qq$, if QCD color symmetry were SU(2) instead of SU(3).

QCD amplitudes

9. The lowest-order QCD diagrams describing the interaction of a quark and an antiquark ($q\bar{q} \to q\bar{q}$) are shown in the Figure below). Calculate the amplitudes for both processes. Show that the difference with respect to QED $e^+e^- \to e^+e^-$ amplitude is in the 'color factor'.

The diagram on the left has been calculated during the lesson (crossing of the $qq \to qq$ process). Assuming that the $q\bar{q}$ pair is in the color singlet configuration, calculate the amplitude of the process shown in the diagram on the right.