Département de physique nucléaire et corpusculaire
Section de Physique
Université de Genève

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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - 4^{th} Assignment

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Deep Inelastic Scattering

1. In inclusive Deep Inelastic Scattering we observe only the scattered lepton (electron or muon). Let E and E' be the energy of the incident and scattered lepton, respectively, and ϑ the scattering angle. Express the observables s, x, y, Q^2, and W^2 in terms of E, E', and ϑ. In the $Q^2 - x$ plane draw the kinematical region accessible for given E, E', y, and ϑ.

2. Calculate the structure function $H_\lambda = \varepsilon^\mu_\lambda W^{\mu\nu} \varepsilon^{*\,\nu}_\lambda$ that describes the absorption of a photon of polarization λ. Start from the decomposition of the hadronic tensor $W^{\mu\nu}$. In lepton – nucleon scattering in the laboratory frame the nucleon is at rest with four-momentum $p^\mu = (M, 0)$. The reference axes are oriented such that the photon four-momentum $q^\mu = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$ (the z-axis is along the photon direction) and the exchanged virtual photon has helicities ±1 (transverse polarization) and 0 (longitudinal polarization), as well. The polarization four-vectors for each helicity are given by

$$\begin{array}{ll} \lambda = 1 & \varepsilon_+ = \frac{1}{\sqrt{2}}(0, 1, i, 0) \\ \lambda = 0 & \varepsilon_0 = \frac{1}{\sqrt{Q^2}}(\sqrt{Q^2 + \nu^2}, 0, 0, \nu) \\ \lambda = -1 & \varepsilon_- = \frac{1}{\sqrt{2}}(0, 1, -i, 0) \end{array}$$

3. If quarks were spin-0 particles, why would $F_1^{ep}(x)/F_2^{ep}(x)$ be zero?

4. By adding up (integrating over) all quarks in a nucleon we obtain the so called **sum rules**. Using isospin symmetry and the quark content of the F_2 structure function, derive the Gottried sum rule, which compares protons and neutrons:

$$I_G = \int_0^1 \frac{dx}{x} \left[F_2^{ep} - F_2^{en}\right] = \left(e_u^2 - e_d^2\right) \int_0^1 dx \left[u_v(x) - d_v(x)\right] = \frac{1}{3} \, .$$

   Prove it! Experiments have found that $I_G = 0.24$, i.e. that this sum rule is violated. What can you can say about this?

Scaling Violations

5. Derive the kinematics for $\gamma^* q \to qg$ (gluon emission diagram) in terms of the parton level Mandelstam variables $\hat{s}$, $\hat{t}$, $\hat{u}$. Show that

$$p_T^2 = \frac{\hat{s}\hat{t}\hat{u}}{(\hat{s}+Q^2)^2}$$

where p_T is the transverse momentum of the outgoing quark w.r.t. the $\gamma^* q$ axis.
Then find $(p_T^2)_{max}$

$$(p_T^2)_{max} = \frac{\hat{s}}{4} = Q^2 \frac{1-z}{4z}$$

with z the parton momentum after gluon emission.

6. Derive the spin and color averaged matrix element for $\gamma^* q \to qg$ (gluon emission diagram)

$$\langle |M|^2 \rangle = 32\pi^2 \left(e_i^2 \alpha\right) \left(\frac{4}{3}\alpha_S\right) \left(-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right)$$

where e_i is the charge of the quark emitting the gluon.

7. Calculate the splitting function

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \; .$$

8. Derive the color factor for $\gamma^* g \to q\bar{q}$ (answer $C_F = \frac{1}{2}$).

9. Show that momentum conservation at the QCD vertex requires ($z < 1$!)

$$P_{q\leftarrow q}(z) = P_{g\leftarrow q}(1-z) \; , \qquad P_{q\leftarrow g}(z) = P_{q\leftarrow g}(1-z) \; , \qquad P_{gg}(z) = P_{gg}(1-z) \; .$$