Département de physique nucléaire et corpusculaire
Section de Physique
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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - 6^{th} Assignment

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Kinematics

1. To describe the kinematics of outgoing hadrons or jets in $p + p$ collisions one uses the *rapidity* y defined as

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) = \tanh^{-1} (p_z/E) \ ,$$

where p_z is the momentum along the collision axis. How does this observable change under a Lorentz boost in the z direction? And the rapidity difference Δy ?

Express y in terms of the polar angle ϑ. Then neglect the masses of hadrons; the variable so obtained is referred to as the pseudorapidity η:

$$\eta = -\ln \tan (\vartheta/2) \ .$$

2-body QCD processes in pp collisions

2. Draw the Feynman diagrams for all two body elementary sub-processes in hadron-hadron collisions between quarks and gluons (all s, t, and u channels). Sketch the angular dependence of the matrix elements as a function of $\cos \vartheta_{\mathrm{CM}}$. Are the matrix elements divergent for $\cos \vartheta_{\mathrm{CM}} \to +1$ or -1? Explain.

3. Draw all leading order QCD Feynman diagrams that involve photons (initial and/or final state). Calculate the color factors for each diagram (sum over final state colors, average over initial state colors).

4. Consider the QCD Compton diagram $gq \to \gamma q$.
In the detector we can measure the (direct–)γ and the away side jet. Derive the partonic centre of mass kinematics, i.e. x_g and x_q, in terms of the γ and jet kinematics.
Is there a QCD sub-process with 2 isolated photons in the final state? If so, draw the corresponding Feynman diagram(s).

5. What is the main difference between a $p-p$ and $p-\bar{p}$ collider at the same c.o.m. energies? Draw the lowest order QCD Feynman diagrams for the process $pp \to \mathrm{di-jets} + X$ and $p\bar{p} \to \mathrm{di-jets} + X$, where X represents the remnants of the colliding hadrons.

The Drell-Yan process

6. The total cross section for the Drell–Yan process $p\bar{p} \to \mu^+\mu^- X$ has been derived during the lesson. Here we assume that only u ($\bar{u}$) and d ($\bar{d}$) quarks are found in the proton:

$$\sigma_{DY} = \frac{4\pi\alpha^2}{81s} \int_0^1 \int_0^1 \frac{1}{x_1 x_2} \left[4u(x_1)u(x_2) + 4\bar{u}(x_1)\bar{u}(x_2) + 4d(x_1)d(x_2) + 4\bar{d}(x_1)\bar{d}(x_2)\right] \mathrm{d}x_1 \mathrm{d}x_2 \ .$$

(i) Let the sea contribution be described by a single PDF $S(x) = \bar{u}(x) = \bar{d}(x)$. Express σ_{DY} in terms of the valence quark PDFs and S.
(ii) Derive the corrisponding expression for $pp \to \mu^+\mu^- X$.

7. The Drell–Yan production of $\mu^+\mu^-$ pairs with an invariant mass Q^2 has been studied, for instance, in $\pi^\pm$ interactions with carbon. Explain why the ratio

$$\frac{\sigma(\pi^+ C \to \mu^+\mu^- X)}{\sigma(\pi^- C \to \mu^+\mu^- X)}$$

tends to unity for small Q^2 and tends to $\frac{1}{4}$ as Q^2 approaches s (s is the c.o.m. energy). Hint: Think in terms of valence quarks and sea quarks.

8. Draw all Feynman diagrams which give an $O(\alpha^2\alpha_s)$ contribution to the lepton pair cross section.