Advanced Particle Physics 2

Strong Interactions and Weak Interactions

L1 – Introduction to the Standard Model

(http://dpnc.unige.ch/~bravar/PPA2/L1)

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zoom https://unige.zoom.us/j/5309798089

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Particle Physics

Study interactions of Nature
that act on particles at distances of the size of the atomic nucleus
These would appear only if we can probe matter at very small distances → high energy.

By the middle of the 20th century
experiments have revealed a series of questions that could not be resolved without the introduction of new particles and new interactions.
Particle physics is therefore an experimental science.

what is radioactivity? → weak interactions

what holds the atomic nucleus together? → strong interactions

what are the protons and neutrons made off? → strong interactions

Today particle physics is described by the Standard Model (of particle physics), which is a mathematically consistent theory of electromagnetic, weak, and strong interactions (though a coherent collection of different ideas and theories) that describes (almost) all know phenomena and which has been verified in detail in different experiments.
With the recent discovery (2012) of the Higgs boson the Standard Model is completed.
Is this all?

This course is an introduction to the Standard Model of Particle Physics.


Le plus important
est invisible.
Le Petit Prince

Q1: What is Matter Made off?

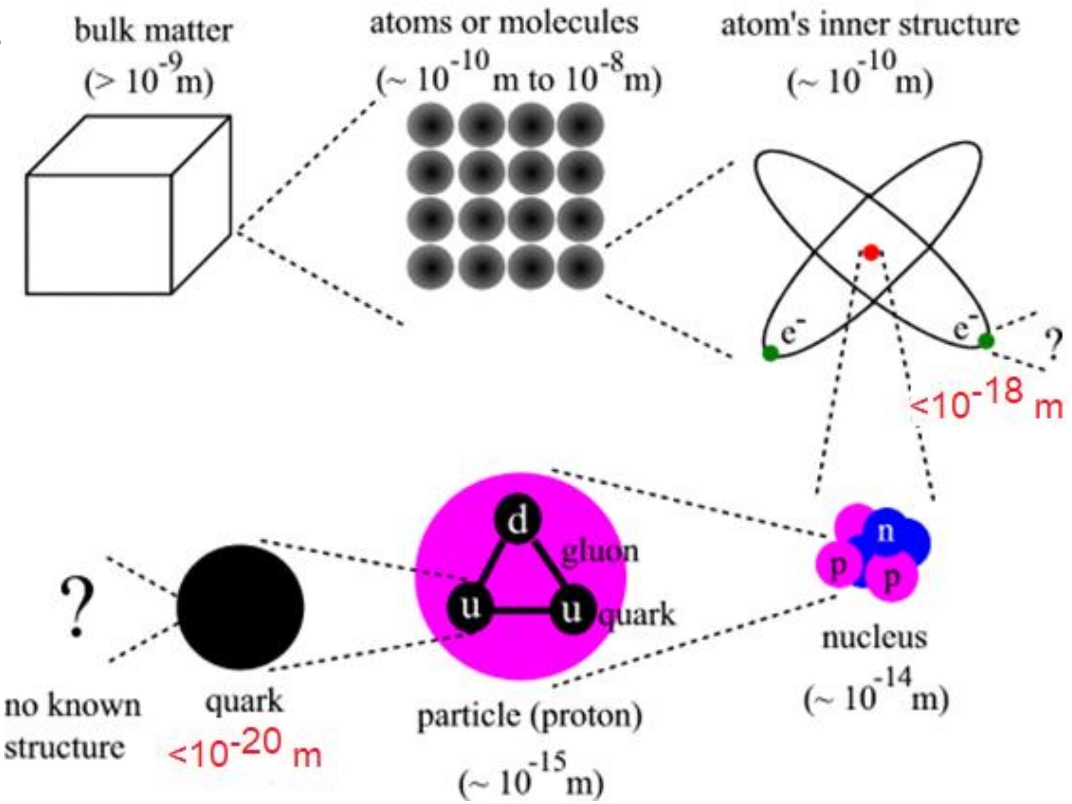
chemical elements and molecules
~10^{-9} m

atom = nucleus + electrons
~10^{-10} m

nucleus = protons + neutrons
~10^{-14} m

proton, neutron = Σ quarks
~10^{-15} m

quarks, leptons < 10^{-20} m
(structureless)



cfr. electron classical radius: $r_e = \dfrac{e^2}{4\pi\varepsilon_0 mc^2} = 2.8\cdot 10^{-15}\,\mathrm{m}$

Ordinary matter : Quark *up* + Quark *down* + Electron (+ Neutrino)

Is that all?

(the volume of an atom corresponds to > 10^{27} times the "volume" of an electron!)

Classically, matter contains a lot of void.

Quantum mechanically, the void is populated by virtual particles.

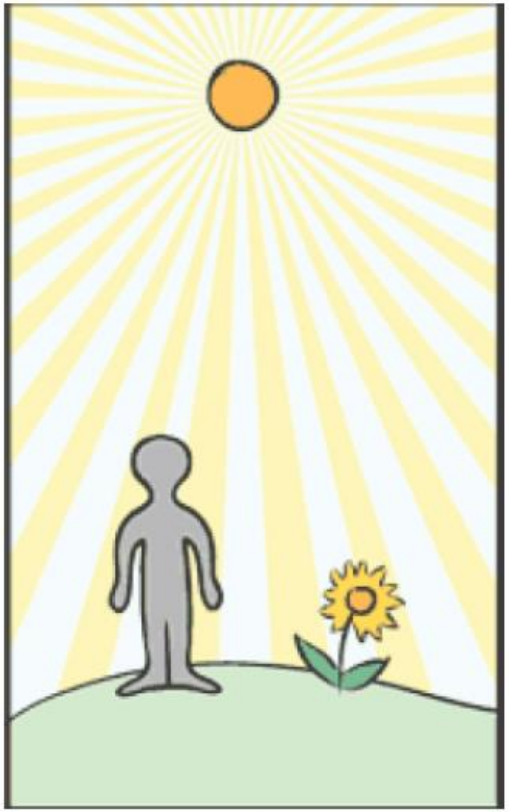
Q2: What are the Forces

SUN = gigantic source of energy

The very observation that the sun is shining for several millennia tells us that the source of energy cannot be classical (i.e. burning of oil).

Today we know that the Sun is more than 4.5 billion years old.

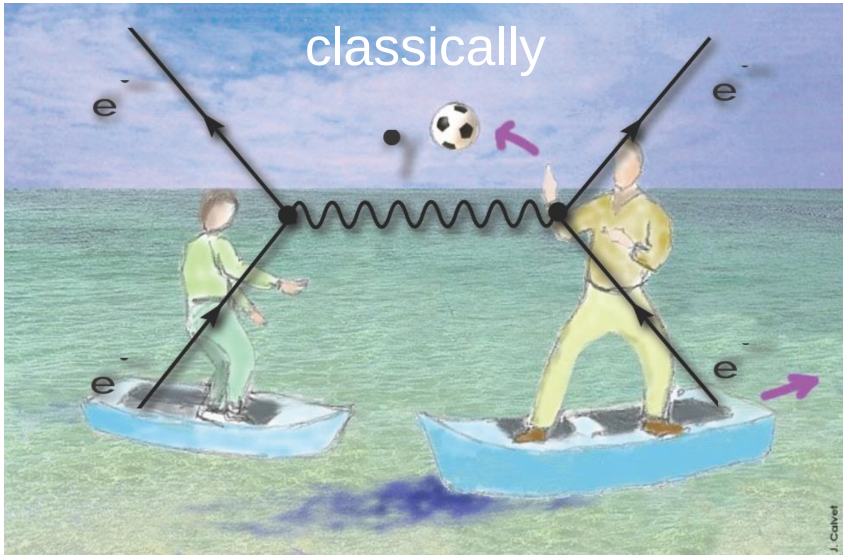
The Four Forces at Work



gravitation	keeps the sun together $R \sim \infty$
electromagnetic	energy transport from the core to the photosphere (several thousand years) light! $R \sim \infty$
strong	nuclear fusion $4p \rightarrow He + \#\gamma$ this is the source of sun's energy $R \sim 10^{-15}$ m
weak	responsible for $p \rightarrow n\, \nu\, e^+$ transitions $[p + p \rightarrow d + \nu + e^+]$ (slow process, otherwise the sun would have burned all of its "fuel" and stopped shining long time ago) $R \sim 10^{-17}$ m

Q3: How do the Particles Interact?

In QFT elementary particles interacts by the exchange of spin-1 gauge bosons.



Classically, the exchange of particles leads to a repulsive force.

The forces between particles result from the transfer of the momentum carried by the exchanged particles (no action at a distance).

The momentum of the exchanged quantum can be positive or negative:

- the force is attractive

+ the force is repulsive

The nature of the force is determined by the propagator:

$$-\frac{ig_{\mu\nu}}{q^2}$$ photon propagator

Plan of the Course

1) Introduction to the Standard Model

2) The Quark Model and the Hadron Spectrum (SU(3) flavor symmetry)

3) Introduction to QCD (QCD Lagrangian, running of α_S, qq scattering)

4) QCD Parton Model (scaling violations and QCD evolution equations)

5) $e^- e^+$ annihilation $\rightarrow$ hadrons ($e^- e^+ \rightarrow q\bar{q}$, $e^- e^+ \rightarrow q\bar{q} + g$, hadronization)

6) Hadron – Hadron Interactions (low energy, $q\bar{q} \rightarrow l\bar{l}$ (Drell-Yan), jet production, HF)

7) Phenomenology of Weak Interactions

8) Weak Decays (beta, μ, π, n)

9) νe scattering and νq scattering (Charged and Neutral Currents)

10a) quark mixing and CKM matrix

10b) Matter – Antimatter oscillations ($K^0 - \overline{K^0}$ and $B^0 - \overline{B^0}$ systems) and CP Violation

11) Electro-Weak Unification and Electro-Weak Interactions (W^+, W^-, Z^0)

12a) Spontaneous Electroweak Symmetry Breaking and the Higgs Mechanism

12b) The Higgs Boson

13) Neutrino Oscillations

References

F. Halzen and A. D. Martin (our main reference)
Quarks & Leptons

M. Thomson (our main reference)
Modern Particle Physics

G.Kane
Modern Elementary Particle Physics (2nd Ed.)

D. Griffiths
Introduction to Elementary Particles (2nd Rev. Ed.)

Recent developments and results
review articles in Annual Reviews of Nuclear and Particle Physics Science
reviews and articles posted on the archive http://arXiv.org

And on Quantum Field Theory
F. Mandel and G. Show
Quantum Field Theory (3rd Ed.)

C. Quigg
Gauge Theories of the Strong, Weak, and Electromagnetic Interactions (2nd Ed.)

I.J.R. Aitchison and A.J.G. Hey
Gauge Theories in Particle Physics (4th Ed.)

Lecture notes: http://dpnc.unige.ch/~bravar/PPA2

Evaluation

Written and Oral exam in June or September (must have passed PPA1)

Active participation in exercise sessions
homeworks!
You must return the exercises, at least 10 series, and solve at least 50% of problems

Halzen & Martin and Thomson textbooks are our main references for the course
(they are almost equivalent, Thomson is more recent)
For topics not covered in Halzen & Martin or Thomson,
we shall provide the reference material (lecture notes).

Evaluation of the course (you) via a questioner at the end of the semester.

During the semester one or two additional question times, if you ask for
(you ask questions, we try to answer them).

ASK QUESTIONS !!!!!

The Particles: Quarks and Leptons

nature is slightly more complicated (not only electrons and protons)!

there are 3 generations of pointlike spin ½ quarks and leptons (fermions)

quarks: $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}$ $\begin{pmatrix} t \\ b \end{pmatrix}$ Q = +2/3e, −1/3e

leptons: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$ $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$ 0e, −1e

each generation is composed of

2 types (flavors) of quarks and 2 types (flavors) of leptons

and their antiparticles

particles in the 2^{nd} and 3^{rd} generation are "copies" of particles in the 1^{st} generation differing only in mass

[missing the right-handed neutrinos]

each of 6 quarks come with 3 different colors (= strong charge)

each generation is heavier than the preceding one and

the particles of the 2^{nd} and 3^{rd} generation are unstable, they decay to the 1^{st} generation

fermions are described by the Dirac equation

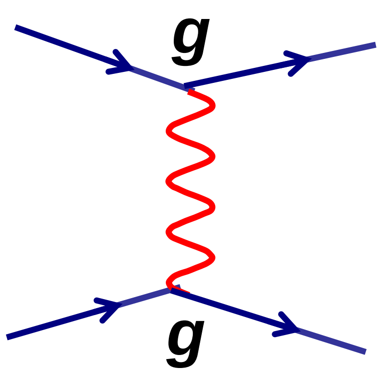
observed hadrons (baryons and mesons) are composite objects made of quarks

there are no composite states made of leptons

The Mediators

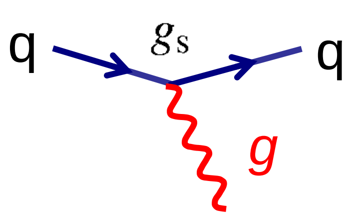
Interactions between fermions are mediated by the exchange of spin-1 Gauge Bosons

Force	Boson(s)	J^P	*m* [GeV]
EM (QED)	Photon γ	1^-	0
Weak	$W^\pm$ / Z	1^-	80 / 91
Strong (QCD)	8 Gluons *g*	1^-	0
Gravity	Graviton?	2^+	0



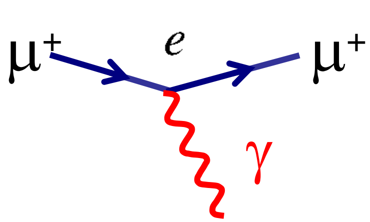
Interactions of gauge bosons with fermions described by SM vertices

STRONG



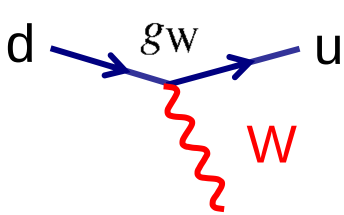
Only quarks
Never changes flavour

EM



All charged fermions
Never changes flavour

WEAK CC



All fermions
Always changes flavour

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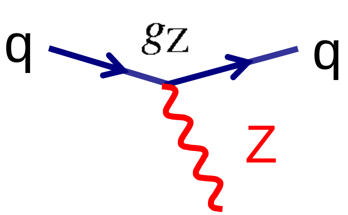
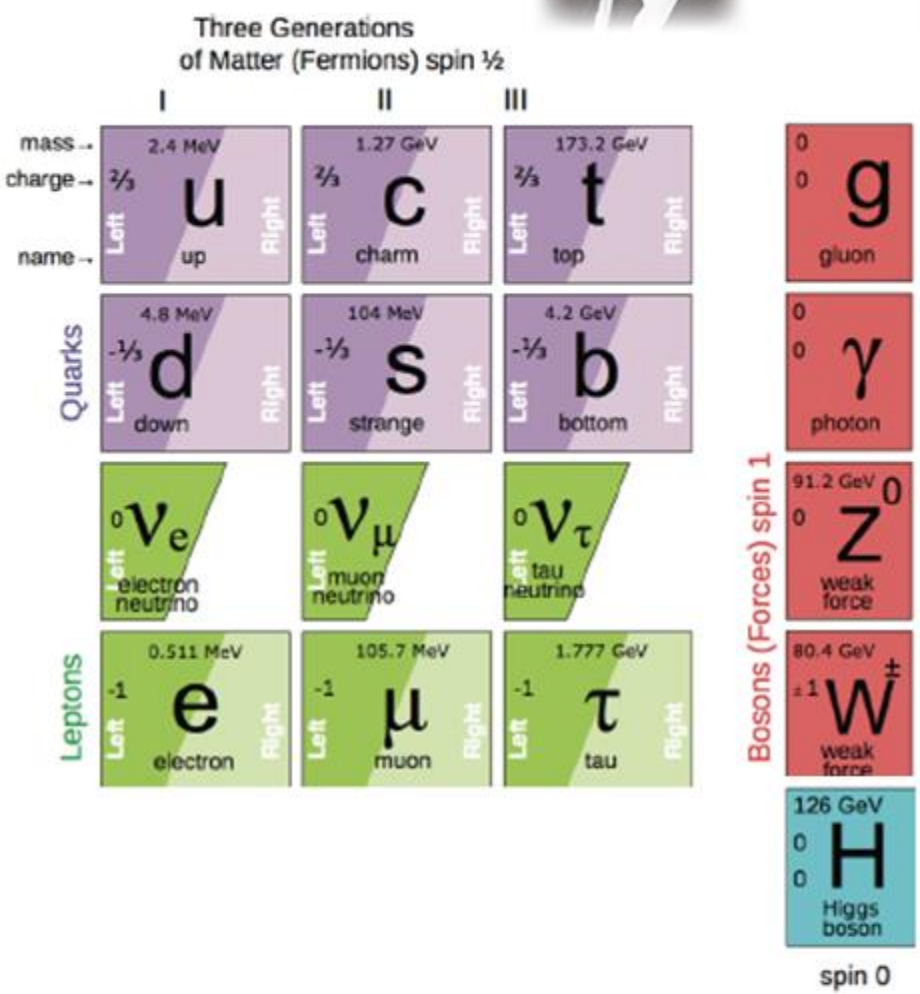


All fermions
Never changes flavour

$\alpha_s \sim 0.1$ $\alpha \sim 1/137$ $\alpha_W \sim 1/40$

The Standard Model (1973 → …)

1979



impressive agreement with data
(mathematically consistent theory of quarks and leptons and their interactions)

using 19 parameters the SM predicts the interactions of electro-weak and strong forces,
the properties of the 12 spin-1/2 fermions
and the 12 spin-1 gauge bosons
carrying the force between the fermions
1 spin-0 Higgs boson (origin of mass)

the SM describes all particle physics data
(almost) no deviations observed so far

Le Petit Prince

Nous écrivons
des choses éternelles.

Particle Physics Goals

1. identify the basic (structureless) constituents of matter
2. identify and understand the nature of forces acting between them
3. understand how they interact

"three" distinct type of particles:

quarks and leptons – spin ½ fermions
gauge bosons – spin 1 bosons, mediate interactions between quarks and leptons
Higgs boson – spin 0 boson, origin of mass

four interactions in Nature:

strong
electromagnetic
weak
gravitational

Symmetries play a central role in particle physics: our knowledge of forces stems from our understanding of the underlying symmetries and the way in which they are broken. One aim of particle physics is to discover the fundamental symmetries.

Link experimental (measurable) observables with calculable quantities:

scattering

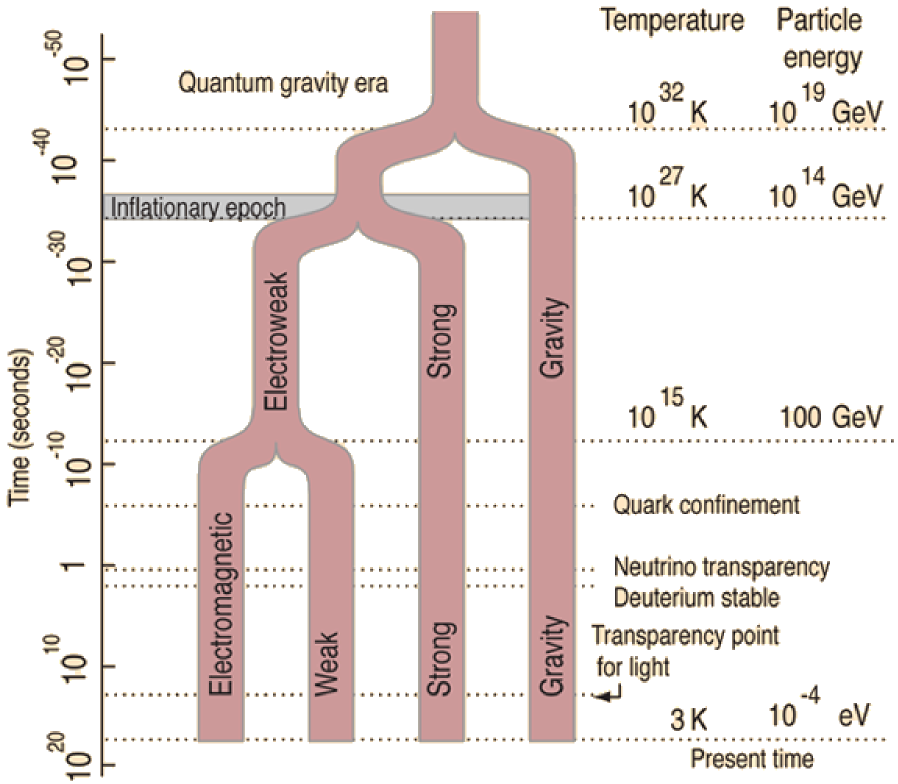
spectroscopy

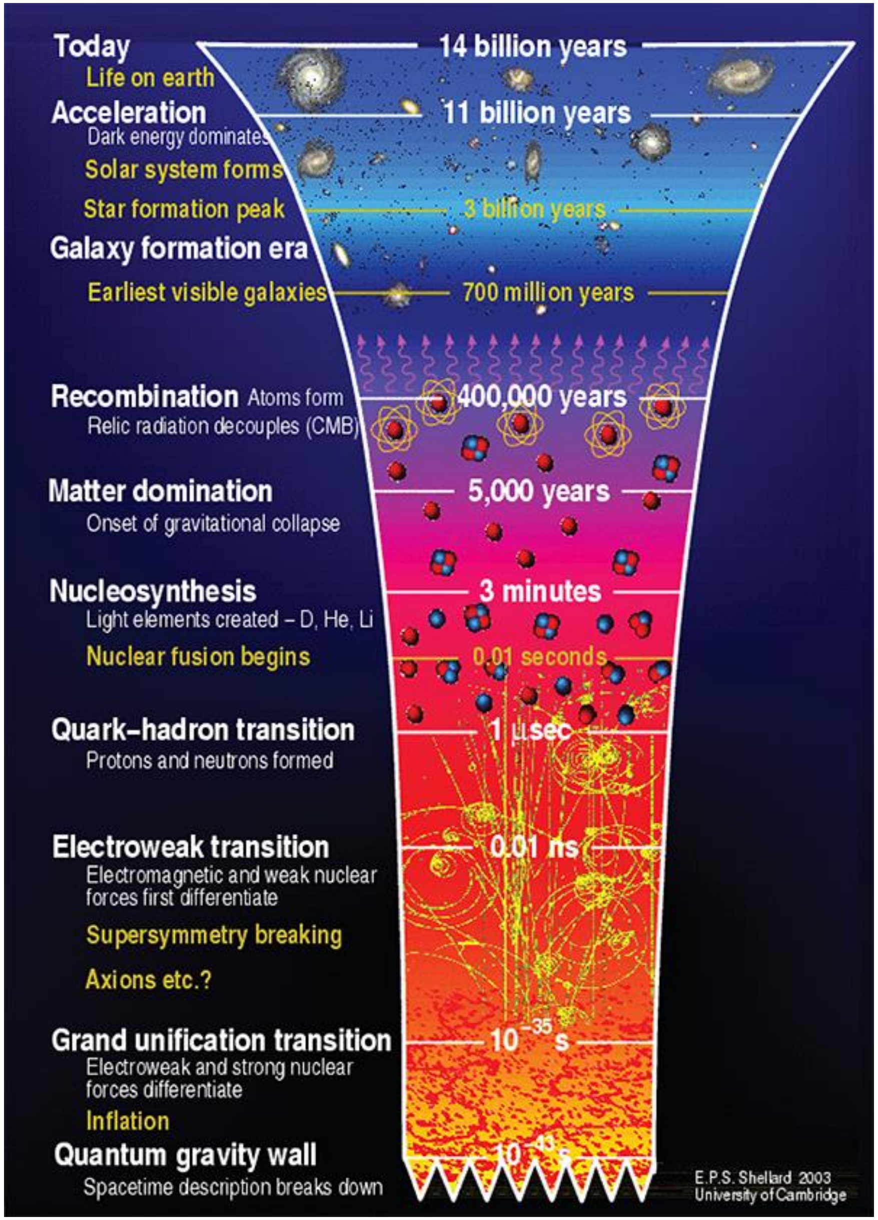
decays

History of the Universe

high energy ~ looking backward in time

energies we can reach in our laboratories
~ energy of the early Universe





Natural Units: $\hbar = c = 1$

S.I. units: kg m s A are a natural choice for "everyday" objects

not very natural in particle physics (1 barn = 10^{-28} m^2), instead use Natural Units

from quantum mechanics - unit of action $\hbar$

from relativity - speed of light c

from particle physics - unit of energy GeV

$$\tau_\mu = \frac{24(2\pi)^3 \hbar^7}{G_F^2 m_\mu^2 c^4}$$

all quantities expressed in powers of GeV

energy	kg m^2 / s^2	GeV	GeV
momentum	kg m / s	GeV / c	GeV
mass	kg	GeV / c^2	GeV
time	s	(GeV / ħ)$^{-1}$	GeV^{-1}
length	m	(GeV / ħ c)$^{-1}$	GeV^{-1}
area	m^2	(GeV / ħ c)$^{-2}$	GeV^{-2}

to simplify algebra set $\hbar = c = 1$ (loose the "tool" of dimensional cross-checks in calc.s)

to convert back to S.I. units, need to restore missing factors of c and $\hbar$

recall $\hbar \times c$ = 197.3 MeV fm (~ 200 MeV fm) $\rightarrow$ 1 fm ≈ 5 GeV^{-1}

Natural Units: Heaviside-Lorentz Units

Different units are used to describe charges and electro-magnetic interactions

	M K S A	Heaviside-Lorentz (rationalized c g s)	c g s
F	$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$	$\frac{1}{4\pi}\frac{q_1 q_2}{r^2}$	$\frac{q_1 q_2}{r^2}$
charge	1 C =	0.845×10^9 esu	2.997×10^9 esu
$\nabla\cdot E$	$\frac{1}{\varepsilon_0}\rho$	ρ	$4\pi\rho$
$\nabla\times E$	$-\frac{\partial B}{\partial t}$	$-\frac{1}{c}\frac{\partial B}{\partial t}$	$-\frac{1}{c}\frac{\partial B}{\partial t}$
$\nabla\cdot B$	0	0	0
$\nabla\times B$	$\mu_o j+\varepsilon_0\mu_o\frac{\partial E}{\partial t}$	$\frac{1}{c}j+\frac{1}{c}\frac{\partial E}{\partial t}$	$\frac{4\pi}{c}j+\frac{1}{c}\frac{\partial E}{\partial t}$
α	$\frac{e^2}{4\pi\varepsilon_0\hbar c}$	$\frac{e^2}{4\pi\hbar c}$	$\frac{e^2}{\hbar c}$

$\approx \frac{1}{137}$ dimensionless! same in all units

$c^2 = 1/\varepsilon_0\mu_0$

conversions (MKSA to cgs)

$\varepsilon_0 \rightarrow 1/4\pi$

$\mu_0 \rightarrow 4\pi/c^2$

$B \rightarrow 1/c\, B$

We will use (rationalized) Heaviside-Lorentz + Natural Units: $\hbar = c = \varepsilon_0 = \mu_0 = 1$

(the 4π appears in the Coulomb force and α rather than in the Maxwell equations)

Cosmological Units

Cosmological units or Planck units : $c = k_B = \hbar = G_N = 1$

In terms of G_N, ħ, c (Planck system) we obtain a unique length scale, mass, and time:

Planck length $$l_P = \left(\frac{G_N \hbar}{c^3}\right)^{1/2} = 1.62 \times 10^{-35}\,\text{m}$$

Planck mass $$M_P = \left(\frac{\hbar c}{G_N}\right)^{1/2} = 2.18 \times 10^{-8}\,\text{kg}$$

Planck time $$t_P = \left(\frac{G_N \hbar}{c^5}\right)^{1/2} = 5.39 \times 10^{-44}\,\text{s}$$

Planck energy $$E_P = M_P c^2 = 1.22 \times 10^{28}\,\text{eV}$$

Planck temperature $$T_P = E_P / k_B = 1.42 \times 10^{32}\,\text{K}$$

Fermi Golden Rule – Factors of 2π

Derived by Fermi to calculate decay rates in non-relativistic Quantum Mechanics

interaction rate per target particle

$$W_{fi} = \frac{2\pi}{\hbar}\overline{\left|T_{fi}\right|^2}\rho(E)$$

transition amplitude (pert. expansion)

$$T_{fi} = \langle f|V|i\rangle + \sum_{k\neq i}\frac{\langle f|V|k\rangle\langle k|V|i\rangle}{E_i - E_k} + \cdots$$

normalized wavefunction to 2 particles per unit volume ($2\,E_A$, $2\,E_1$, …)

Lorentz-invariant amplitude

$$M_{fi} = \left(2E_A \cdot 2E_1 \cdot 2E_2 \cdots 2E_n\right)^{1/2} T_{fi}$$

Lorentz-invariant phase space (LIPS)

$$\rho(E) = \frac{c\mathrm{d}^3 p_1}{(2\pi)^3\, 2E_1}\cdots\frac{c\mathrm{d}^3 p_n}{(2\pi)^3\, 2E_n}(2\pi)^4\,\delta^4\left(p_A - p_1 - \cdots - p_n\right)$$

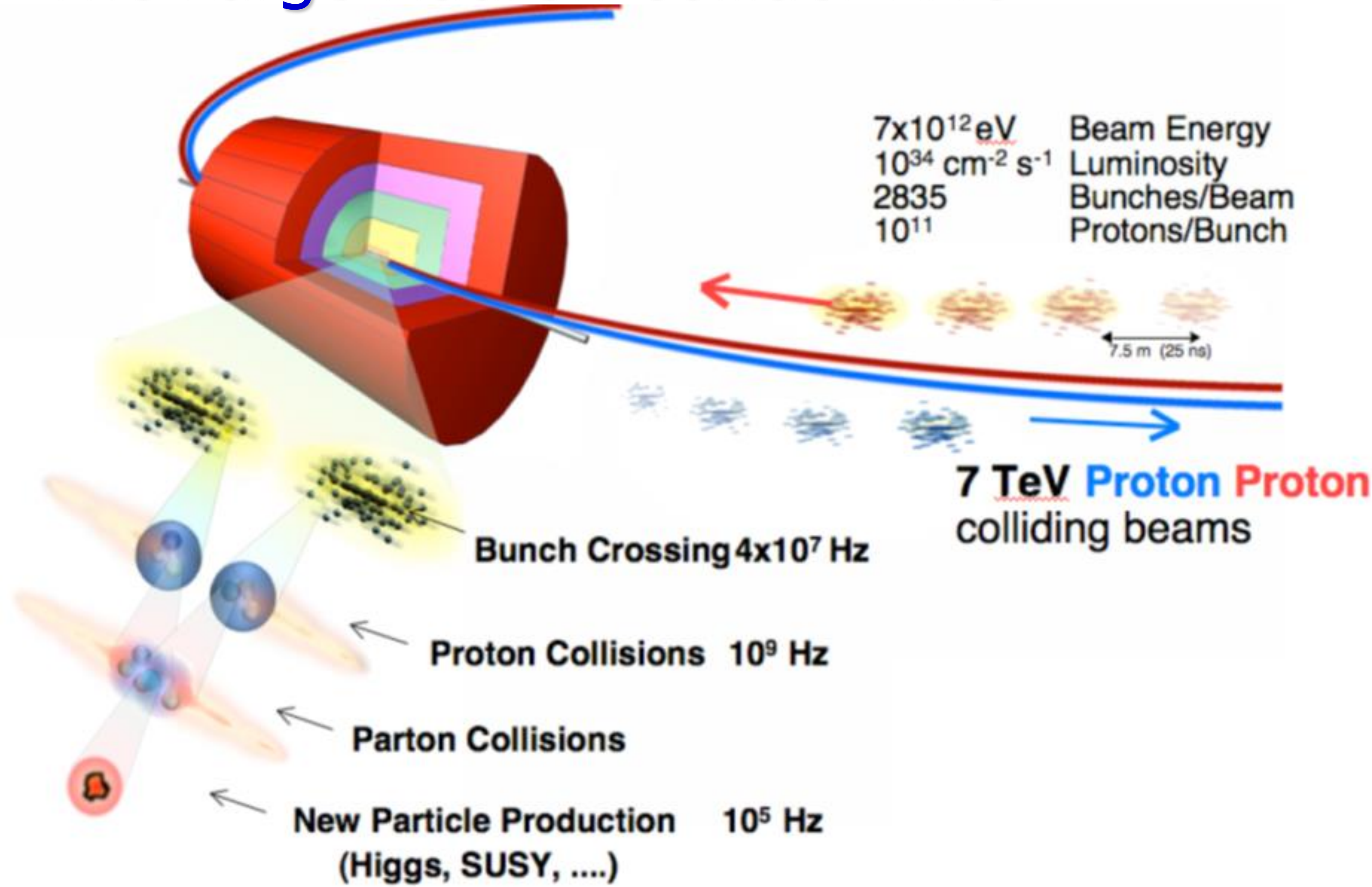
decay rate

$$\Gamma = \frac{1}{2\hbar E_A}\int\overline{\left|M_{fi}\right|^2}\frac{c\mathrm{d}^3 p_1}{(2\pi)^3\, 2E_1}\cdots\frac{c\mathrm{d}^3 p_n}{(2\pi)^3\, 2E_n}(2\pi)^4\,\delta^4\left(p_A - p_1 - \cdots - p_N\right)$$

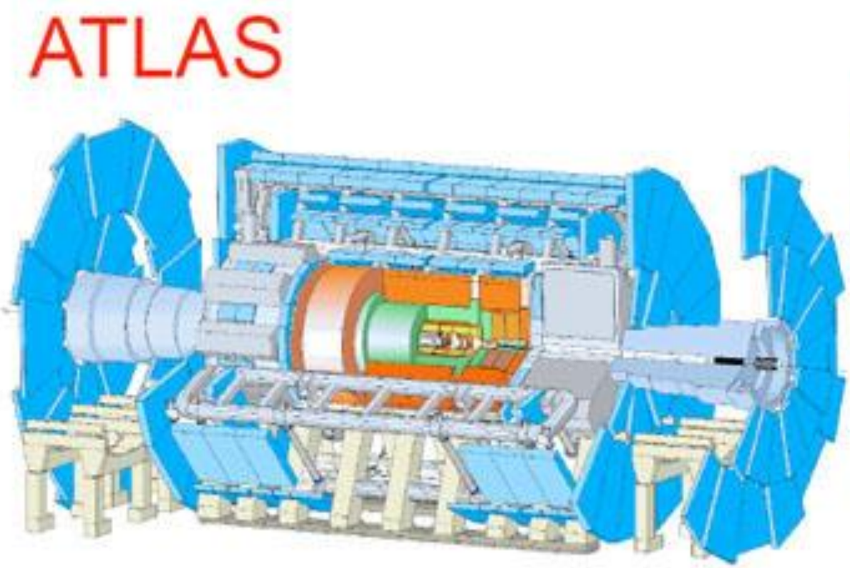
Decay $A \rightarrow 1 + 2$:

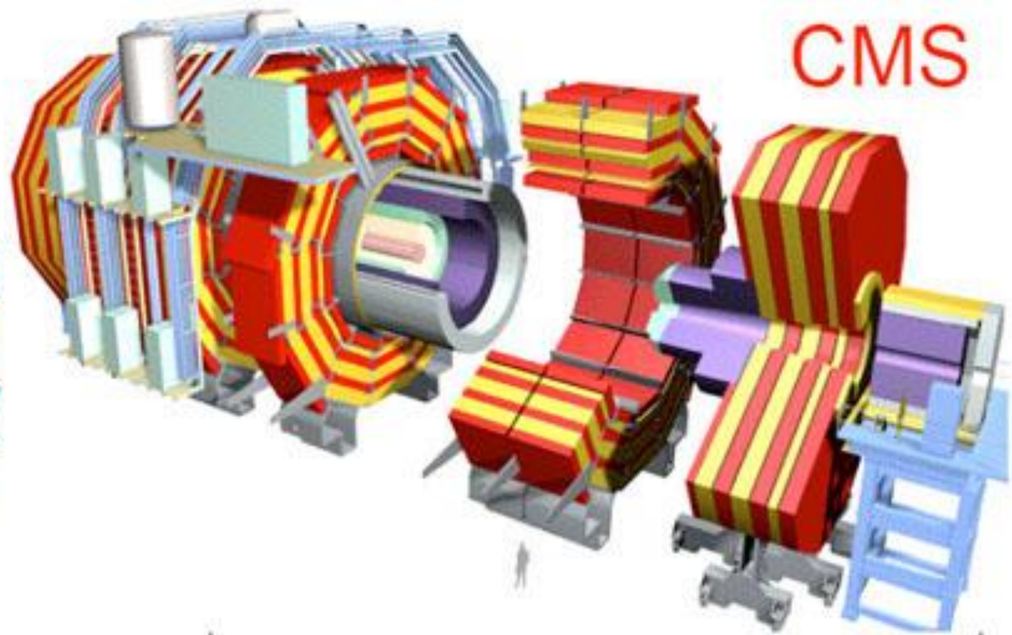
$$\Gamma(A\rightarrow 1+2) = \frac{p^*}{8\pi\hbar m_A^2 c}\int\overline{\left|M_{fi}\right|^2}\mathrm{d}(\cos\vartheta)$$

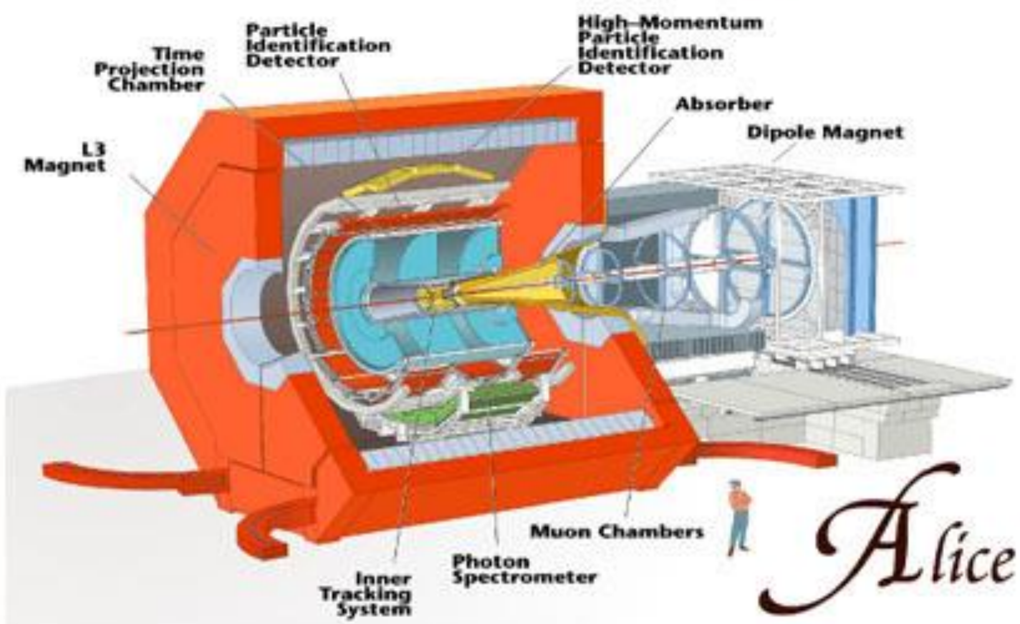
Energy Frontier:
The Large Hadron Collider LHC

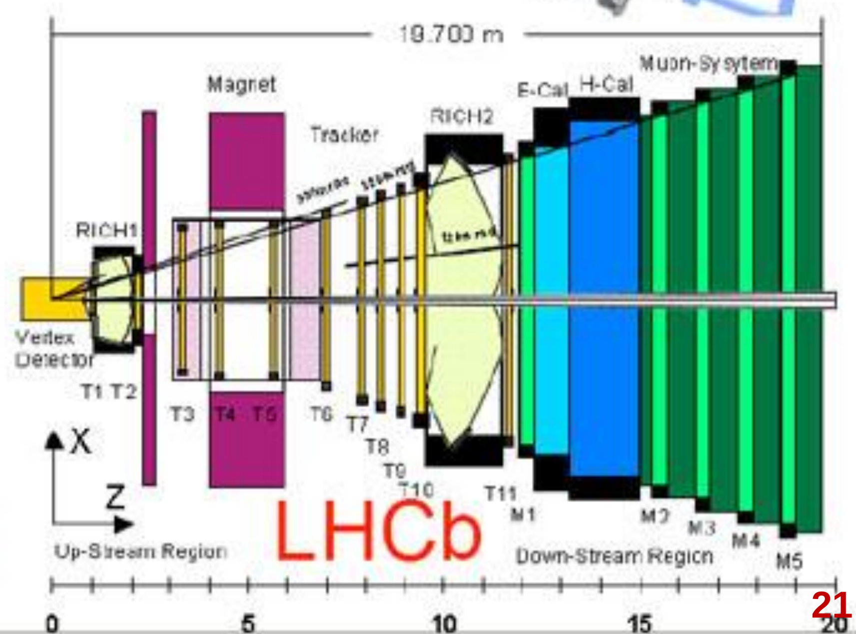


The Four Big Experiments at LHC

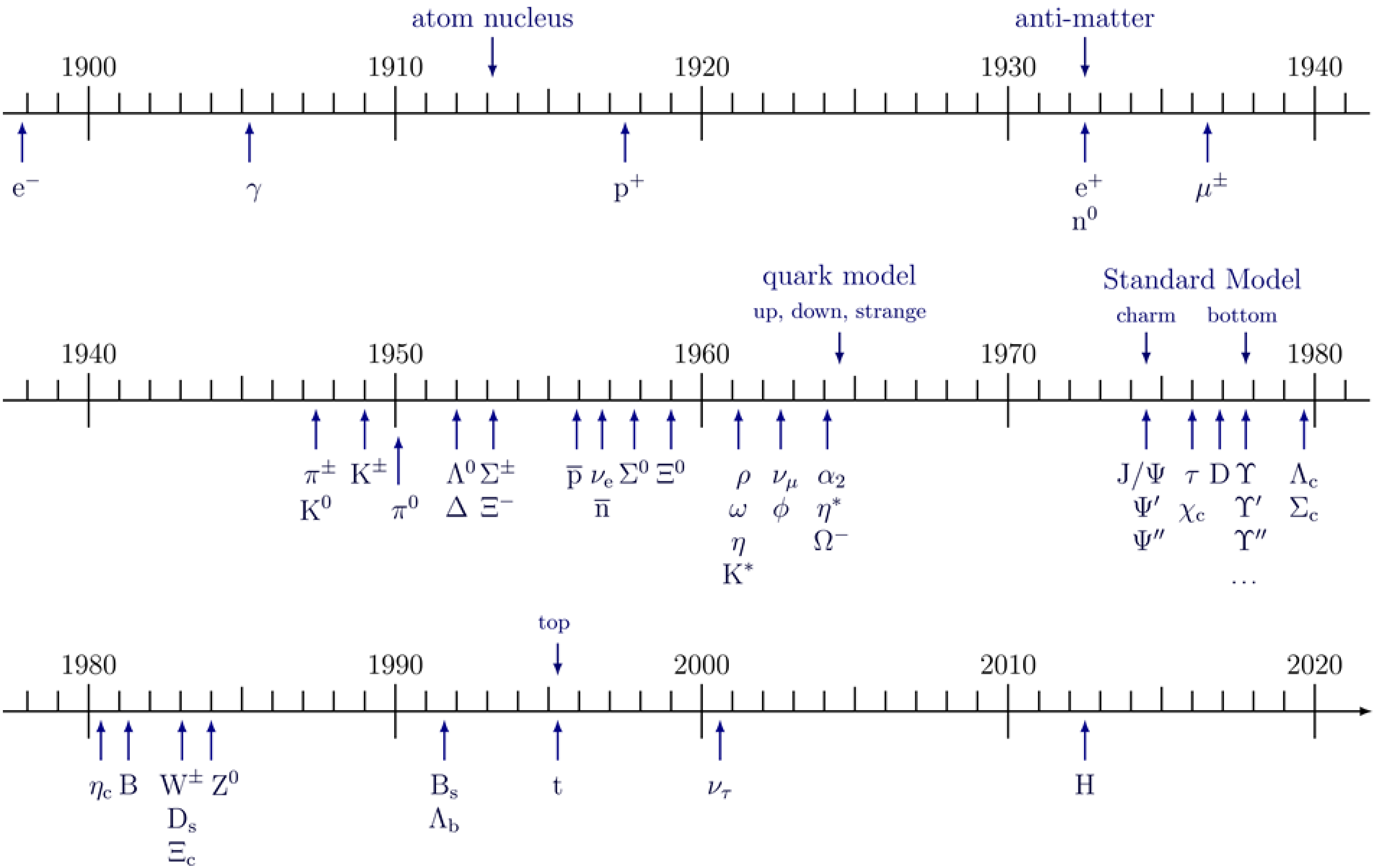








A Bit of History of Particle Discoveries



Relativistic Quantum Field Theory

The language that describes the interactions between elementary particles is Quantum Field Theory.

The quantization of fields allow for the possibility that the number of particles changes as in the creation or annihilation of electron – positron pairs, or in the weak decay of a neutron ($n \rightarrow p \, \overline{\nu} \, e$).

General framework to study fundamental interactions, which leads to a unified treatment of all interactions.

Relativistic QFT combines quantum theory
relativity
the concept of field

The quantization of any classical field introduces the quanta of the field, which are particles with well defined properties.

The electron and the positron themselves can be thought of as the quanta of an electron-positron field. That allows the number of particles to change, like the creation or annihilation of an electron-positron pairs.

The electrons are the source of yet another field – the electromagnetic field, whose quanta are yet other particles – the photon.

The interaction between an electron and a positron is mediated by the electromagnetic field (action at a distance, classical) or due to an exchange of photons (local, quantum).

These processes occur through the interaction of fields.
→ the solution of the equations of the quantized interacting fields is extremely difficult
→ if the interaction is sufficiently weak one can employ perturbation theory, like in electrodynamics (α ~1/137).

Major difficulty when calculating beyond leading order is the (almost unavoidable) appearance of UV divergences (e.g. in loop diagrams) → renormalization

Classically: the electron is an elementary constituent of matter and source of the electromagnetic field which carries energy

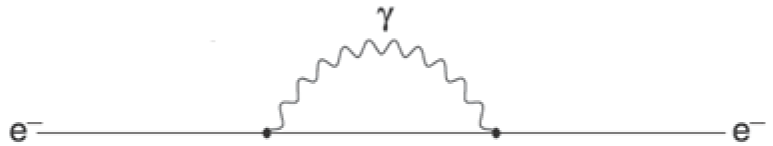
$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}$$

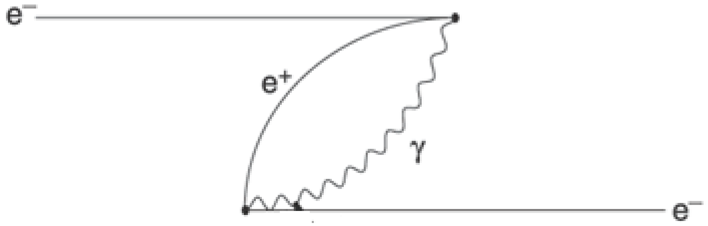
The field interacts back with the electron and contributes to its mass $\delta mc^2 = \Delta E_{\text{Coulomb}}$

$$\delta m \sim m_e \to r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \sim 10^{-13}\,\text{m}$$

electron classical radius

Quantum Electro-Dynamics: the electron emits and absorbs the quanta of the EM field





$$\Delta E = \frac{3\alpha}{4\pi} m_e c^2 \log\frac{\hbar}{r_e m_e c}$$

Lagrangians in Particle Physics

Formulate particle physics by giving the Lagrangian (density) $L = T - V = L_{free} + L_{\text{int}}$

The equations of motions follow from variational principles (Euler-Lagrange).
Example: spin-1/2 fermion of mass m (Dirac Lagrangian)

$$L = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \quad \rightarrow \quad \left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

Using QFT rules all observables can be calculated, i.e. the Lagrangian defines the theory.
The kinetic energy part describes the motion of free particles → L_{free} .
The potential energy part specifies the theory, i.e. the fundamental interactions of the theory (the forces) → L_{int} .

Why Lagrangians?

The Hamiltonian corresponds to a conserved quantity, the total energy, while the Lagrangian does not. Hamiltonians however are not Lorentz invariant.
The Lagrangian is a single real function that determines the dynamics,
and must be a scalar invariant under Lorentz transformations, since the action is invariant.
Lorentz invariance → all predictions of the theory are Lorentz invariant.

Symmetry transformations of the fields are readily expressed via the invariance of *L*.
If the Lagrangian is invariant under some transformation (more precisely the action),
then there is a corresponding conserved current (Noether's theorem).

Interactions – Yukawa Theory


1949

To describe interactions, construct the interaction Lagrangian L_{int}, a Lorentz scalar, which couples the field to its source (particle).

Electromagnetic interaction: couple the spinor field Ψ to the vector field A^μ :
$\bar{\psi}\gamma_\mu\psi$ is a 4-vector, and the scalar product $\lambda\bar{\psi}\gamma_\mu\psi A^\mu$ is a Lorentz scalar.

This is the simplest interaction Lagrangian that we can construct $L_{int} = \lambda\bar{\psi}\gamma_\mu\psi A^\mu$

To see how particles and fields are thought of, let's look at Yukawa theory.
Introduce an interaction term for the field ϕ coupled to a "source" ρ

$$L_{\text{int}} = -\phi\rho(\vec{x},t)$$

The wave equation for the field ϕ is modified to include the source term

$$\partial^\mu\partial_\mu\phi + m^2\phi = \rho(\vec{x},t)$$

By analogy to electrodynamics we think of ρ as the source of the field ϕ (~ charge dist.).
Let ρ be a static, pointlike charge at the origin

$$\rho(\vec{x},t) = g\delta(\vec{x})$$

then the wave equation reduces to $(-\nabla^2 + m^2)\phi = g\delta(\vec{x})$

which we solve in momentum space (via a Fourier transform).

Taking the Fourier transform

$$\phi(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \; e^{i\vec{k}\cdot\vec{x}} \tilde{\phi}(\vec{k})$$

(the inverse transformation is given by $\tilde{\phi}(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int d^3x \; e^{-i\vec{k}\cdot\vec{x}} \phi(\vec{x})$)

can rewrite the wave equation in momentum space $(\vec{k}^2 + m^2)\tilde{\phi}(\vec{k}) = g/(2\pi)^{3/2}$

and solve for $\tilde{\phi}(\vec{k})$

Then we can get back $\phi(\vec{x})$

$$\phi(\vec{x}) = \frac{g}{(2\pi)^3} \int d^3k \, \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2 + m^2}$$

Had we not taken a static source,
the denominator would have been $-k_0^2 + \vec{k}^2 + m^2 = m^2 - k^2$

Let's do the integral ($\vec{k}\cdot\vec{x} = kr\cos\vartheta$)

$$\int d^3k \, \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2 + m^2} = \int_0^\infty dk \, \frac{k^2}{k^2 + m^2} \int_0^{2\pi} d\varphi \int_{-1}^{+1} d(\cos\vartheta) e^{ikr\cos\vartheta} = \frac{\pi}{ir} \int_{-\infty}^{+\infty} dk^2 \, \frac{e^{ikr}}{k^2 + m^2}$$

which finally gives (residue at $k = im$)

$$\phi(|\vec{x}| = r) = \frac{g}{4\pi} \frac{e^{-mr}}{r}$$

Yukawa identified ϕ as the meson field with the nucleon as the source.
The effects of the field are transmitted by mesons (particles!) ⇒ strong interaction.
For a massive field, the force has a range of $r \sim 1/m$ (ϕ decreased exponentially).

To complete the picture, let's see how two nucleons interact by sensing its meson field.

The interaction Hamiltonian between two nucleons, the first generating the field ϕ and the second described by $\rho_2(\vec{x})$ is

$$H = -\int d^3x\ \phi(\vec{x})\rho_2(\vec{x})$$

To make it symmetrical, put back $\rho_1(\vec{x})$ in ϕ

$$\phi(\vec{x}) = \frac{1}{4\pi}\int d^3x'\ \rho_1(\vec{x}')\frac{e^{-m|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

then

$$H_{12} = -\frac{1}{4\pi}\int d^3x\ d^3x'\ \rho_1(\vec{x})\rho_2(\vec{x}')\frac{e^{-m|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

Finally, the potential can be written as $V(r) = -\frac{1}{4\pi}\frac{e^{-mr}}{r}$ (Yukawa potential)

This leads to the interpretation in QFT that
all interactions are due to the exchange of field quanta.

In momentum space the quantity representing the exchanged particle of mass *m* is

$$1/(k^2 - m^2)$$

which is referred to as the propagator.
The propagator gives the amplitude for a particle to *propagate* from point A to point B.


Le Petit Prince
Dessine-moi un mouton.

Feynman Calculus

1965

Relativistic quantum Field theory is a highly formal and sophisticated discipline.

In this course we will follow a heuristic approach (as in H&M), based on solutions of relativistic wave equations for free particles and treat the interaction as a perturbation, but in agreement with rigorous QFT calculations, i.e. short circuit the formalism and reach the calculational stage more quickly by emphasizing the relevant physics aspects.

Antiparticles are introduced following the Stückelberg – Feynman prescription: negative energy particle solutions going backward in time describe positive-energy antiparticles going forward in time ($\exp^{-i(-E)(-t)} = \exp^{iEt}$!).

$\Rightarrow$ Feynman diagrams

NB They are much more than simple pictorial representations of the fundamental phys. processes
They tell us how to calculate the interactions (physics)!

The Feynman rules allow a systematic diagrammatic representation of the terms in the perturbative expansion of the transition amplitude M_{fi} between an initial state *i* and final state *f*.

Their use is quite simple and intuitive in deriving important particle physics results.

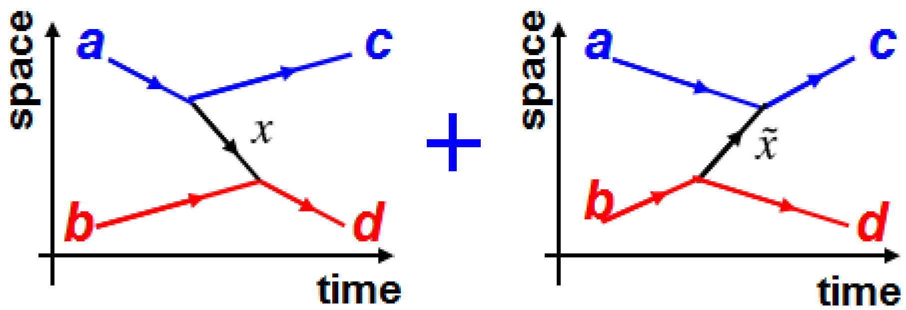
In Quantum Electro-Dynamics a complete agreement exists between theory and experiment to an incredibly high degree of accuracy.

QED is the prototype theory for all other interactions (strong and weak).

The Feynman Diagrams

The “language” we use to describe these processes are the Feynman diagrams

“Time-ordered QM”



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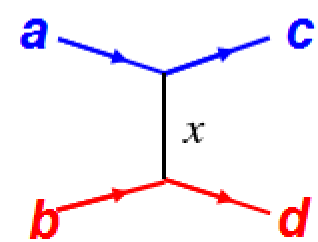
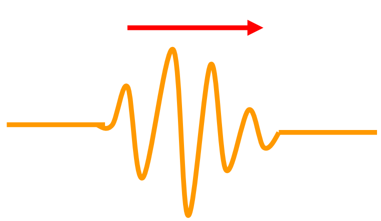
The sum over all possible time-orderings is represented by a Feynman diagram



momentum conserved at vertices
energy not conserved at vertices
⇒ exchanged particle “on mass shell”

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

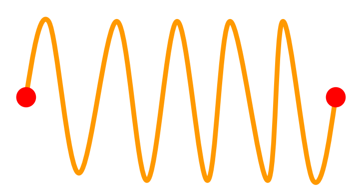
REAL PARTICLE



energy AND momentum are conserved at interaction vertices
⇒ exchanged particle (“propagator”) “off mass shell”

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

VIRTUAL PARTICLE



Feynman Rules for QED

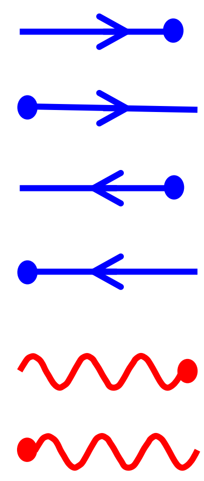
External Lines

spin 1/2

incoming particle	$u(p)$	
outgoing particle	$\bar{u}(p)$	
incoming antiparticle	$\bar{v}(p)$	
outgoing antiparticle	$v(p)$	

spin 1

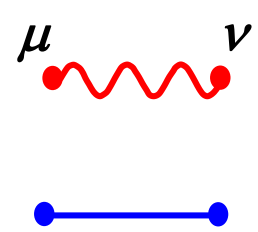
incoming photon	$\varepsilon^{\mu}(p)$	
outgoing photon	$\varepsilon^{\mu}(p)^{*}$	



Internal Lines (propagators)

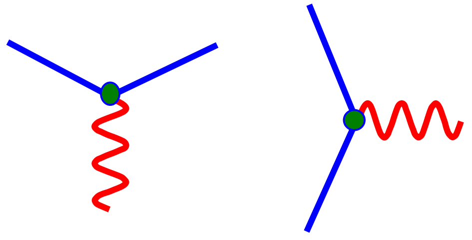
spin 1 photon $\quad -\dfrac{ig_{\mu\nu}}{q^2}$

spin 1/2 fermion $\quad \dfrac{i}{\left(\gamma^{\mu}q_{\mu}-m\right)}$



Vertex Factors

spin 1/2 fermion (el. charge -|e|) $\quad -ie\gamma^{\mu}$



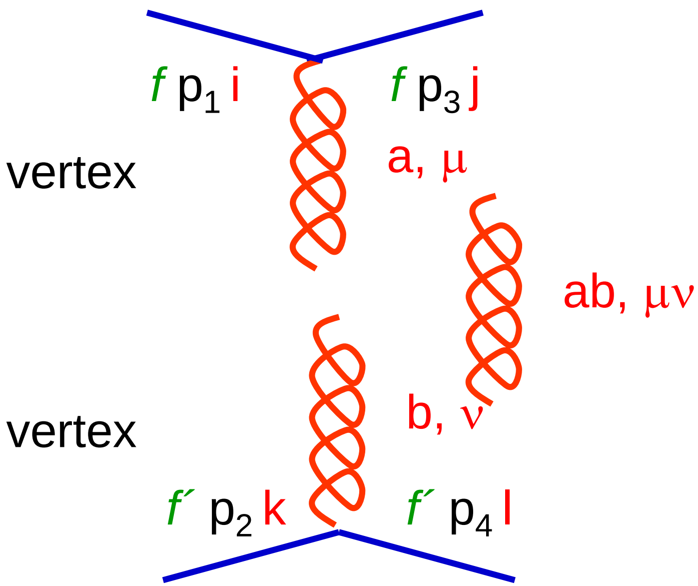
Matrix Element -iM = product of all factors

The Feynman diagrams can be obtained by combining several vertices.

At each vertex one has to verify the conservation laws arising from Lorentz invariance, internal symmetries, gauge invariance, … :

- conservation of four-momentum
- conservation of spin
- conservation of electric charge
- conservation of lepton and quark flavor (except for the weak charged currents)
- conservation of color charge

An example: $qq' \rightarrow qq'$ Scattering in QCD



gluon propagator

vertex factor

$$M_t = \left[\bar{u}_3^{\,j}\left(-ig_S\gamma^{\mu}\frac{\lambda^a_{ij}}{2}\right)u_1^i\right]\times \frac{-ig_{\mu\nu}\delta_{ab}}{t}\times \left[\bar{u}_4^{\,l}\left(-ig_S\gamma^{\nu}\frac{\lambda^b_{kl}}{2}\right)u_2^k\right]$$

f, f´ quark flavors (i.e. $ud \rightarrow ud$)

i, j, k, l quark colors

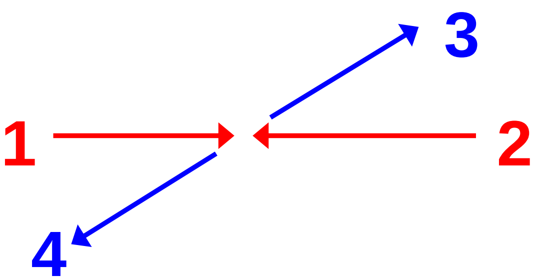
a, b, c gluon color combinations

the propagator imposes the same color δ_{ab} and same helicity $g_{\mu\nu}$ to the exchanged gluon at the interaction vertices

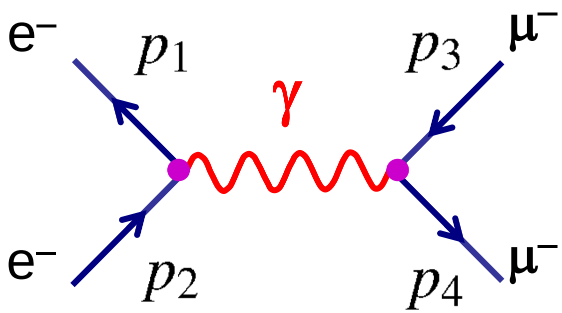
Mandelstam Variables s, t, u

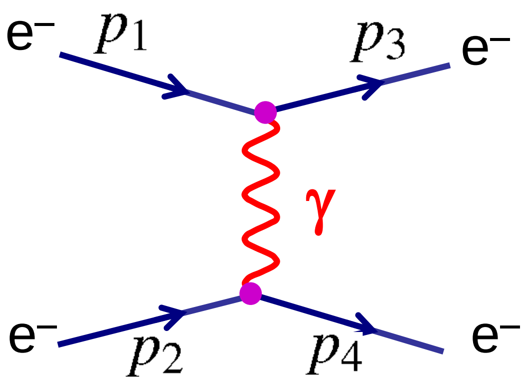
Particularly useful Lorentz invariant kinematical quantities

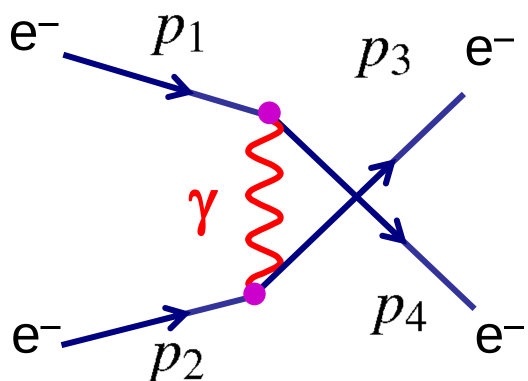
Consider the scattering process $1 + 2 \rightarrow 3 + 4$



The scattering processes (described via Feynman diagrams) can be categorized according to the four-momentum of the exchanged particles







Can define three kinematic variables from the four-momenta of interacting particles

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

equivalent to q^2 of the exchanged boson

scalar products of 2 four-vectors → Lorentz invariants

note $s = E_{CM}^2 = (E_1^* + E_2^*)^2$, $\quad s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

Gauge Invariance in Classical EM

The electromagnetic fields **E** and **B** can be expressed in terms of a potential $A^\mu = (A_0, \mathbf{A})$

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{E} = -\partial \mathbf{A} / \partial t - \nabla A_0$$

and the Maxwell equations follow from the field tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu} = 0 \quad \text{and} \quad \partial_\mu F^{\mu\nu} = -J^\nu \quad \Rightarrow \quad \partial_\mu J^\mu = 0$$

The potentials, however, are not unique, since a gauge transformation of the form

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x) = (\phi + \partial\Lambda / \partial t, \vec{A} - \nabla\Lambda)$$

leaves the Maxwell equations invariant ($\Lambda(x)$ is an arbitrary differentiable scalar field). Because of the gauge ambiguity, the potential A^μ , corresponding to particular **E** and **B** fields, is not uniquely defined, i.e. the potential contains "too much" information and it is not observable!

1. The electromagnetic current $\partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = -J^\nu$

however is conserved $\partial_\nu J^\nu = -\partial_\nu \partial_\mu F^{\mu\nu} = 0$

2. Each component of the vector potential satisfies a Klein-Gordon equation for massless particles $\partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu = \Box A^\nu = 0$

and can be identified with the photon field.

To understand the meaning of this gauge transformation, let's introduce a Lorentz invariant Lagrangian,

$$L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - J^{\mu} A_{\mu}$$

which will give us back the Maxwell equations via the Hamilton's principle (δS =0). Note that we have taken the potentials as the basic fields of the theory, not **E** and **B**.

Under a gauge transformation the action S acquires an additional term

$$\Delta S = -\int J_{\mu} \partial^{\mu} \Lambda \mathrm{d}^4 x = \int (\partial^{\mu} J_{\mu}) \Lambda \mathrm{d}^4 x$$

ΔS is zero for arbitrary Λ if, and only if $\partial_{\mu} J^{\mu} = \partial^{\mu} J_{\mu} = 0$

Thus the gauge invariance of the action requires, and follows from, the conservation of electric charge.

Phase Invariance in Quantum Mechanics

Suppose that we know the Schrödinger equation but not the laws of electrodynamics.
Can we guess Maxwell's equations from a gauge (symmetry) principle?
Yes! But …

QM observables are unchanged under global phase transformations of the wave function

$$\psi(x) \rightarrow \psi'(x) = e^{i\vartheta}\psi(x)$$

The absolute phase of the wave function cannot be measured and relative phases (like in interference experiments) are unaffected by this transformation.
Can we chose freely the phase in Geneva and Paris?
In other words, is QM invariant under local phase transformations?

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

Yes! But …

QM equations always involve derivatives

$$\partial_\mu\psi(x) \rightarrow \partial_\mu\psi'(x) = e^{i\alpha(x)}\left[\partial_\mu\psi(x) + i\left(\partial_\mu\alpha(x)\right)\psi(x)\right]$$

The additional term spoils the local phase invariance. Note that $\partial_\mu\alpha(x)$ is a vector!

Local phase invariance can be restored if the equations of motion and observables involving derivatives are modified by introducing a vector field A^{μ} (the EM field).

The gradient ∂_{μ} is replaced everywhere by the covariant derivative

$$\partial_{\mu} \to D_{\mu} = [\partial_{\mu} + iqA_{\mu}(x)]$$

such that also the D_{μ} transforms in the same was as Ψ

$$D_{\mu}\psi(x) \to D'_{\mu}\psi'(x) = e^{i\alpha(x)} D_{\mu}\psi(x)$$

Then quantities such as $\psi^{*}(x) D_{\mu}\psi(x)$ are invariant under local phase transformations.

Let's find out how the field A^{μ} transforms by writing out explicitly the various terms

$$D^{\mu\prime}\psi'(x) = (\partial^{\mu} + iqA^{\mu\prime}) e^{i\alpha(x)}\psi = e^{i\alpha(x)}(\partial^{\mu} + iqA^{\mu})\psi$$

and solve for A^{μ}

$$iqA^{\mu\prime} e^{i\alpha(x)}\psi = -\partial^{\mu}\left(e^{i\alpha(x)}\psi\right) + e^{i\alpha(x)}\partial^{\mu}\psi + iqe^{i\alpha(x)}A^{\mu}\psi = -\partial^{\mu}\left(e^{i\alpha(x)}\right)\psi + iqe^{i\alpha(x)}A^{\mu}\psi$$

Since each term acts on an arbitrary state Ψ, we can drop Ψ and

$$A^{\mu}(x) \to A^{\mu\prime}(x) = A^{\mu}(x) - \frac{1}{q}\partial^{\mu}\alpha(x)$$

We reestablished the invariance under local phase transformations at the price of introducing a vector field A^{μ} and a local interaction $\Psi^{*} qA^{\mu} \Psi$, that will be constructed to be electromagnetism.

The required transformation law for A^{μ} is precisely the same as in classical EM, i.e. up to a gradient of a scalar field, $\partial_{\mu}\alpha(x)$, and the covariant derivative corresponds to the minimal substitution $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$ of EM.

The form of the coupling between the EM field and matter is suggested by $D_{\mu}\Psi \to \Psi^{*} qA^{\mu}\Psi$.
We used a local gauge invariance as dynamical principle which led us to modify the equations of motion, i.e. we have built the interaction term D_{μ} and arrived at an interacting theory.

Note that Maxwell by imposing local charge conservation was led to modify Ampere's law by the addition of the displacement current dE/dt.

Aharonov-Bohm Effect

Electrodynamics is invariant under gauge transformations of the vector potential

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x) = (\phi + \partial\Lambda/\partial t, \vec{A} - \nabla\Lambda)$$

without affecting any physical laws,
which implies that the potential $A^\mu(x)$ is not a physical observable
(**E**, **B**, $F^{\mu\nu}$ are gauge invariant, A^μ is not, only potential differences are observable).

Are potentials physical or just calculational tools?
The vector potential does have a significance in quantum physics, as shown by Aharonov and Bohm (1959).

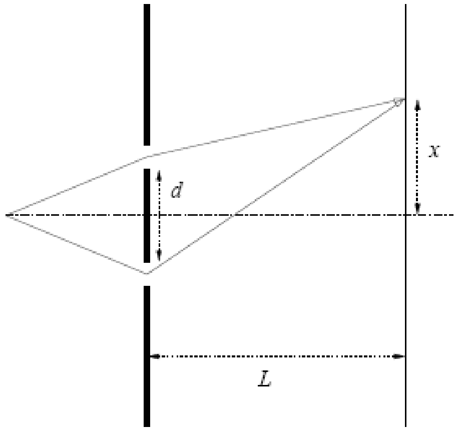
Let's imagine a two split experiment (i.e. split a coherent beam of charged particles in two parts), and let's observe the interference pattern on a far screen.

The wavefunction at a given point on the screen has the form

$$\psi \sim \psi_0 \left[1 + \exp\left(i\delta\varphi\right)\right]$$

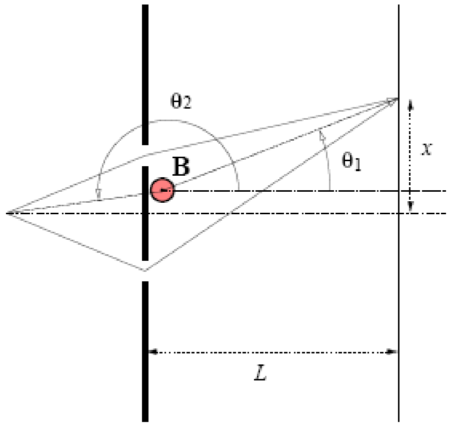
with

$$\delta\varphi = \frac{2\pi}{\lambda}(d_2 - d_1) = \frac{2\pi}{\lambda}\frac{xd}{L}$$



Young's experiment

Now introduce an infinite solenoid in the experiment.
There is no magnetic field outside of the solenoid (B = 0),
B is confined inside the solenoid, however **A** ≠ 0 everywhere



Aharonov-Bohm's experiment

$$\vec{A} = \begin{cases} \frac{1}{2} B r \hat{\varphi} & r < R \\ \frac{1}{2} B \frac{R^2}{r} \hat{\varphi} = \frac{\Phi_B}{2\pi r} \hat{\varphi} & r > R \end{cases}$$

What happens to a non-relativistic charged particle moving through a static vector potential that corresponds to a vanishing magnetic field?

If Ψ_0(**x**,t) is the solution of the Schrödinger equation for A = 0,
the solution of the Schrödinger equation in the presence of the vector potential **A**

$$\frac{\left(-i\hbar\nabla - q\mathbf{A}\right)^2}{2m} \psi(\vec{x},t) = i\hbar \frac{\partial \psi(\vec{x},t)}{\partial t}$$

is $\psi(\vec{x},t) = \psi_0(\vec{x},t) \exp(iS / \hbar)$ with $S = q\int d\vec{x} \cdot \mathbf{A}$

The phase shift experienced by the particle is the change in its classical action.
The fact that the new solution differs from the unperturbed one simply by a phase factor implies that there is no change in any physical result.

By analogy with the Young's experiment, the "perturbed" wave function is

$$\psi(\vec{x},t) = \psi_{0,1}(\vec{x},t)\exp(iS_1/\hbar) + \psi_{0,2}(\vec{x},t)\exp(iS_2/\hbar)$$

The phase difference at the screen between the two paths becomes

$$\exp\left(i\delta\varphi + iq\int_2 \vec{A}\cdot \mathrm{d}\vec{x} - iq\int_1 \vec{A}\cdot \mathrm{d}\vec{x}\right) = \exp\left(i\delta\varphi + iq\oint \vec{A}\cdot \mathrm{d}\vec{x}\right)$$

The interference of the two components of the recombined beam will depend on the phase difference

$$\frac{S_1 - S_2}{\hbar} = \frac{q}{\hbar}\oint \mathrm{d}\vec{x}\cdot \mathbf{A} = \frac{q}{\hbar}\Phi_B$$

because the two beams followed different paths through the potential **A**.

The result is gauge independent, since $\oint \vec{\nabla}\Lambda \cdot \mathrm{d}\vec{x} = 0$

Since it is not possible to eliminate **A** in the empty space outside of the solenoid with a gauge transformation, the phase shift $\Delta\varphi_B = q\Phi_B$ becomes observable.
The vector potential does induce a physical observable effect.
This implies that the link between the phase transformation of the electron wavefunction and the gauge degree of freedom of the electromagnetic field is fundamental and goes beyond the classical predictions.
The Aharonov-Bohm effect has been confirmed experimentally in 1986.

QED: Dirac + EM Fields

We start with the Lagrangian for a free Dirac field

$$L_0 = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$$

The EM field is introduced as in classical physics via the minimal substitution $p \rightarrow p - qA$:

$$\partial_\mu \rightarrow D_\mu = [\partial_\mu + iqA_\mu(x)]$$

where A_μ is the electromagnetic potential.
We assume that this substitution introduces correctly the EM field into the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = q\gamma^\mu A_\mu(x)\psi(x)$$

The resulting Lagrangian acquires an interaction term L_{int}

$$L = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) = L_0 - q\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x) = L_0 + L_{int}$$

The interaction term L_{int} couples the conserved current

$$j^\mu(x) = q\bar{\psi}(x)\gamma^\mu\psi(x)$$

to the electromagnetic field A_μ.
To complete the Lagrangian we add a term L_{rad} describing the radiation field

$$L_{rad} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

with $F_{\mu\nu}$ the EM energy-momentum tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Only the EM fields **E** and **B** have physical significance, not the potential A_μ itself, therefore the theory must be invariant under gauge transformations of the potentials

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$$

where $\alpha(x)$ is an arbitrary real scalar differentiable function.
Before quantum theory this step could be argued to be a mathematical reformulation of Maxwell classical EM theory with no physical consequences.
The resulting Lagrangian

$$L \to L' = L + \bar{\psi}(x)\gamma^\mu \psi(x)\partial_\mu \alpha(x)$$

however, is not invariant.
Invariance can be restored by demanding that the Dirac fields transform as

$$\psi(x) \to \psi'(x) = \psi(x)e^{-iq\alpha(x)}$$
$$\bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x)e^{+iq\alpha(x)}$$

i.e. undergo a local phase transformation.

We started by introducing the EM interactions in the simplest way $p \to p - qA$ and required that the resulting Lagrangian is invariant under gauge transformations of the EM potential A_μ. This requires the local phase invariance of the Dirac fields.
Now that we have identified a powerful invariance principle, we can proceed the other way by requiring that the Lagrangian is invariant under local phase transformations.

Gauge theory: any theory invariant under such coupled transformations.
QED is the simplest example of such theories.

Gauge Fields

Let's start by requiring the invariance of the free Lagrangian L_0

$$L_0 = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$$

under global phase transformations

$$\psi(x) \rightarrow \psi'(x) = \psi(x) e^{-i\alpha}$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x) e^{+i\alpha}$$

L_0 is invariant and this invariance ensures that current and charge are conserved:

$$j^\mu(x) = q\bar{\psi}(x)\gamma^\mu\psi(x) \qquad Q = q\int \mathrm{d}^3x\, \psi^\dagger(x)\psi(x)$$

Next we demand invariance under more general local phase transformations

$$\psi(x) \rightarrow \psi'(x) = \psi(x) e^{-iq\alpha(x)}$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x) e^{+iq\alpha(x)}$$

The resulting Lagrangian

$$L_0 \rightarrow L_0' = L_0 - q\bar{\psi}(x)\gamma^\mu\psi(x)\partial_\mu\alpha(x)$$

is not invariant (not a surprise!).

To restore the invariance of L_0 we add an interaction term L_{int} by associating matter fields to the gauge field A_μ, which must transforms according to (A_μ itself is not gauge invariant!)

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu\alpha(x)$$

The interaction between matter and gauge fields is introduced via the minimal substitution in the free Lagrangian L_0 by replacing the ordinary derivative with the covariant derivative

$$D_\mu \psi(x) = [\partial_\mu + iqA_\mu(x)]\psi(x)$$

The free Lagrangian transforms into

$$L = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) = L_0 - q\bar{\psi}(x)\gamma^\mu \psi(x) A_\mu(x) = L_0 + L_{int}$$

where L_{int} describes the interaction between the Dirac field and the gauge field A_μ, known also as minimal gauge interaction.

Note that also the covariant derivative transforms in the same way as the Dirac fields

$$D_\mu \psi(x) \rightarrow D'_\mu \psi'(x) = e^{-iq\alpha(x)} D_\mu \psi(x)$$

Hence the resulting Lagrangian is invariant.

To complete the whole we add a term L_{rad} describing the gauge field (for completeness, one would need to show that also this term is gauge invariant)

$$L_{QED} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) - q\bar{\psi}(x)\gamma^\mu \psi(x) A_\mu(x) - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x)$$

To summarize: by requiring local gauge invariance (phase invariance) of the Dirac fields, we are led to introduce a gauge field A_μ to preserve the invariance of the resulting Lagangian. By doing so we developed the full QED Lagrangian.

Can try to extend the gauge symmetry principle (local phase invariance) to other forces. . .

Generalization

Suppose we want to build a theory, which is invariant under some transformation *U*(x) (the transformation group U in general is non-abelian)

$$\psi'(x) = U(x)\psi(x)$$

We define the covariant derivative

$$D^\mu = \partial^\mu - igA^\mu(x)$$

and introduce the interacting vector field $A^\mu(x)$ to make the theory invariant.
g is the coupling constant to be determined from the experiment.
We want that the covariant derivative transforms in the same way as the spinor fields

$$D^{\mu\prime}\psi' = U(x)\left(D^\mu\psi\right)$$

Explicitly

$$\left(\partial^\mu - igA^{\mu\prime}\right)U\psi = U\left(\partial^\mu - igA^\mu\right)\psi$$

and solve for A^μ to obtain the transformation properties of the vector field $A^\mu(x)$

$$-igA^{\mu\prime}U\psi = -\partial^\mu\left(U\psi\right) + U\partial^\mu\psi - igA^\mu\psi = -\left(\partial^\mu U\right)\psi - igUA^\mu\psi$$

Since each term acts on an arbitrary state ψ (and *U* is not necessarily abelian)

$$A^{\mu\prime} = -\frac{i}{g}\left(\partial^\mu U\right)U^{-1} + UA^\mu U^{-1}$$

Gauge Theories

Is there a symmetry principle powerful enough to dictate the form or the interaction?

The form of the interaction in QED is known from classical theory of Maxwell et *al*.
There are no classical counterparts for the Strong and Weak interactions.
In general, guess a suitable form of the interaction and confront it with experiment (particle spectrum, known symmetries and conservation laws, cross-sections, decays, ...)

Quite generally, the form of interaction is restricted by requiring

- Lorentz invariance
- locality
  (interaction Lagrangian involves products of fields evaluated at same space-time point)
- renormalizability

QED is a gauge field theory and renormalizable theories are gauge field theories, i.e. possessing local phase invariance.
Elementary particle physics is almost exclusively concerned with such theories:
QCD and GWS are both gauge filed theories, remarkable generalizations of QED.

Strong interactions – quantum chromodynamics QCD
characterized by an apparently simple Lagrangian, but physical properties very difficult to deduce because of technical problems in formulating perturbation theory and the need of higher order corrections (α_S not so small)

ElectroWeak interactions – GSW model
very complicated Lagrangian, but easy to deal with in perturbation theory

Gauge Theories: QED and Yang-Mills

EM U(1) $\phi \to e^{i\alpha}\phi$ but $\partial_\mu \phi \to e^{i\alpha}(\partial_\mu \phi) + \underbrace{i(\partial_\mu \alpha)}\phi$

U(1) symmetry Lee group

≠0 if local transformations

EM field and covariant derivative $\partial_\mu \phi + ieA_\mu \phi \to e^{i\alpha}(\partial_\mu \phi + ieA_\mu \phi)$

if $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$

the EM field keep track of the phase in different points of the space-time

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

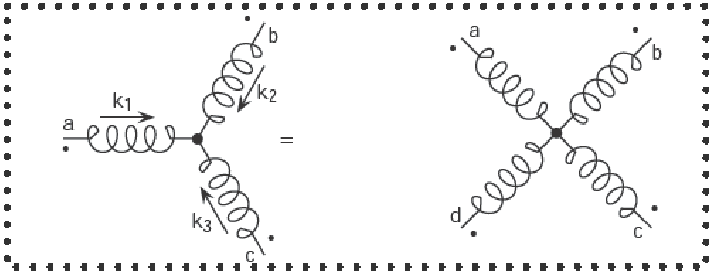
Yang-Mills : non-abelian transformations $\phi \to U\phi$

SU(2) and SU(3) symmetry Lee groups

$\partial_\mu \phi + igA_\mu \phi \to U(\partial_\mu \phi + igA_\mu \phi)$ if $A_\mu \to UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1}$

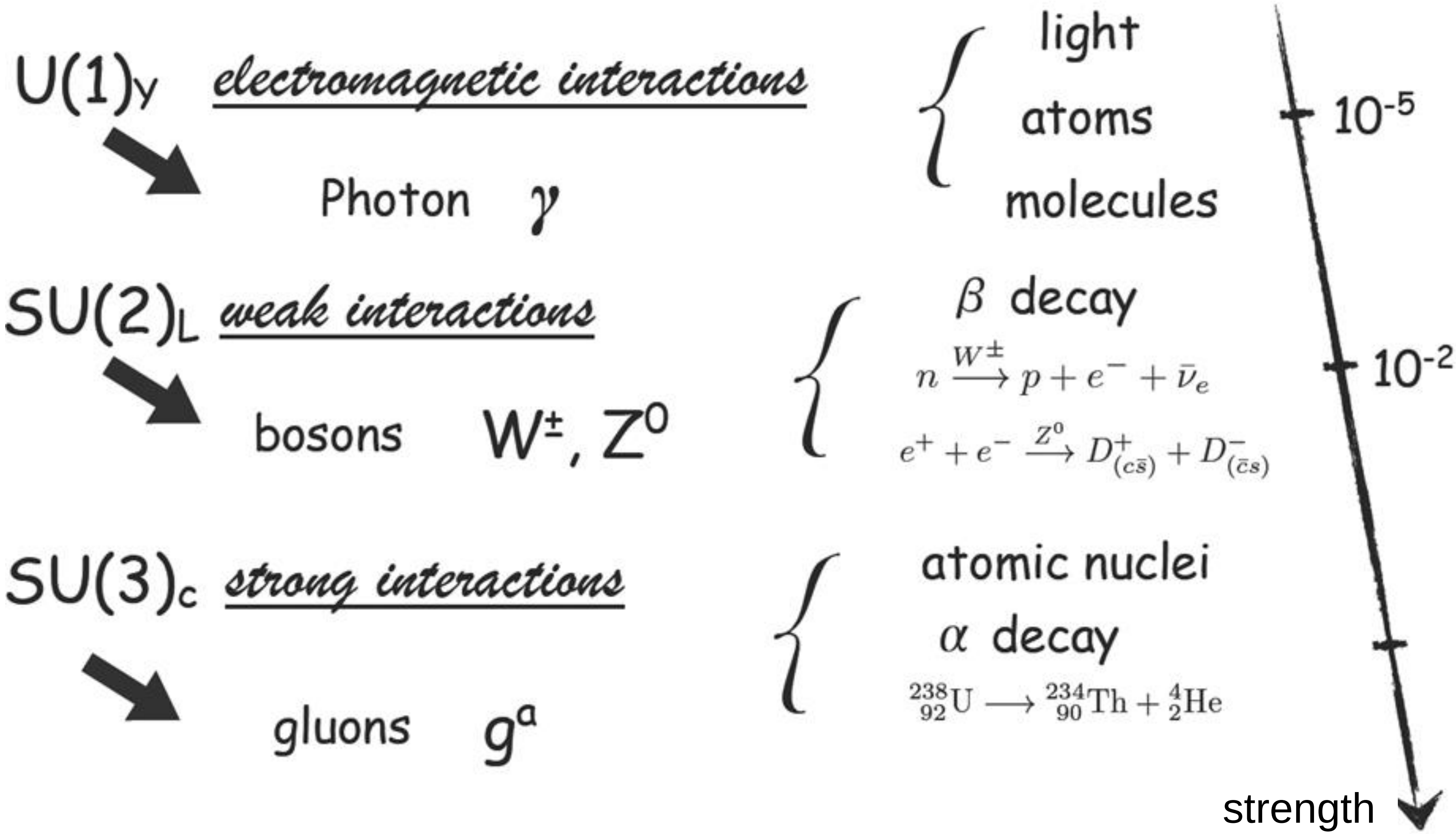
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{ig[A_\mu, A_\nu]}$$

non-abelian int.



The Standard Model Interactions

Our knowledge of these forces stems from our understanding of the underlying symmetries and the way in which they are broken

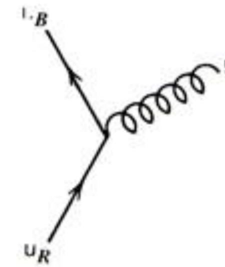
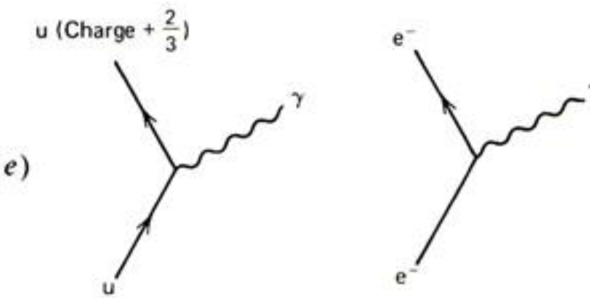
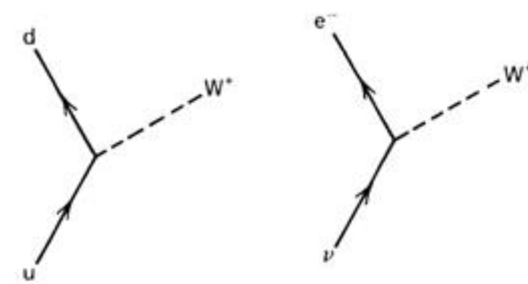


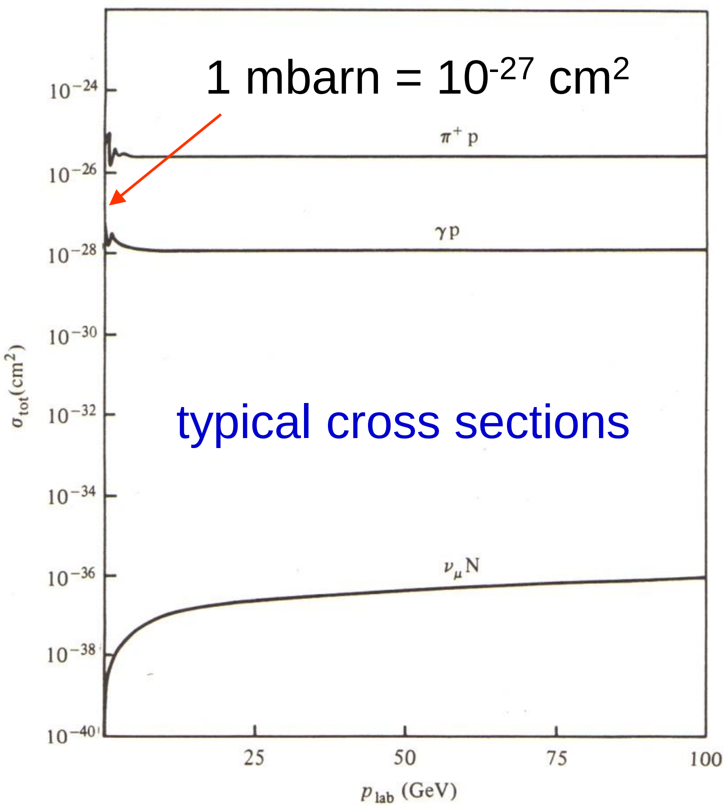
The Forces

Classification of interactions into a hierarchy of

- strong
- electromagnetic, and
- weak

is a convenient framework.

Interaction	Charge	Diagrams
Strong	Color	
Electromagnetic	Electric charge (e)	
Weak	Weak charge (g), giving $u \rightarrow d$ or $\nu \rightarrow e^-$ flavor-changing transitions	



$$\sigma_{strong} : \sigma_{EM} : \sigma_{weak} = 1 : 10^{-2} : 10^{-13}$$

A scattering or decay process can receive contributions from more than one of the Standard Model forces. See for instance the scattering process $e^+e^- \rightarrow \mu^+\mu^-$, which at low $\sqrt{s}$ is dominated by the electromagnetic interaction.
Weak effects are visible through interference effects only, while at the Z^0 pole, the scattering is dominated by the weak interaction (Z^0 exchange).

The classification is most meaningful if one of these interactions dominates.
As the energy increases the classification becomes less distinct
(running of the coupling constants).

p-p scattering at $\sqrt{s} \sim 10$ GeV is mediated by the strong interaction,
however at $\sqrt{s} \sim 10^{15}$ GeV this might not be the case.

π^0 decays electromagnetically into two photons
(no competing strong decay because the π^0 is the lightest hadron).

The weak processes become observable when both strong and electromagnetic decays are suppressed or forbidden (i.e. neutrino interactions).

It would be nice to have a single theory which describes all of the fundamental interactions in Nature (Grand Unified Theories, $\sqrt{s} \sim 10^{15}$ GeV).

Typical Scales

Interaction	Range (m)	Lifetime (sec)	Cross-section (mb)	Coupling strength
STRONG	10^{-15} (~proton radius)	10^{-23}	10	1
ELECTRO MAGNETIC	∞	10^{-20}-10^{-16}	10^{-3}	1 / 137
WEAK	$1 / M_W \sim 10^{-18}$	$10^{-12} – 10^{3}$ very wide range	10^{-11}	10^{-6}

	u	d	s	c	b	t
quark charges	+2/3	-1/3	-1/3	+2/3	-1/3	+2/3
quark masses (MeV)	3	5	100	1270	4190	172000

Particle Lifetimes

Large range in lifetimes, due to the intensity of the force responsible for the decay.
Weak decays : the density of final states (Q-value) determines the lifetime of a particle.

Decay examples :

strong int.

$\rho^0 \rightarrow \pi^+ \pi^-$

$\Delta^{++} \rightarrow p\, \pi^+$

EM int.

$\pi^0 \rightarrow \gamma\, \gamma$

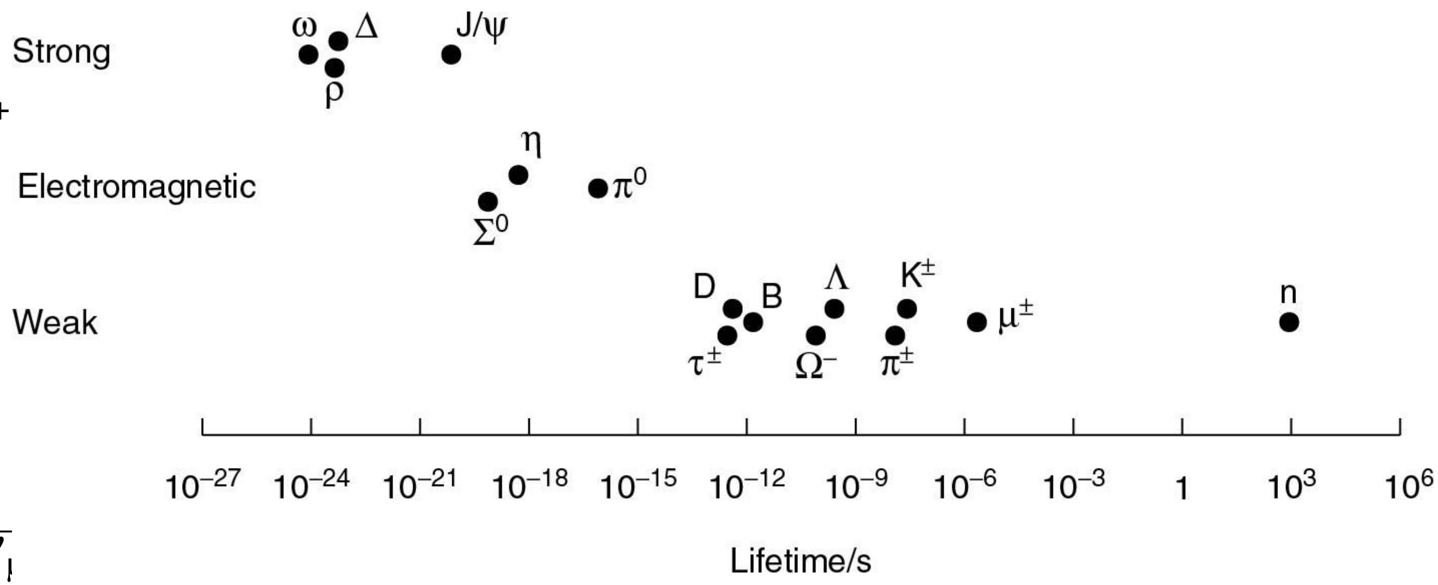
$\Sigma^0 \rightarrow \Lambda\, \gamma$

weak int.

$\pi^+ \rightarrow \mu + \overline{\nu}_\mu$

$\mu^- \rightarrow e^-\, \nu_e\, \nu_\mu$

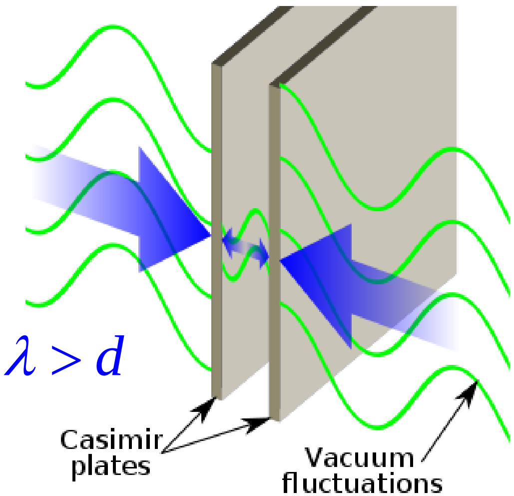
$K^+ \rightarrow \pi^+\, \pi^0$

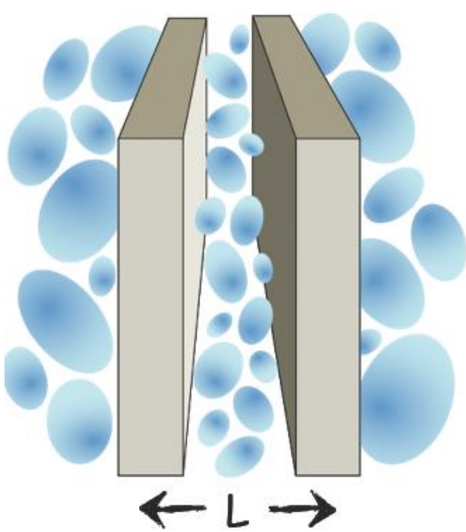


Weak decays become observable when both strong and electromagnetic decays are suppressed (i.e. $K^+ \rightarrow \pi^+\, \pi^0$, strong and EM forbidden, strangeness conservation !).

Casimir Effect (1948)

vacuum fluctuations





S » L^2 : no boundary effect

Casimir effect

attractive force between 2 conductive plates

ideal conductors

∞ electrical conductivity

no electric charge (the planes are “grounded”)

energy of virtual photons between plates

E ↓ when L ↓ → attractive force

Force per unit area

$$\frac{\mathrm{d}F}{\mathrm{d}S} = -\frac{\pi^2}{240}\frac{\hbar c}{L^4}$$

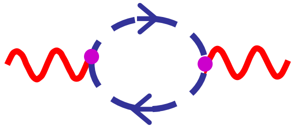
pressure of ~ 1 atm for 10 nm separation

The quantum vacuum is not empty

Divergences

vacuum polarization (i.e. loops)

$$iM =$$



$$\sim \int k \, \mathrm{d}k = \infty$$

What to do about divergences?
This loop is not by itself measurable. As long as we compute measurable quantities, the answer will be finite.
In practice it is impossible to compute physical observables all along (perturbation theory).
Deform the theory such to make the integrals finite with some regulating parameters, such that when all integrals are put together the result turns out to be independent of the regulator and the regulator disappears or it can be removed (renormalization).

Let's start with the simplest divergence, the one in the free Hamiltonian of a scalar field

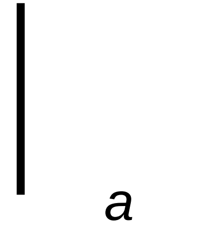
$$H = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \hbar \omega_k \left(a_k^\dagger a_k + \frac{1}{2} V \right)$$

The contribution to the vacuum energy of the field zero mode (zero-point energy) is

$$E = \langle 0 | H | 0 \rangle = \frac{1}{2} V \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \hbar \omega_k = \infty$$

While infinite, it is also not observable. Only energy differences matter and the absolute energy is unphysical (with the exception of the cosmological constant).

Consider the zero-point energy in a box of size *a*.

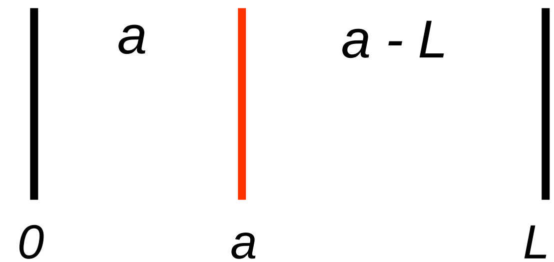


If the energy changes with separation *a*, we can calculate the force $F = -\mathrm{d}E / \mathrm{d}a$ acting on the walls of the box.

In this case we have a natural low-energy (IR) cutoff: $k > 1/a$, but not a high-energy (UV) cutoff which diverges at large *k*.

There is however a finite residual dependence on *a*, giving an observable force, because the boundary conditions of a system can affect the zero-point energy of the system
→ Casimir force (1948)

Let's add a second wall at *L* ($L \to \infty$).
The zero-point energy in the one-dimensional box *a* is



$$E(a) = \langle 0 | H | 0 \rangle = \frac{1}{2} \sum_{n=1}^{\infty} \hbar \omega_n = \frac{\pi}{2} \frac{\hbar c}{a} \sum_{n=1}^{\infty} n$$

and the total energy of the system (left side plus right side) is

$$E_{tot}(a) = E(a) + E(L-a) = \frac{\pi}{2} \hbar c \left(\frac{1}{a} + \frac{1}{L-a} \right) \sum_{n=1}^{\infty} n$$

The force acting on the wall is then given by the derivative

$$F(a) = -\frac{\mathrm{d}E_{tot}(a)}{\mathrm{d}a} = \frac{\pi}{2} \hbar c \left(\frac{1}{a^2} + \frac{1}{(L-a)^2} \right) \sum_{n=1}^{\infty} n \xrightarrow{L \to \infty} \infty$$

which is diverging … what are we missing?

The boundaries at *0*, *a*, and *L* impose the quantization of the electromagnetic waves due to interactions with the atoms in the wall. In reality, for the very high energy photons the walls become transparent (UV cutoff) and should be irrelevant since we are interested in the modes that affect the walls.

Here we have a natural UV cutoff. We can introduce a regulator in our calculations, for example an exponential attenuation of the modes into the walls with parameter Λ (heat-kernel regularization):

$$\exp\left(-\omega_n / \pi c\Lambda\right) = \exp(-n/\Lambda a) \qquad \Lambda a >> 1$$

Develop E(a)

$$E(a) = \frac{1}{2}\sum_{n=1}^{\infty}\hbar\omega_n e^{-\omega_n/\pi c\Lambda} = \frac{\pi}{2}\frac{\hbar c}{a}\sum_{n=1}^{\infty} n e^{-n/\Lambda a} = \frac{\pi}{2}\frac{\hbar c}{a}\sum_{n=1}^{\infty} n e^{-n\varepsilon}, \qquad \varepsilon = \frac{1}{\Lambda a} << 1$$

and note that

$$\sum_{n=1}^{\infty} n e^{-n\varepsilon} = -\frac{d}{d\varepsilon}\sum_{n=1}^{\infty} e^{-n\varepsilon} = -\frac{d}{d\varepsilon}\left[\frac{1}{(1-e^{-\varepsilon})} - 1\right] = \frac{e^{-\varepsilon}}{(1-e^{-\varepsilon})^2} = \frac{1}{\varepsilon^2} - \frac{1}{12} + \frac{\varepsilon^2}{240} + O(\varepsilon^4)$$

which gives us

$$E(a) = \frac{\pi}{2}\frac{\hbar c}{a}\left[\Lambda^2 a^2 - \frac{1}{12} + \frac{1}{240\Lambda^2 a^2} + \ldots\right]$$

Finally, the Casimir force is

$$F(a) = -\frac{d}{da}\left[E(a) + E(L-a)\right] = -\frac{\pi}{2}\hbar c\frac{d}{da}\left[\Lambda^2 L - \frac{1}{12}\left(\frac{1}{L-a} + \frac{1}{a}\right) + \ldots\right]$$

$$= \frac{\pi}{24}\hbar c\left(\frac{1}{(L-a)^2} - \frac{1}{a^2}\right) + \ldots \xrightarrow[L\to\infty]{} -\frac{\pi}{24}\frac{\hbar c}{a^2}$$

This is a finite result and depends only on *a*; the regulator Λ has disappeared!

Casimir Force

The result for the force acting on the wall is non-zero and finite

$$F(a) = -\frac{\pi}{24}\frac{\hbar c}{a^2}$$

This is an attractive force. It is purely quantum mechanical in origin (it is proportional to ħ).
The result is independent of the particular regulator used and cutoff Λ !

The Casimir force is independent of any regulator.

The Casimir force is an infrared effect.

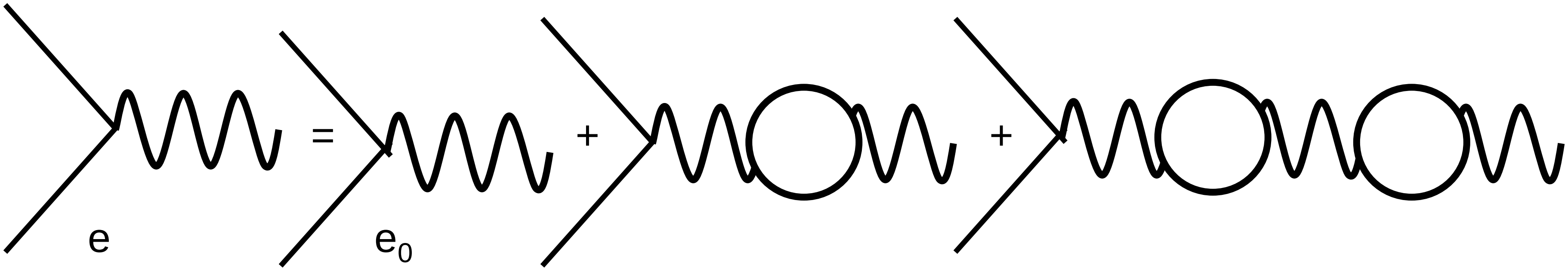
(for a fermion field, the force changes sign)

In three dimensions, accounting for the two photon polarizations the Casimir Force is

$$F(a) = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}A$$

where *A* is the area of the walls of the box.
Although predicted by Casimir in 1948, the force has not been conclusively observed until 1997.

Renormalization in QED



physical or effective charge

bare charge

bare charge screened by e^+e^- loops

$$\alpha\left(Q^2\right)=\alpha_0\left\{1+\frac{\alpha_0}{3\pi}\log\frac{Q^2}{M^2}+\frac{1}{2}\left(\frac{\alpha_0}{3\pi}\log\frac{Q^2}{M^2}\right)^2+\ldots\right\}=\frac{\alpha_0}{1-\frac{\alpha_0}{3\pi}\log\frac{Q^2}{\mu^2}}$$

large Q^2 leading log sum (M cutoff on loop momentum)

Subtract

$$\frac{1}{\alpha\left(Q^2\right)}=\frac{1}{\alpha_0}-\frac{1}{3\pi}\log\frac{Q^2}{M^2}+\cdots$$

$$\frac{1}{\alpha\left(\mu^2\right)}=\frac{1}{\alpha_0}-\frac{1}{3\pi}\log\frac{\mu^2}{M^2}+\cdots$$

$$\frac{1}{\alpha\left(Q^2\right)}-\frac{1}{\alpha\left(\mu^2\right)}=-\frac{1}{3\pi}\left(\log\frac{Q^2}{M^2}-\log\frac{\mu^2}{M^2}\right)=-\frac{1}{3\pi}\log\frac{Q^2}{\mu^2}$$

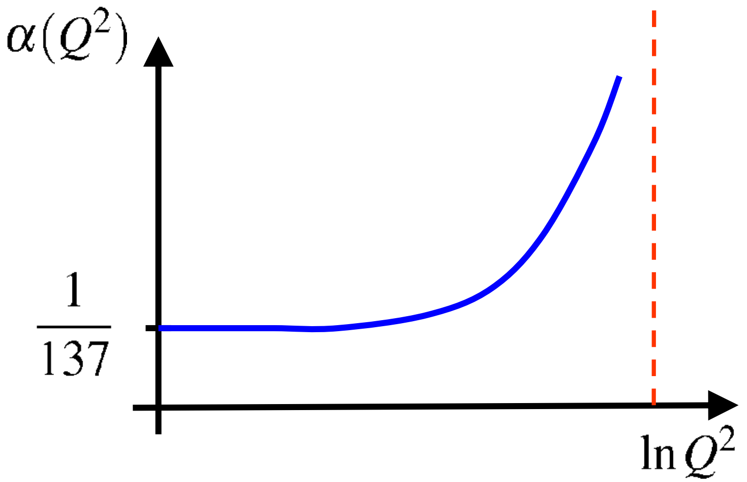
running coupling constant

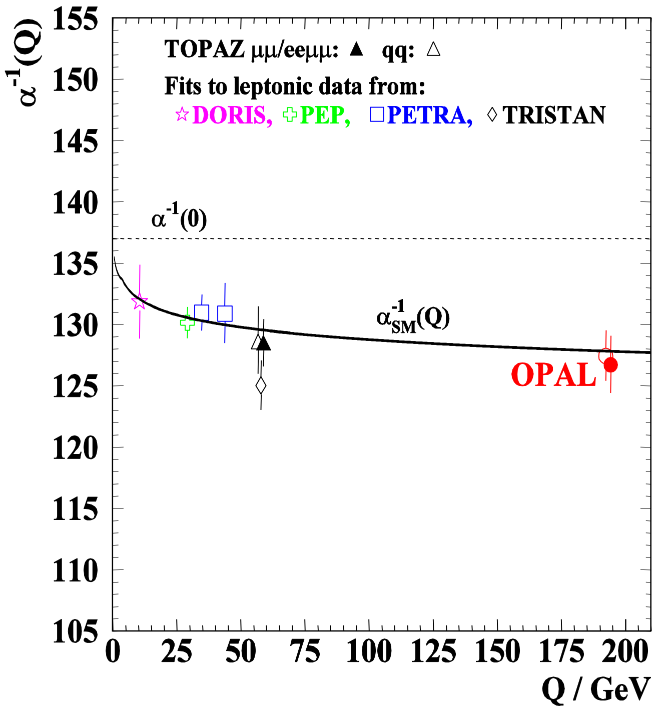
$$\alpha\left(Q^2\right)=\frac{\alpha\left(\mu^2\right)}{1-\frac{\alpha\left(\mu^2\right)}{3\pi}\log\frac{Q^2}{\mu^2}}$$

infinities removed at the price of introducing the renormalization scale μ^2

$\alpha(\mu^2 \to 0$ i.e. $4m_e^2)$ = measured = 1/137

Running of α_{EM}





Might worry that coupling becomes infinite at

$$\log\left(\frac{Q^2}{Q_0^2}\right) = \frac{3\pi}{1/137}$$

i.e. at $Q^2 \sim 10^{52}$ GeV^2

But quantum gravity effects would come in way below this energy scale and it is highly unlikely that QED "as it is" would be valid in this regime.

In QED, running coupling increases very slowly

atomic physics ($Q^2 \sim 0$):

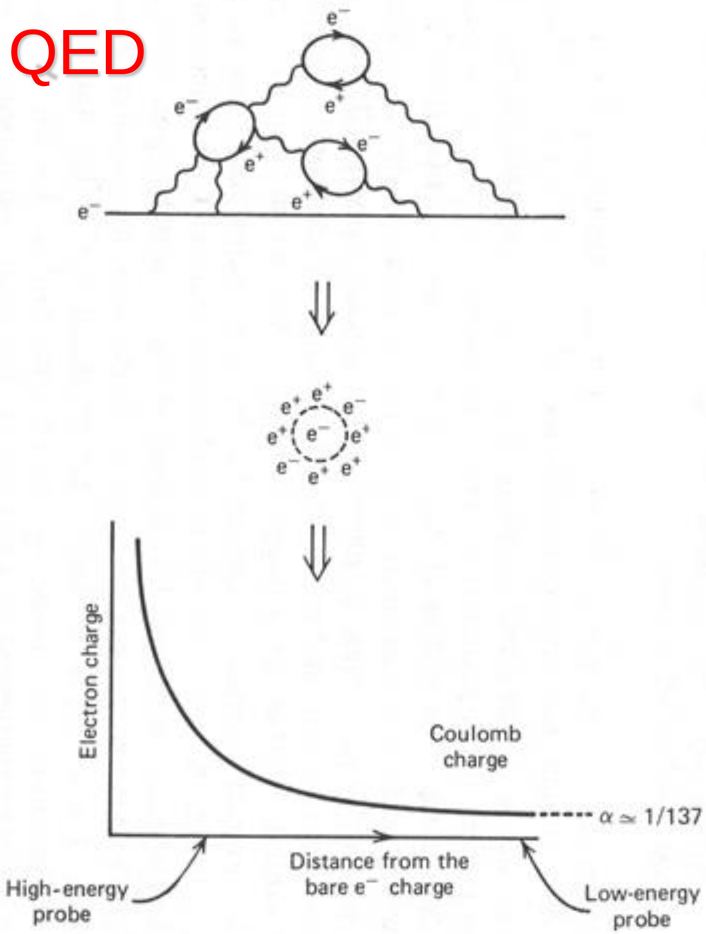
$1/\alpha = 137.03599976(50)$

high energy physics:

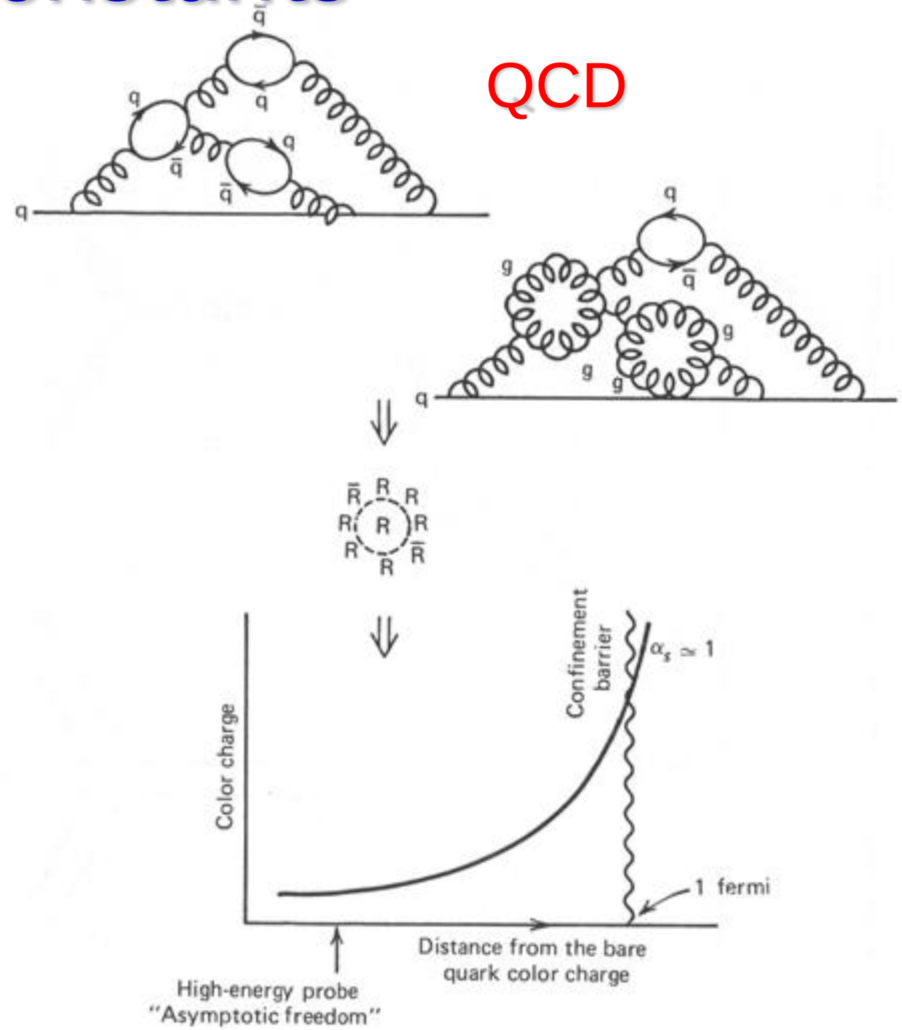
$1/\alpha\,(M_Z) = 128$

Running Coupling Constants

QED



QCD



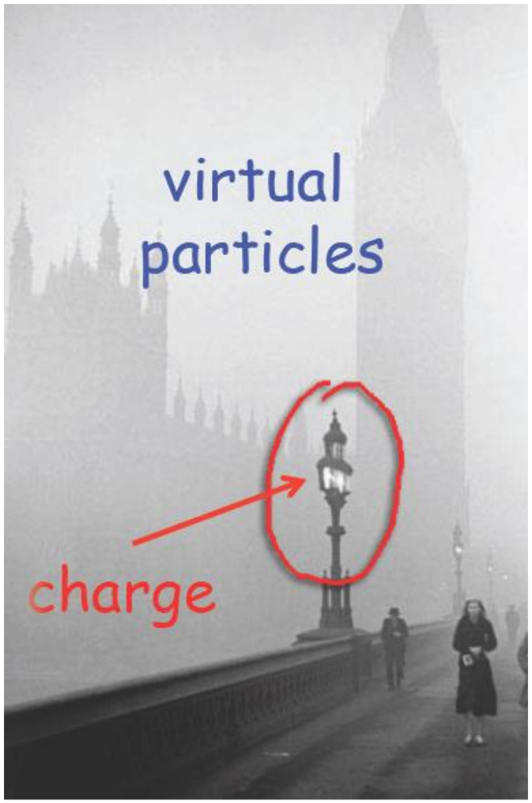
$$\alpha_{EM}\left(Q^2\right)=\frac{\alpha_{EM}\left(\mu^2\right)}{1-\frac{1}{3\pi}\alpha_{EM}\left(\mu^2\right)\log\frac{Q^2}{\mu^2}}$$

$$\alpha_S\left(Q^2\right)=\frac{\alpha_S\left(\mu^2\right)}{1+\frac{33-2N_f}{12\pi}\alpha_S\left(\mu^2\right)\log\frac{Q^2}{\mu^2}}$$

Evolution of Coupling Constants

Classical physics: the forces depend on distance

Quantum Physics: the charges depend on distance



QED : virtual particles (electrons and photons) screen the electric charge:

$\alpha \downarrow$ distance $\uparrow$

QCD : virtual particles (quarks and gluons) anti-screen the strong charge:

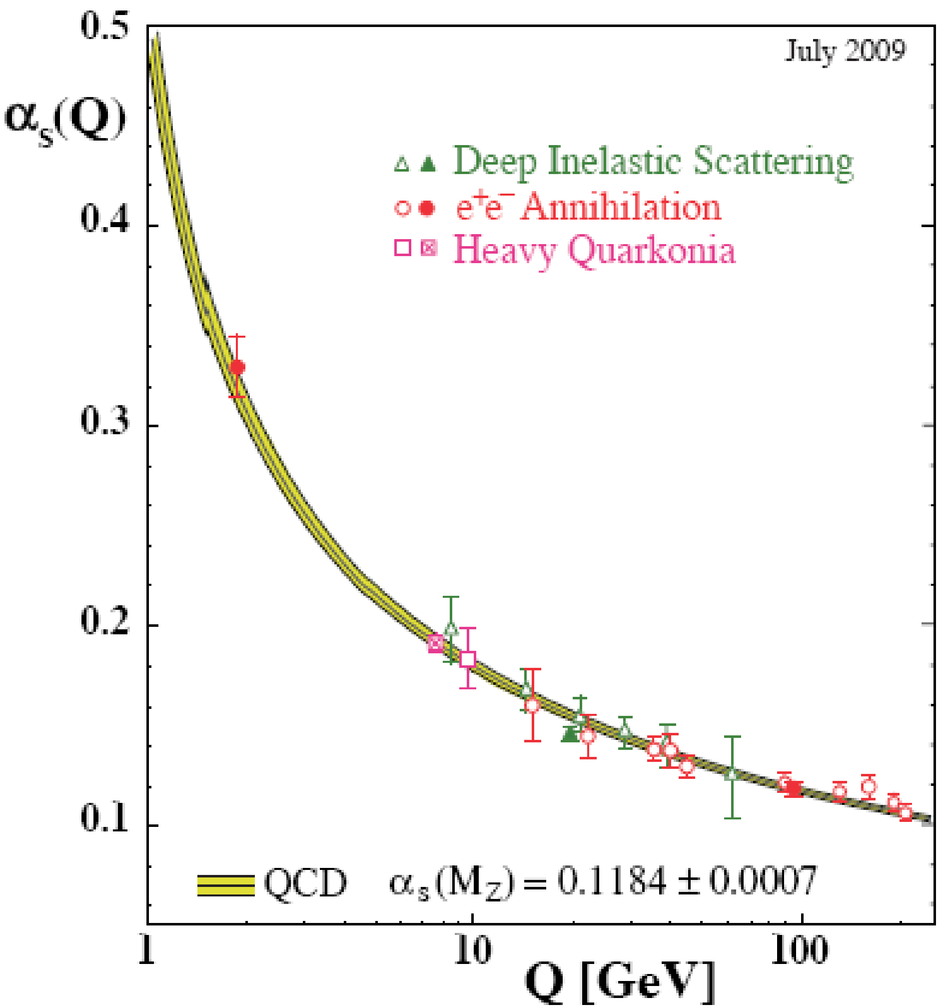
$\alpha_S \uparrow$ distance $\uparrow$

(*asymptotic freedom*)

$$\frac{\partial \alpha_S}{\partial \log E} = \beta(\alpha_S) = \frac{\alpha_S^2}{\pi}\left(-\frac{11N_c}{6} + \frac{N_f}{3}\right)$$

2004

Running of α_S

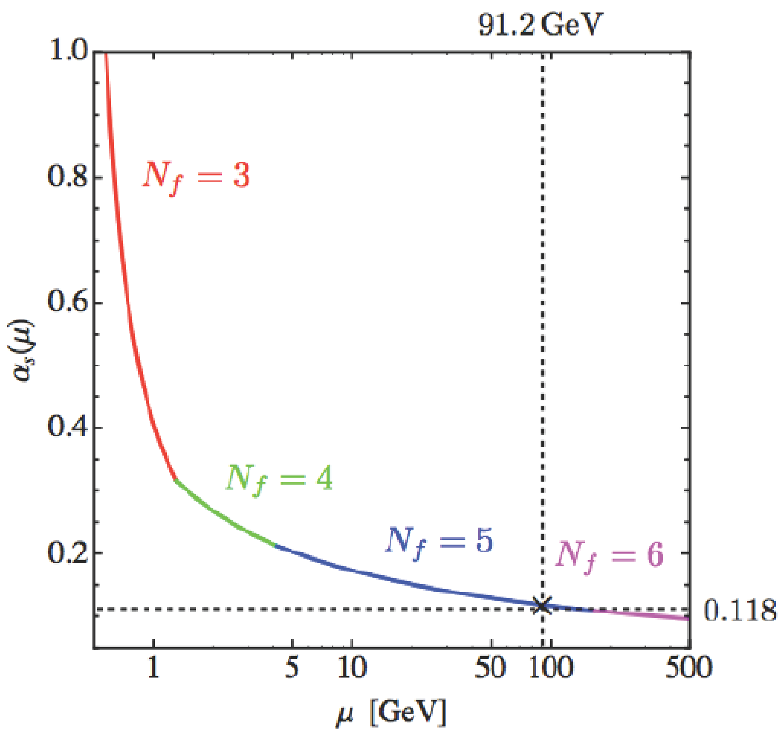


$\alpha_S(\mu)$ at the μ of measurement

active flavors and running of α_S

$$\alpha\left(Q^2\right) = \frac{12\pi}{\left(33 - 2n_f\right)\ln\left(Q^2/\Lambda_{QCD}^2\right)}$$

at leading order in QCD



world average:

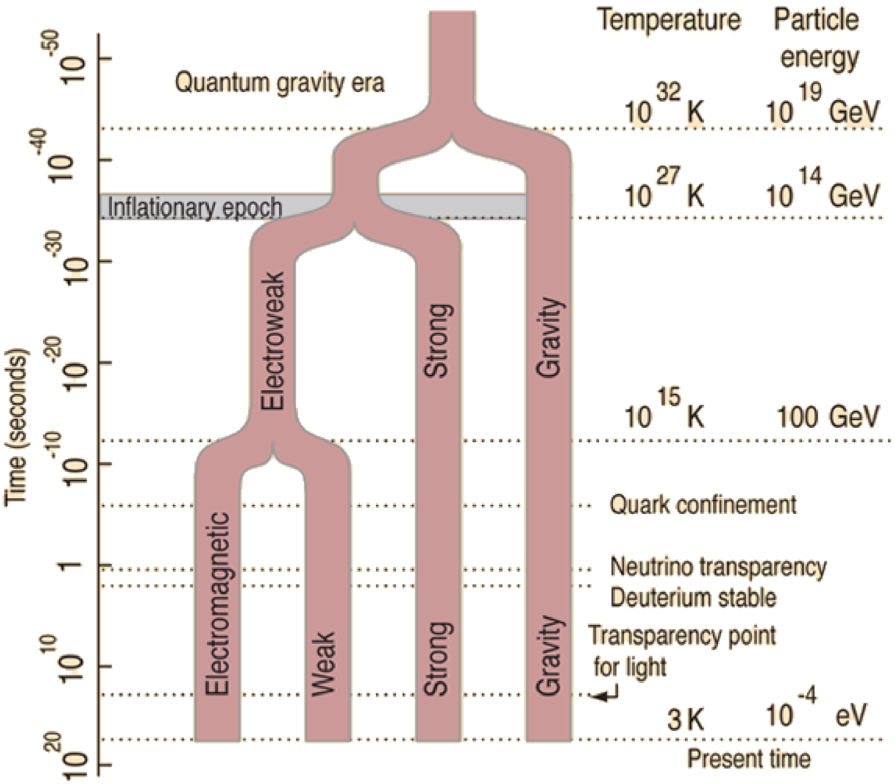
$$\alpha_S\left(M_Z^2\right)^{\overline{MS}} = 0.1184 \pm 0.0007$$

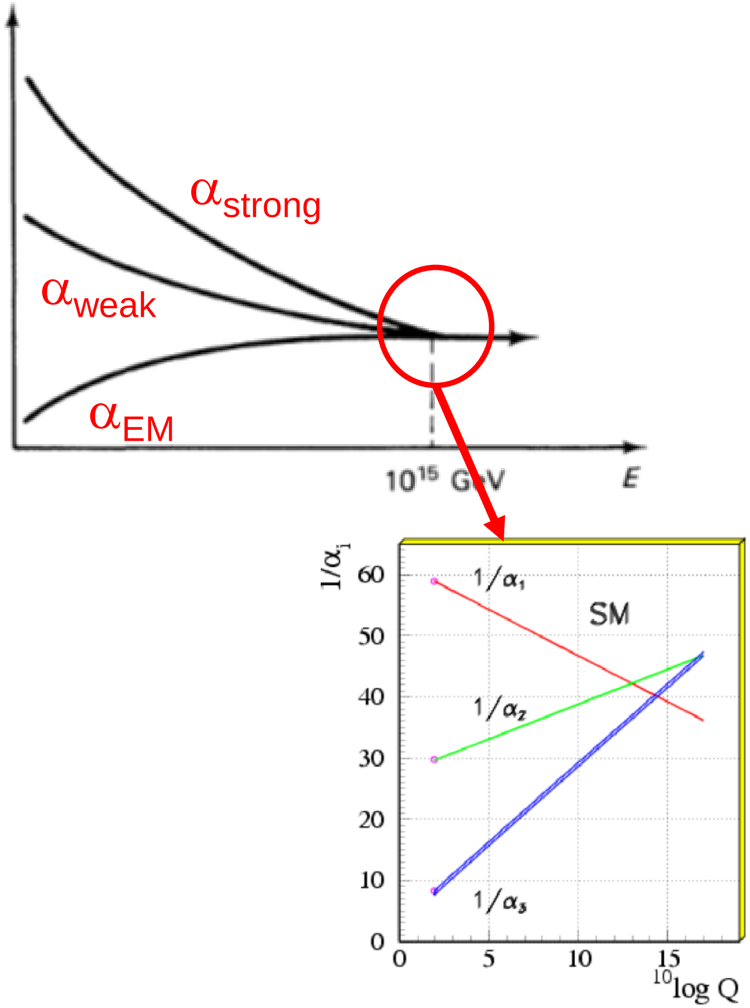
$$\alpha_{EM}\left(M_Z^2\right)^{\overline{MS}} = 0.00781$$

Grand Unification

running of coupling constants
extrapolate to very high energies ~ 10^{15} GeV
→ a single fundamental interaction?
→ only one single bare charge?
→ a singe form of matter?

(one force to rule them all)





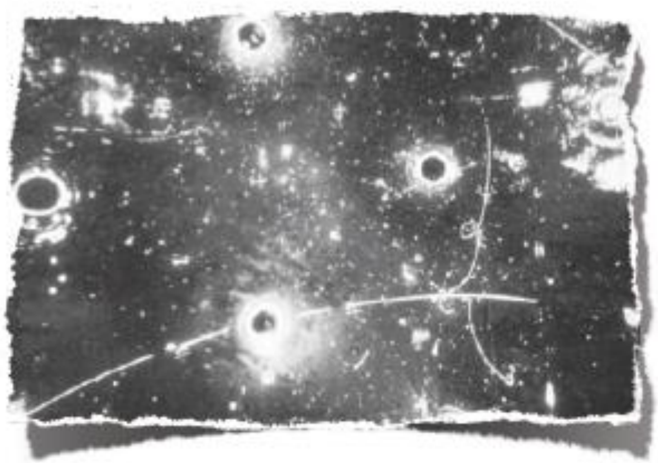
In reality they do not meet
at the same point:
new physics between the
electroweak scale and GUT scale?

The Standard Model

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

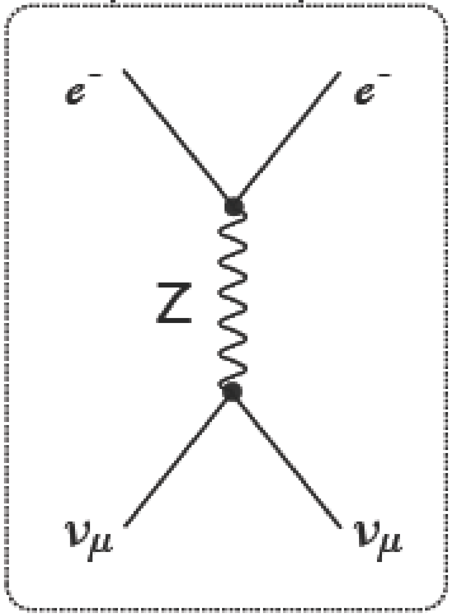
this is a *product* of several symmetry groups and not of a single gauge group, requiring 3 different coupling constants: the interactions are not unified yet!



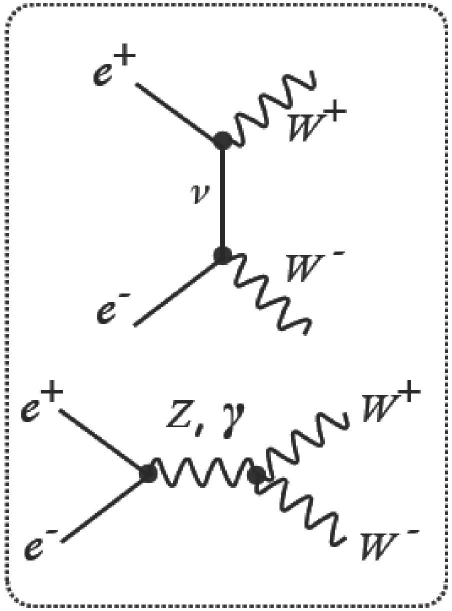
Gargamelle 1973

first neutral current ν e $\rightarrow$ ν e event

$\nu_\mu e^- \rightarrow \nu_\mu e^-$



$e^+e^- \rightarrow W^+W^-$



gauge invariance
requires all these diagrams

The Underlying Principles of the SM

With the discovery of the Higgs boson the Standard Model is now complete!
Is that all? Is there the "desert" up to GUT scales?

The beauty of the Standard Model comes form the identification of a unique dynamical principle → local gauge symmetry describing strong, electromagnetic, and weak interactions that seem so different from each other, but are sufficiently similar to be developed in the same framework

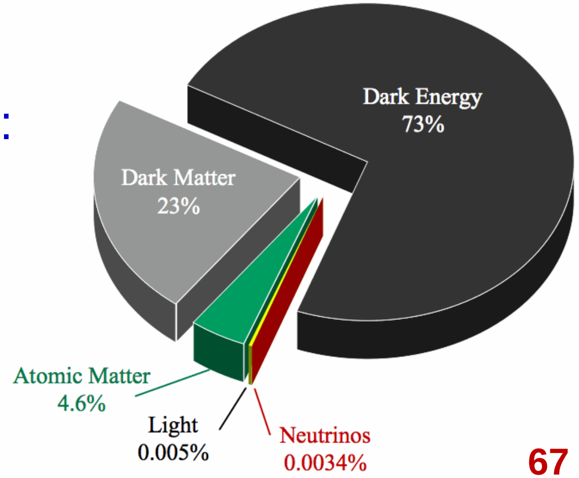
gauge theory = spin–1 bosons

at the same time a particular and predictive structure still leaving room for a rich variety of phenomena

And certainly there is physics beyond the standard model yet to be discovered

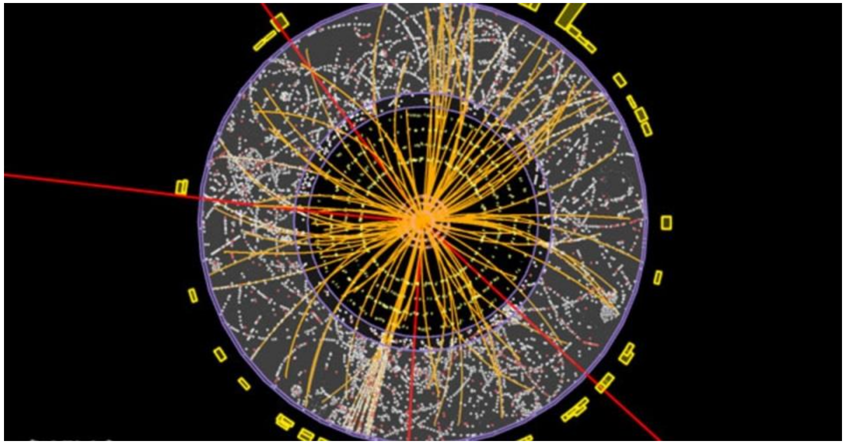
Today there are THREE compelling and firmly established observational facts that the Standard Model does not explain:

- neutrino masses
- the existence of dark matter
- matter over anti-matter abundance (baryon asymmetry)

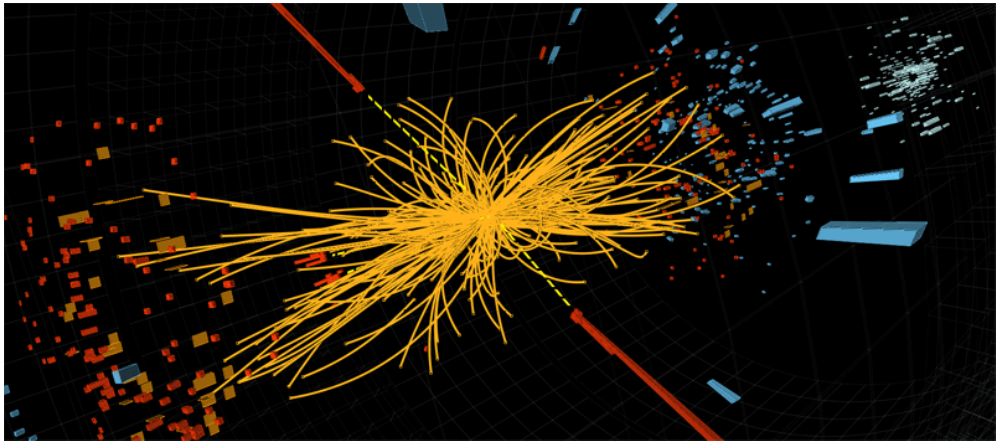


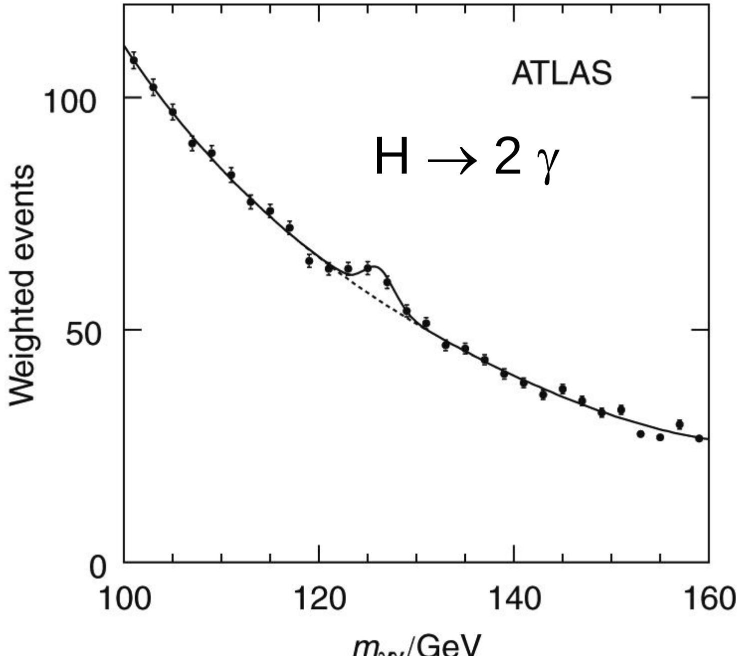
Higgs Boson Discovery

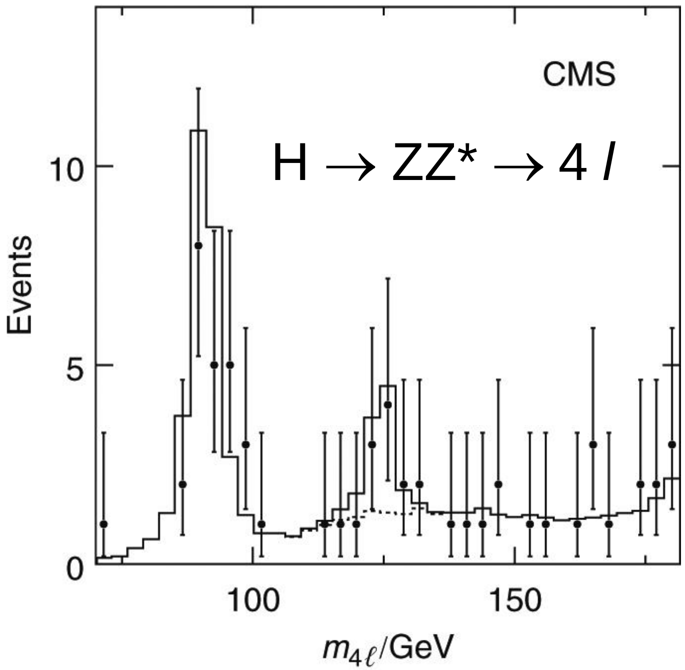
Atlas : H → ZZ* → 4 *l*



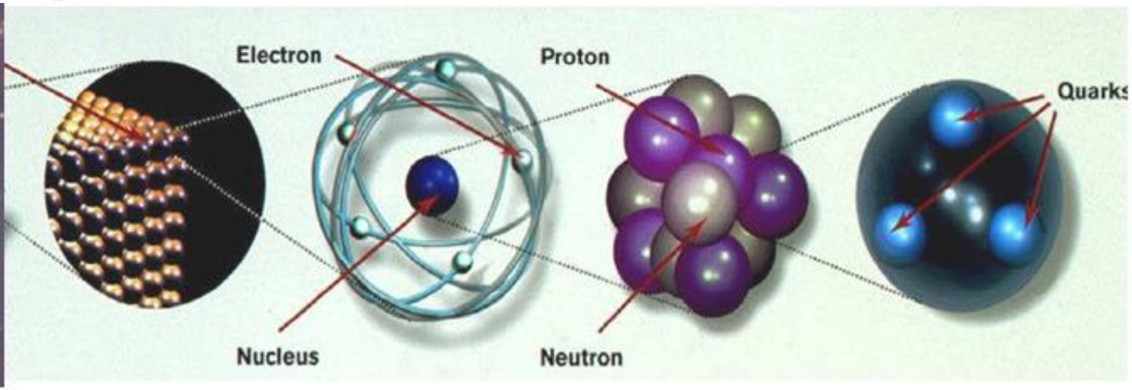
CMS : H → 2 γ







The Dark Mystery of Matter



What stuff is the Universe made of ??

- **Elementary Particles** 0.1%
  - ⇨ 12 **matter particles** (quarks, leptons)
    - only 4 relevant today (u, d, e, ν)
  - ⇨ 13 **force particles** (3 massive, 10 massless)
- **Composite Particles (hadrons)** 5%
  - ⇨ hundreds…
    - only 2 are relevant (p,n), making nuclei
- **Dark Matter** 25%
  - ⇨ made of **unknown particles**
- **Dark Energy** 70%
  - ⇨ **vacuum energy**
    - of completely unknown origin
  - ⇨ should be infinite or exactly 0

Higgs mechanism

QCD chiral symmetry breaking

We don't know **how** and **why** for ~ 5%

We don't even know **what** for the other 95%

For Next Week

Study the material and prepare / ask questions
Read the introductory chapter in any Particle Physics textbook

Do the homeworks!

Next week we will study the non-relativistic Quark Model
- have a first look at the lecture notes, you can already have questions
- read ch. 2 in Halzen & Martin and / or ch. 9 in Thomson