Département de physique nucléaire et corpusculaire
Section de Physique
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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - $7^{\rm th}$ Assignment

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1. Express the Fermi constant $G_F = 1.166 \times 10^{-5}\mathrm{GeV}^{-2}$ in SI units.

2. Indicate which forces (EM, strong, weak) do conserve or do violate these symmetries:
   a) isospin
   b) isospin's 3^{rd} component I_3
   c) flavor
   d) parity
   e) charge conjugation
   and for each of these symmetries indicate an experiment, which established its violation.

3. Consider the following reactions among particles or particle decays:
   a) $\pi^- + p \to \pi^0 + n$
   b) $\pi^0 \to \gamma + \gamma + \gamma$
   c) $\mu^- \to e^- + e^+ + e^-$
   d) $\pi^+ \to \mu^+ + \nu_\mu$
   e) $\pi^- \to \mu^- + \nu_\mu$
   f) $\Lambda^0 \to p + e^- \bar{\nu}_e$
   g) $p + \bar{p} \to \gamma$
   h) $n \to p + e^-$
   i) $n \to p + \pi^-$
   j) $\Lambda^0 \to K^0 + \pi^0$
   k) $\nu_\mu + e^- \to e^- + \nu_e$
   l) $\nu_\mu + e^- \to \nu_\mu + e^-$
   Indicate for each case, if the process is allowed or forbidden, reason if forbidden, and type of interaction if allowed.

4. How thick should be an iron absorber to reduce the flux of a 1 GeV neutrino beam by two? and to reduce the flux of a 1 GeV photon beam by two?

5. In the decay ${}^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^{**} + e^- + \bar{\nu}_e$, the *intensity* of the emitted electrons has the form

$$I(\beta, \vartheta) = 1 + \alpha\, \beta\, \cos\vartheta$$

where $\beta = v/c$ is the speed of the emitted electron and ϑ is the angle between the electron direction an the spin of ^{60}Co. Deduce the value of α.

6. Show that the left-handed and right-handed spinors

$$u_L(p) = \frac{1}{2}(1-\gamma_5)u(p,-) \qquad\qquad u_R(p) = \frac{1}{2}(1+\gamma_5)u(p,+)$$

are eigenstates of the helicity operator $\lambda = \vec{s}\cdot\vec{p}/|\vec{p}|$ in the massless limit.
The same calculation should show that

$$\frac{1}{2}(1+\gamma_5)u(p,-) \qquad\qquad u_R(p) = \frac{1}{2}(1-\gamma_5)u(p,+)$$

in the same limit.

7. Show that the charge-raising weak current

$$(J^\mu)^+ = \bar{u}_\nu \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) u_e$$

couples an ingoing negative helicity electron (positive helicity antineutrino) to an outgoing negative helicity neutrino (positive helicity positron). Neglect the mass of the electron.

8. The particle decay sequences

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \text{and} \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

show evidence of parity non conservation. What observable quantity can be measured to show this effect?

9. Draw the Feynman diagrams for the following decays and reactions (label correctly all incoming and outgoing particles):
   a) $\mu^- \to e^-\,\bar{\nu}_e\,\nu_\mu$
   b) $\tau^+ \to e^+\,\nu_e\,\bar{\nu}_\tau$
   c) $\nu_\mu\,e^- \to \nu_\mu\,e^-$
   d) $\mu^+\,e^- \to \bar{\nu}_\mu\,\nu_e$
   Write down the corresponding amplitude (do not calculate it) in the Fermi theory (no propagator).

10. For a particle described by a spinor $u(p,\lambda)$ we can define the polarization four-vector $s_\mu(p,\lambda)$ as

$$s_\mu(p,\lambda) = \frac{1}{2m}\bar{u}(p,\lambda)\gamma_\mu\gamma_5 u(p,\lambda)\ .$$

Show that

$$s\cdot p = 0$$

and that

$$s^2 = -1\ .$$

Then calculate s_μ for a particle at rest ($\vec{p}=0$) with $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$.
Suppose that for a particle at rest the polarization four-vector is given by $s^\mu = (0,\vec{\eta})$ with $\vec{\eta}^2 = 1$. Show that in a frame, where the particle moves with momentum $\vec{p}$, the spin vector s^μ is given by

$$s^0 = \frac{\vec{\eta}\cdot\vec{p}}{m} \qquad\qquad \vec{s} = \vec{\eta} + \frac{\vec{p}(\vec{\eta}\cdot\vec{p})}{(E+m)m}$$