Département de physique nucléaire et corpusculaire
Section de Physique
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ADVANCED PARTICLE PHYSICS II

http://dpnc.unige.ch/∼bravar/PPA2

Exercises - $9^{\rm th}$ Assignment

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Cabibbo *decays*

1. Show a Cabibbo allowed and a Cabibbo suppressed semileptonic decay of the charm quark. Sketch three Cabibbo allowed and three Cabibbo suppressed decays of D^+ mesons.

2. Estimate the relative decay rates for the following three decay modes of the D^0 meson: $D^0 \to K^+\pi^-$, $D^0 \to \pi^+\pi^-$, and $D^0 \to K^-\pi^+$. First draw the corresponding lowest order Feynman diagrams.

CKM matrix

3. Consider the decays $B^0 \to D^- + \pi^+$, $B^0 \to D^- + K^+$, $B^0 \to \pi^- + K^+$, and $B^0 \to \pi^- + \pi^+$. Find the valence quark composition of each meson, establish the dependence of the partial decay rates on the CKM mixing matrix elements and sort them in decreasing order of the decay rates. Compare your findings with the values reported by the PDG.

4. Show that the CKM matrix is unitary for any (real) numbers ϑ_{12}, ϑ_{23}, ϑ_{13}, and δ.

5. For the case of two, three, and four quark generations, state:
(i) the number of free parameters in the corresponding $n \times n$ unitary flavor mixing matrix;
(ii) how many of these parameters are real and how many are phases;
(iii) how many of these phases can be absorbed in the redefinition of the quark fields;
(iv) whether CP violation can be accommodated in the quark mixing.

6. From the measured values

$$|V_{ud}| = 0.97425 \pm 0.00022 \ \text{ and } \ |V_{ub}| = (4.155 \pm 0.49) \times 10^{-3} \ ,$$

$$|V_{cd}| = 0.230 \pm 0.011 \ \text{ and } \ |V_{cb}| = 0.041 \pm 0.001$$

calculate the length of the corresponding side of the unitary triangle and its uncertainty. By stretching this constraint and that from the measured value of β, obtain approximate constraints on the values of ρ and η.

The $K^0 - \bar{K}^0$ system

7. Does the K^0 meson participate in charge exchange processes with neutrons? Does the $\bar{K}^0$ do so?
   A charge exchange process is of the type $K^0 + n \to B^\pm + M^\mp$, where B is a charged baryon and M a charged meson, i.e. one unit of electric charge is excahnged between the interacting particles.

8. Show that the $\pi^+\pi^-$ or the $\pi^0\pi^0$ state in the S wave is an eigenstate of CP with eigenvalue $C = +1$. Then, show that by adding a π^0 in S wave, we obtain CP eigenstates $\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$ with eigenvalue $C = -1$.

9. A π^- beam is sent onto a target producing neutral K mesons and Λ hyperons. Consider the component of the resulting K beam with momentum $p = 10$ GeV. What is the ratio between K_S and K_L at the production point? What is it at a distance $L = 10$ m from the target? Determine the fraction of decays into 2π that would be observed in the absence of CP violation.

10. In the neutral kaon system, time reversal violation can be expressed in terms of the asymmetry
    $$A_T = \frac{\Gamma(\bar{K}^0 \to K^0) - \Gamma(K^0 \to \bar{K}^0)}{\Gamma(\bar{K}^0 \to K^0) + \Gamma(K^0 \to \bar{K}^0)} \ .$$
    Time reversal conservation requires $A_T = 0$. Show that this is equivalent to
    $$A_T = \frac{\Gamma(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e) - \Gamma(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e) - \Gamma(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e)}$$
    and therefore
    $$A_T \approx 4|\epsilon| \cos\phi \ .$$