Département de physique nucléaire et corpusculaire
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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - 5^{th} Assignment

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e^+e^-annihilation into hadrons

1. In a small electron-positron collider of radius $R = 10$ m, each beam has a current $I = 10$ mA and a transverse area $S = 0.1\ \text{cm}^2$. The cross-section for the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ at the peak of the ω resonance is $\sigma = 1.5\ \mu$b. Draw a Feynman diagram to illustrate such an event. Assuming that there is only one electron and one positron bunch and that the beams collide head-on two times per revolution, calculate the rate (number of events per second) for this process.

2. Electron and positron beams with an energy E of 4 GeV each collide head on in a storage ring. What production rate for $\mu^+\mu^-$ pairs would you expect at a luminosity of $10^{32}\ \text{cm}^{-2}\text{s}^{-1}$? What production rate for events with hadronic final states would you expect?

3. Consider the reaction
$$e^+ + e^- \to q + \bar{q}$$
at a collider with a CM energy $\sqrt{s} = 20$ GeV. Give a typical value of the hadronic jet opening angle in a two-jet event (typical transverse momenta of quarks, compatible with confinement, are 200 – 300 MeV/c). Let ϑ be the angle of the common jet direction w.r.t. the colliding beams. Find the ratio between the counting rates at $\vartheta = 90°$ and $\vartheta = 30°$

Fragmentation functions

4. Show that
$$\sum_h \int_0^1 z \cdot D_q^h(z)\,\mathrm{d}z = 1$$
where the sum runs over all hadrons produced in the event (this result basically says that all initial quarks fragments to hadrons).

5. Show that

$$\sum_q \int_{z_{min}}^{1} \left[D_q^h(z) + D_{\bar{q}}^h(z)\right] \, \mathrm{d}z = n_h$$

where the sum runs over all quarks produced in the event and n_h is the avarage multiplicity of hadron h. (this result basically says that the number n_h of hadrons of type h is given by the probabilities of obtaining h from all possible parents).

Gluon emission

6. Show that (calculate) the gluon emission kinematics is described by the following variables

$$x_T^2 = \frac{4}{x_q^2}(1 - x_{\bar{q}})(1 - x_q)(1 - x_g)$$

$$x_{\bar{q}} = \frac{2(1 - x_q)}{2 - x_q - x_q \cos\vartheta}$$

where x_T is the anti-quark transverse momentum relative to the quark direction, x_q, x_g, and $x_{\bar{q}}$ are the quark, gluon, and anti-quark fractional momenta and ϑ the angle between the quark and anti-quark directions.

7. Derive the following expression for the gluon emission cross section in $e^+e^- \to q\bar{q}g$ (see the derivation for $\gamma^* + g \to q + g$ from last week):

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\bar{q}}\mathrm{d}x_T^2} = \frac{\alpha_s}{2\pi}\frac{1}{x_T^2}P_{q\leftarrow q}(x_{\bar{q}}) \; .$$

$P_{q\leftarrow q}$ is the splitting function already encountered in the QCD evolution equations.

8. Show that $\mathrm{d}\sigma/\mathrm{d}x_T$ can be transformed in $\mathrm{d}\sigma/\mathrm{d}\vartheta$ for ϑ not too large, where ϑ is the angle between the q and $\bar{q}$ directions

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\vartheta} \approx \frac{8\alpha_S}{3\pi}\frac{1}{\vartheta}\log\frac{1}{\vartheta^2} \; .$$

The exact result can be obtained starting from

$$\frac{1}{\sigma}\frac{d\sigma}{dx_q\, dx_{\bar{q}}} = \frac{2\alpha_S}{3\pi}\frac{x_q^2 + x_{\bar{q}}^2}{(1 - x_q)(1 - x_{\bar{q}})} \; .$$