Département de physique nucléaire et corpusculaire
Section de Physique
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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - $8^{\rm th}$ Assignment

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Tritium decay

1. Tritium is unstable and undergoes beta decay: $^3\mathrm{H} \rightarrow\ ^3\mathrm{He} + e^- + \bar{\nu}_e$. Rewrite the decay in terms of nucleons and quarks. What are the ^{3}H lifetime and the energy released in the decay?

2. The invariant amplitude for ^{3}H decay (following the notation and approximations made in the class) is
$$M = G_F\, \bar{u}(p_e)\, \gamma^0 \left(1 - \gamma^5\right) \nu(p_\nu)\, 2\, m_N\ .$$
Show that
$$\sum_{spins} \left|\bar{u}\, \gamma^0 \left(1 - \gamma^5\right) \nu\right|^2 = 8\, E_e\, E_\nu\ (1 + \cos\vartheta)$$
and calculate $|M|^2$.

3. Calculate the electron energy spectrum $\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e}$.

4. Finally calculate the ^{3}H lifetime ($G_F = 1.136 \times 10^{-5}\ \mathrm{GeV}^{-2}$) and compare the result with the measured ^{3}H lifetime.

5. Show that the polarization of electrons emitted in β decays is $-\beta$, where $\beta = v/c$.

Muon decay

6. What is the minimum electron momentum from a muon decaying at rest? And what is the maximum electron momentum?
Show that (same notation as in the class)
$$2\,(k \cdot p')\,(k' \cdot p) = \left(m^2 - 2m\omega'\right) m\omega'\ .$$

7. How would you measure the polarization of a muon beam? Muons are produced in the decay of a pion beam: $\pi^+ \rightarrow \mu^+ + \nu_\mu$.

8. Consider the decay at rest $\tau^- \to \pi^- \nu_\tau$. What is the branching ratio for this decay channel (find it in the PDG)? And its partial width? Assume that the τ is polarized in the z direction and that the ν_τ and π^- are emitted along the z axis. Sketch the allowed spin configurations assuming that the form of the weak charged current interaction is
   (a) $V - A$,
   (b) $V + A$.

Pion decay

9. Predict the ratio of the following K^- decay rates

$$\Gamma(K^- \to e^- \bar{\nu}_e) \,/\, \Gamma(K^- \to \mu^- \bar{\nu}_\mu) \;=\; (2.488 \pm 0.012) \times 10^{-5}\,(\text{exp.})\;.$$

   Estimate the K decay constant f_K and comment on the assumptions that you made for obtaining this result, and the result itself.

10. Repeat the pion decay calculations for a scalar theory and show that

$$\Gamma(\pi^- \to e^- \bar{\nu}_e) \,/\, \Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \;\approx\; 5.5\;.$$