Advanced Particle Physics 2

Strong Interactions and Weak Interactions

L4 – The QCD Improved Parton Model

(http://dpnc.unige.ch/~bravar/PPA2/L4)

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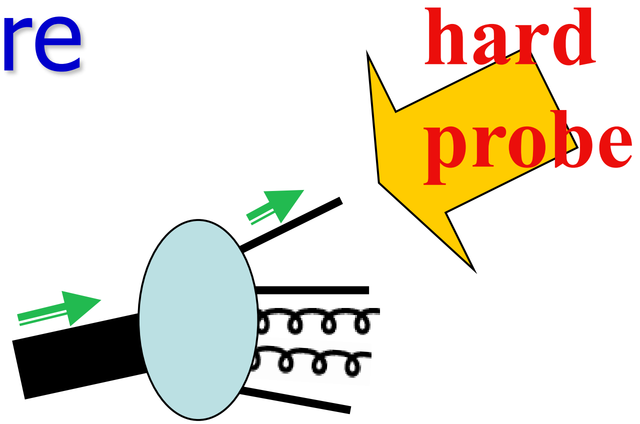
Probing the Proton Structure



EM interaction

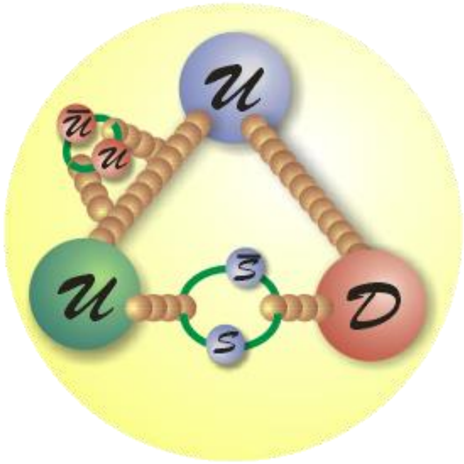
Photon

sensitive to (electric charge)^2
insensitive to color charge
(almost) insensitive to quark flavor
cannot distinguish between quarks and anti-quarks

Weak interaction

Weak Bosons

sensitive to weak charge ~ flavor
insensitive to color
can separate between quarks and anti-quarks



Strong interaction

Gluons

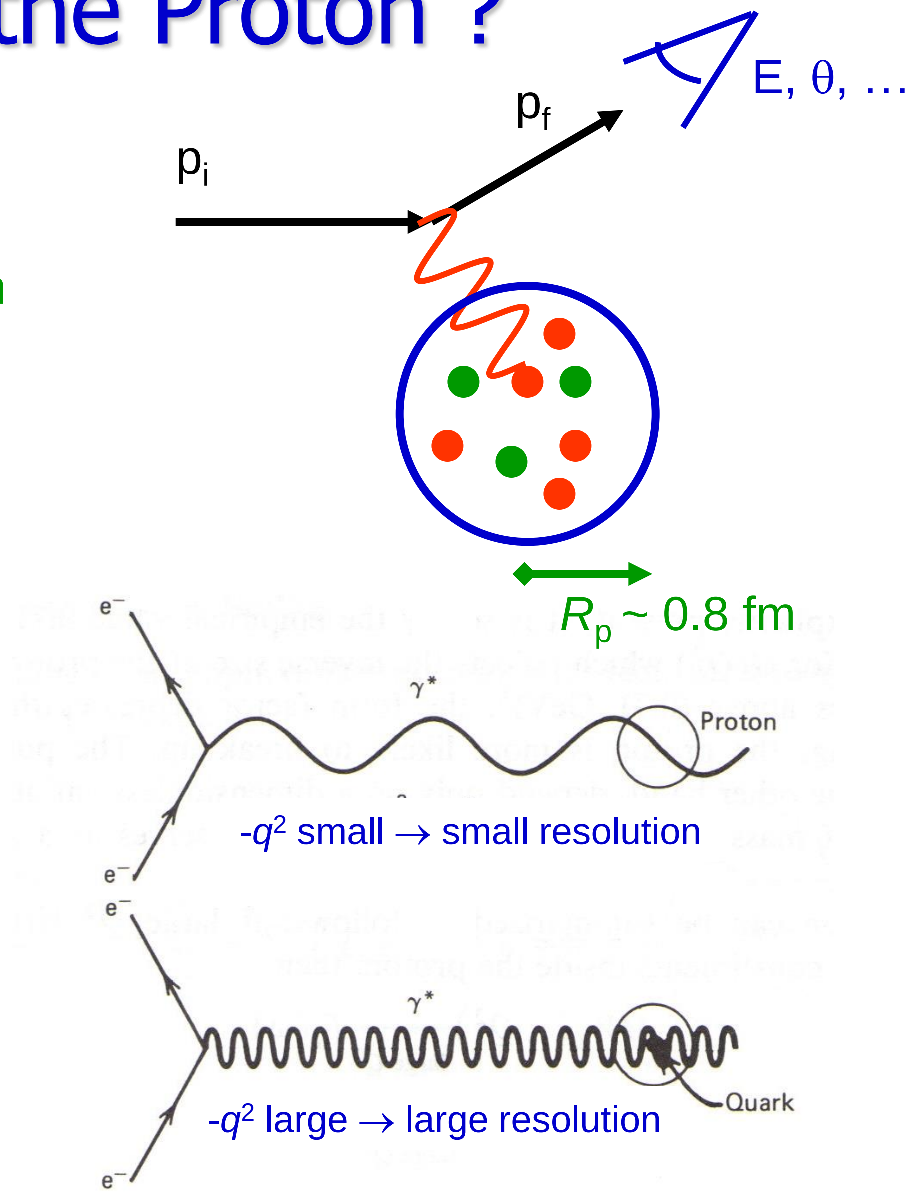
sensitive to color charge
insensitive to flavor
insensitive to electric charge

All three probes are needed to disentangle the proton structure in detail.

How to Look Inside the Proton ?

Deep Inelastic Scattering

scatter a high energy electron off a proton
with a large momentum transfer q
and measure the scattered electron



$q = p_i - p_f \quad q^2 < 0$

def $Q^2 = -q^2$

Q ~ photon momentum

λ ~ photon wave length

$$\lambdabar \approx \frac{\hbar c}{Q} \approx \frac{200\ \mathrm{MeV \cdot fm}}{Q}$$

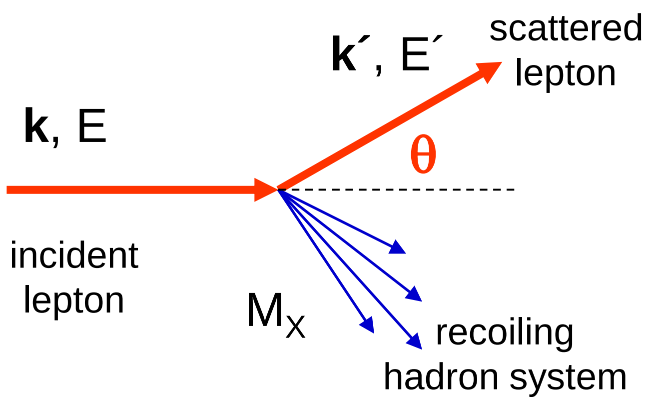
required resolution $\lambda < R_p$

i.e. $Q^2 > 1\ \mathrm{GeV}^2$

Kinematics

inclusive:
only scattered lepton is observed and measured
$\rightarrow$ scattering angle θ and energy E'
semi-inclusive:
some hadrons are observed and measured

in the laboratory frame
where the proton is at rest



$$s = (p+k)^2$$

$$q = k - k' \rightarrow Q^2 = -q^2 \xrightarrow{lab} Q^2 = 4EE' \sin^2 \frac{\vartheta}{2} > 0$$

$$x = Q^2 / 2q \cdot p \xrightarrow{\text{lab}} x = Q^2 / 2M_p \nu$$

$$\nu = q \cdot p / M_p \xrightarrow{\text{lab}} \nu = E - E'$$

$$y = q \cdot p / k \cdot p = \frac{1}{2}\left(1 - \cos \vartheta^*\right) \xrightarrow{\text{lab}} y = \nu / E = 1 - \frac{E'}{E}$$

$$M_X^2 = W^2 = (p+q)^2 \xrightarrow{\text{lab}} W^2 = M_p^2 + 2M_p^2 \nu - Q^2$$

all kinematic variables
are measurable

Elastic scattering: $W^2 = M^2 + 2M\nu - Q^2 = M^2 \Rightarrow Q^2 = 2M\nu \Rightarrow x = 1$
only one of the two variables Q^2 and ν is independent

Inelastic scattering: $W^2 > M^2$ and $0 < x < 1$
two out of three variables Q^2, ν, and W^2 are independent

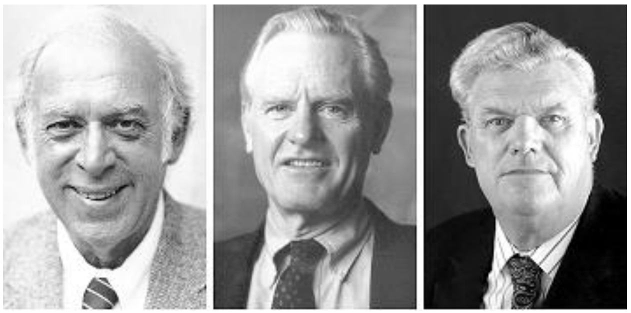
Work in 1-γ exchange approx. (Born); for $Q^2 < 1000$ GeV^2, Z^0 (EW) effects are negligible

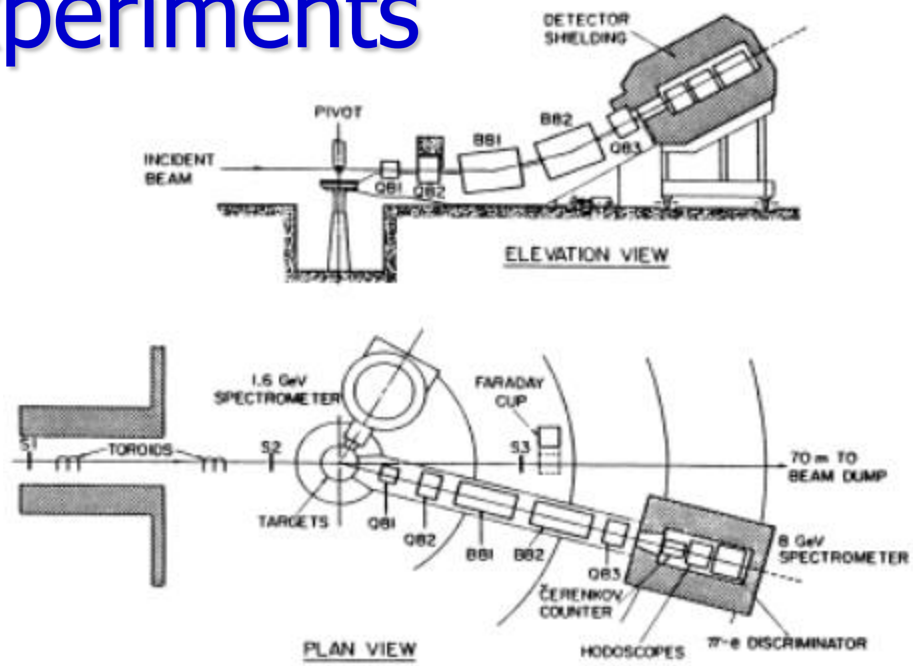
Older Fixed Target Experiments

$e + p \rightarrow e' + X$ (unobserved)

early DIS SLAC (~´70s) experiments

1990

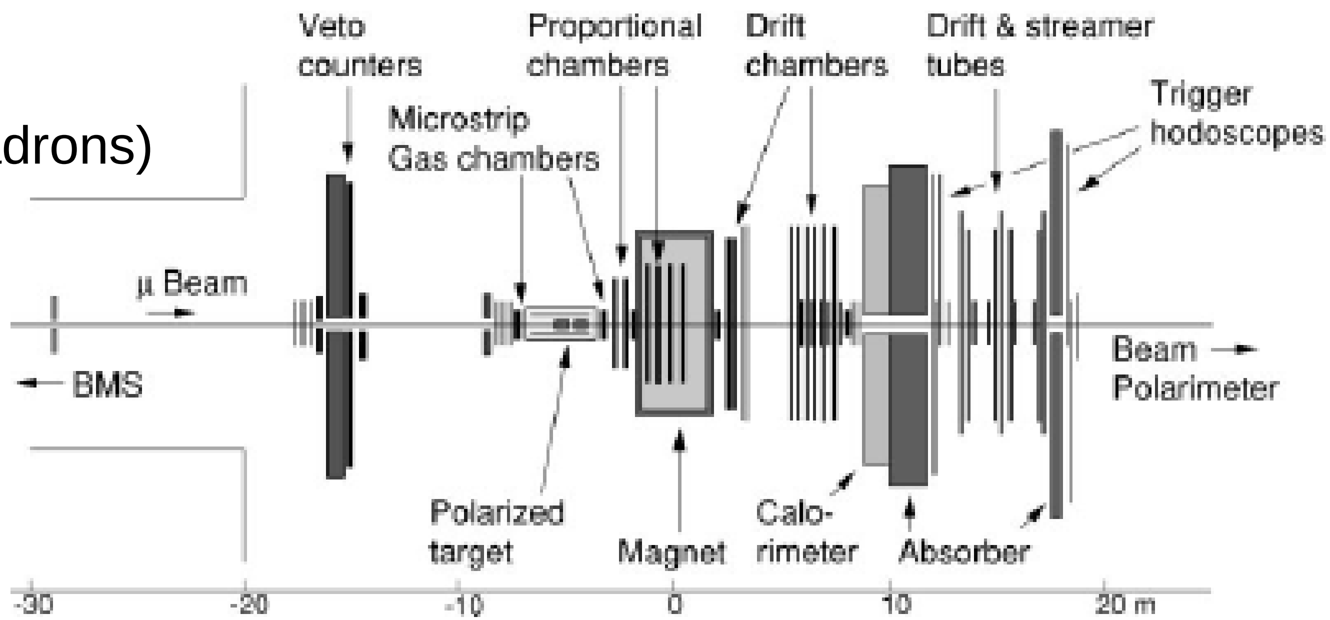




in order to map the proton's $x - Q^2$ structure
change angle θ and beam energy E

$\mu + p \rightarrow \mu' + X$ (hadrons)

European Muon Collaboration (EMC) at CERN (~'80s)

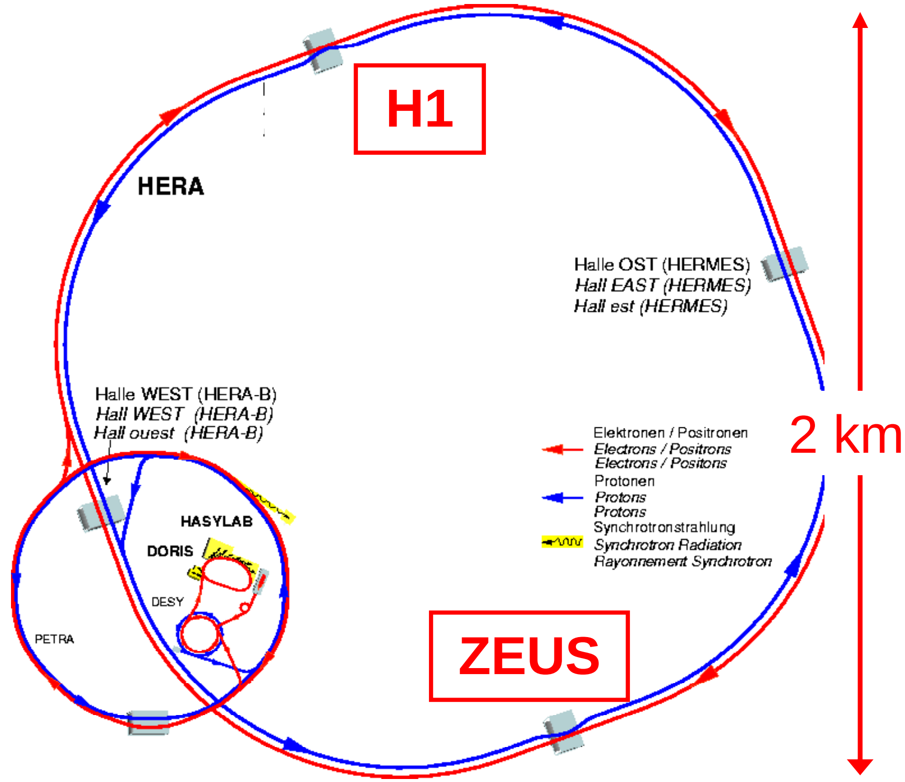


The HERA e± Collider (1991 – 2007)

DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany

$e^{\pm}$ 27.5 GeV → ← 920 GeV p

$\sqrt{s}$ = 300 GeV



Two large experiments : H1 and ZEUS

Probe proton at very high Q^2 and very low x

Collider Experiments (H1)

event kinematics determined from scattered electron angle and energy

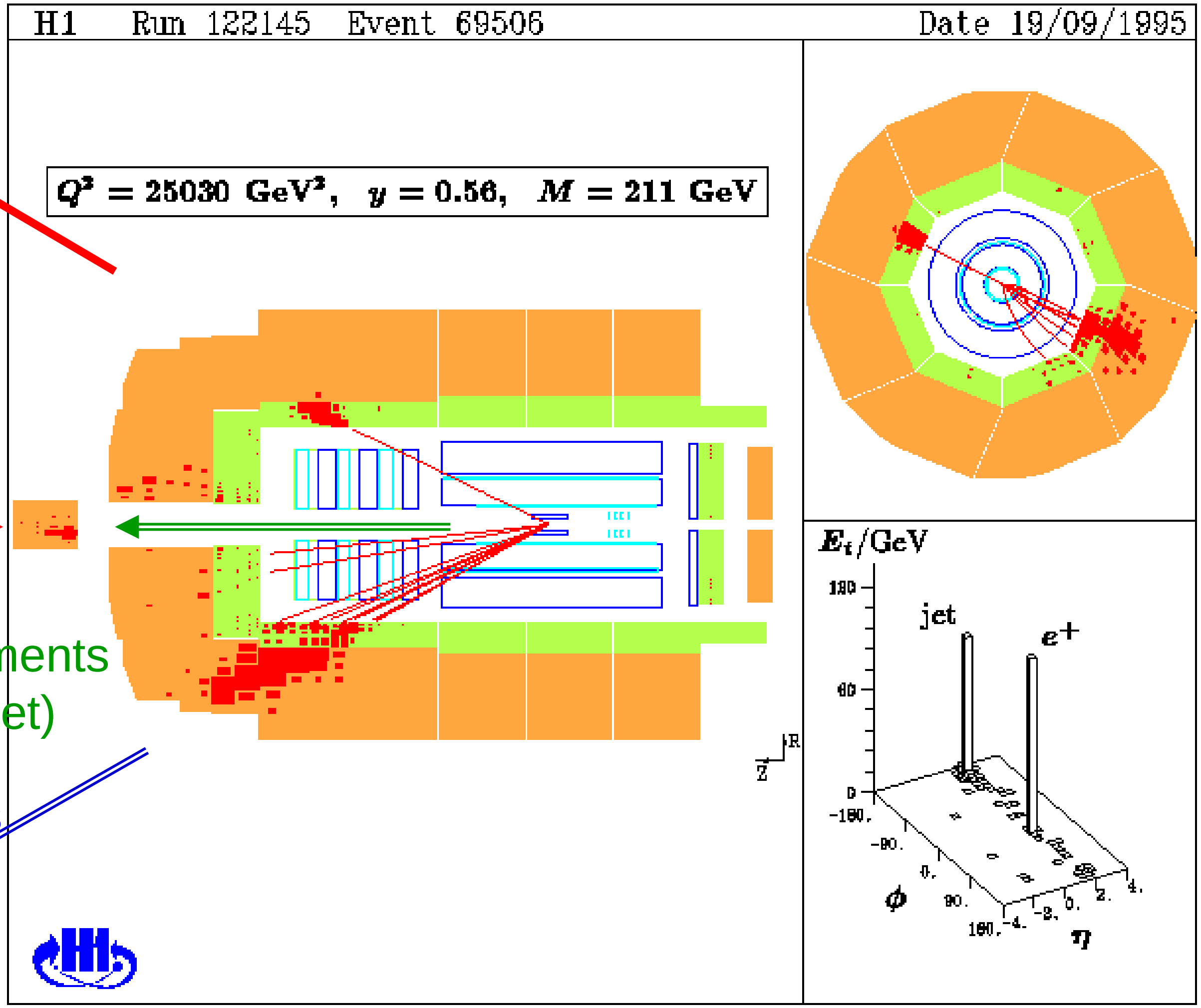
scattered electron

27.5 GeV

electron

proton fragments (forward jet)

quark fragments (jet)



920 GeV

proton

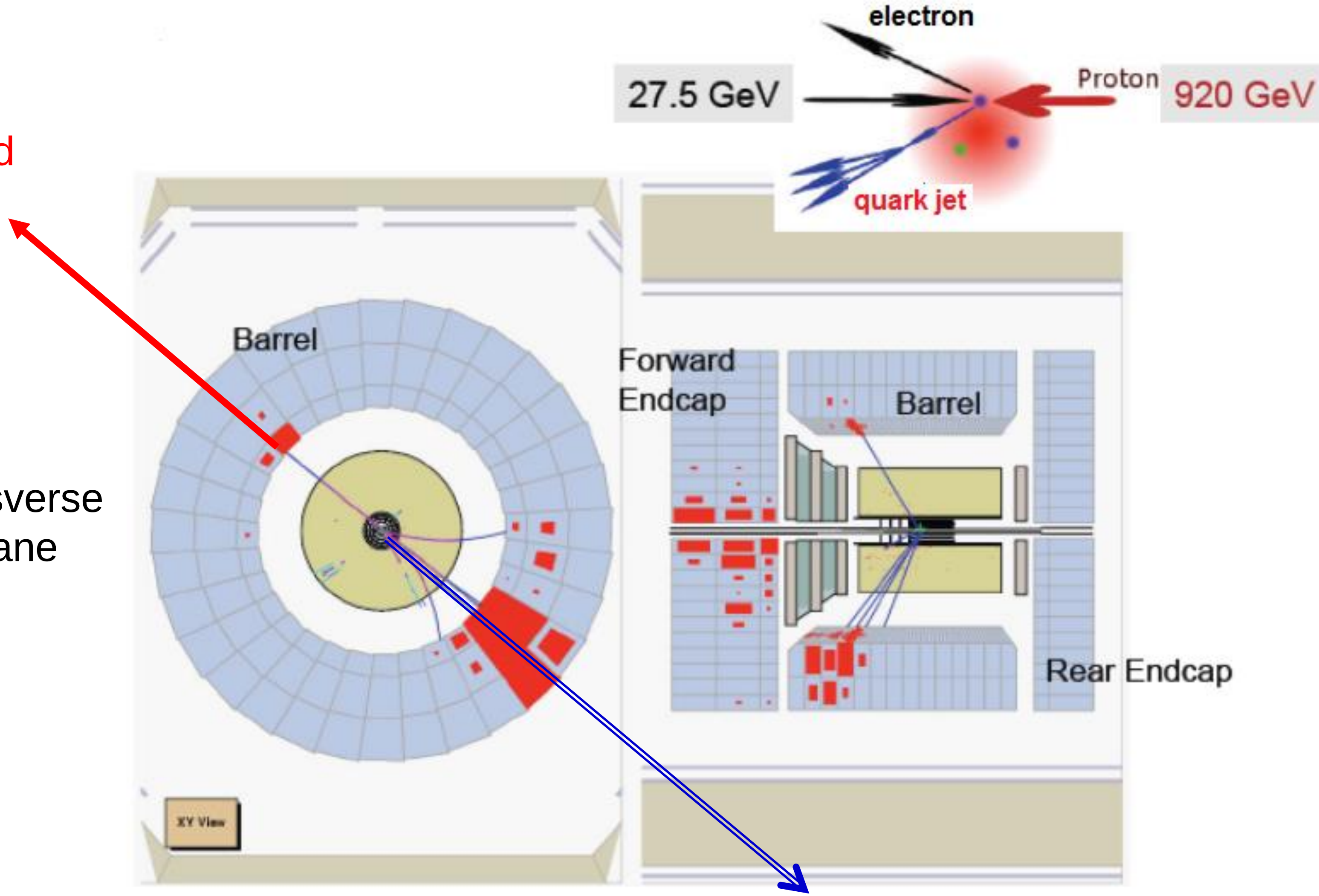
also measure calorimetrically the hadronic system (not as precisely)

Collider Experiments (ZEUS)

event kinematics determined from scattered electron angle and energy

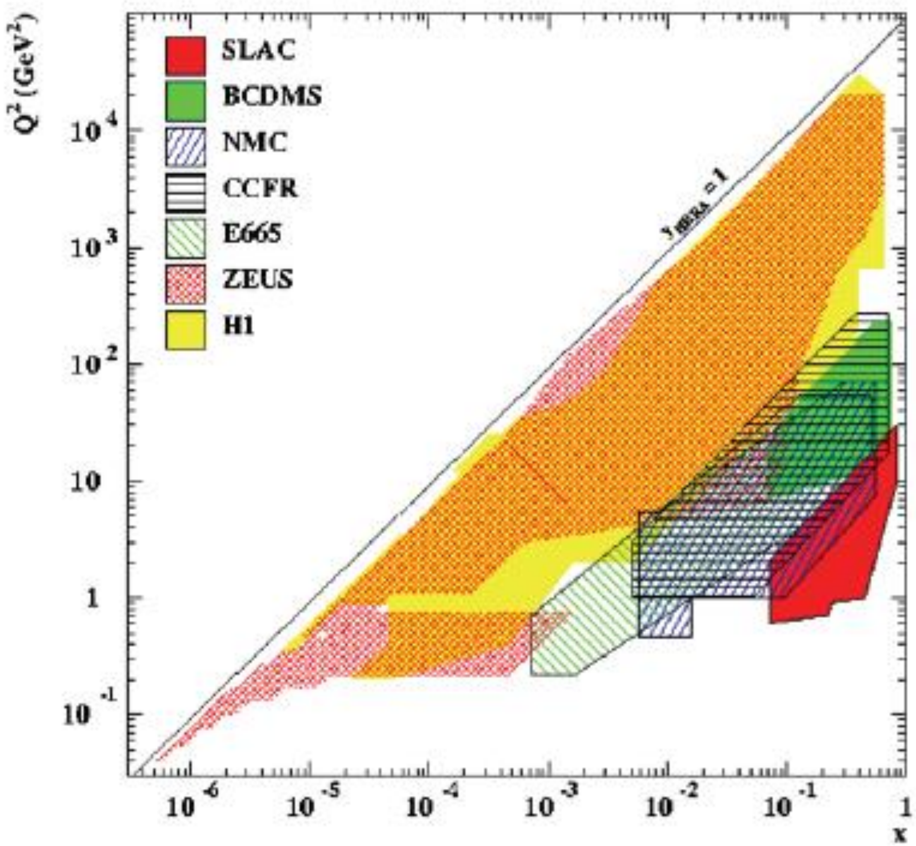
scattered electron

transverse plane



quark fragments (jet)

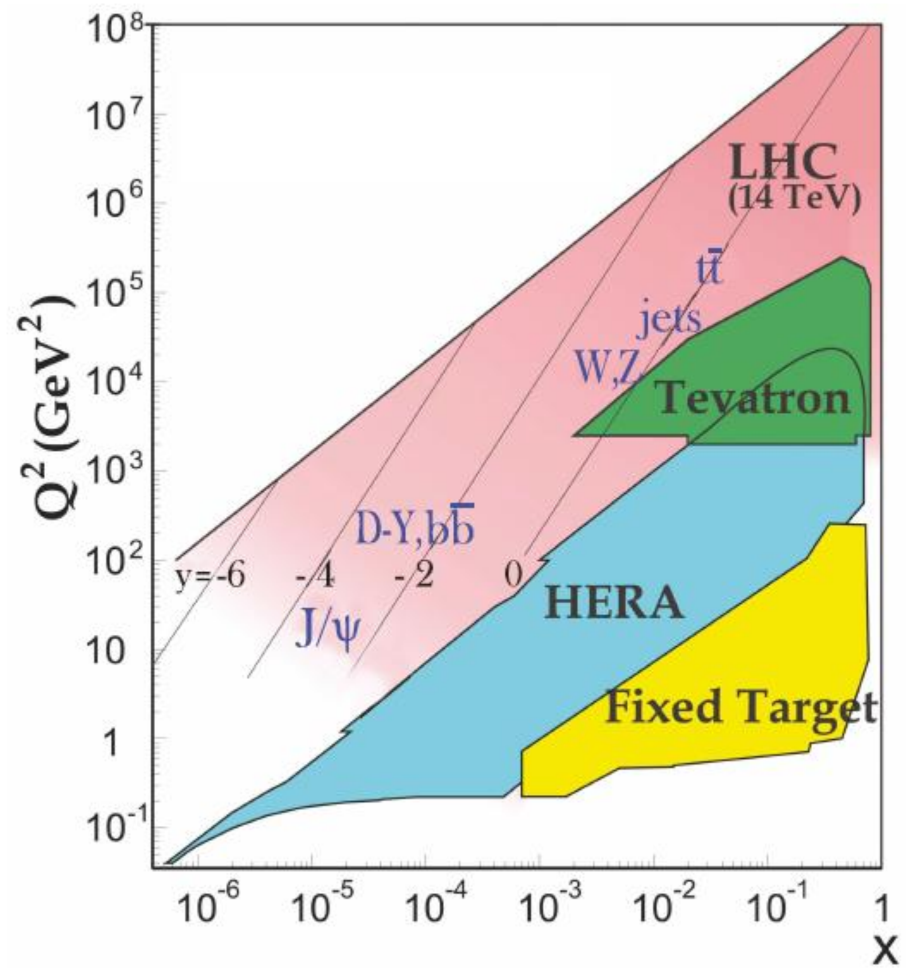
Kinematical Range of DIS Experiments



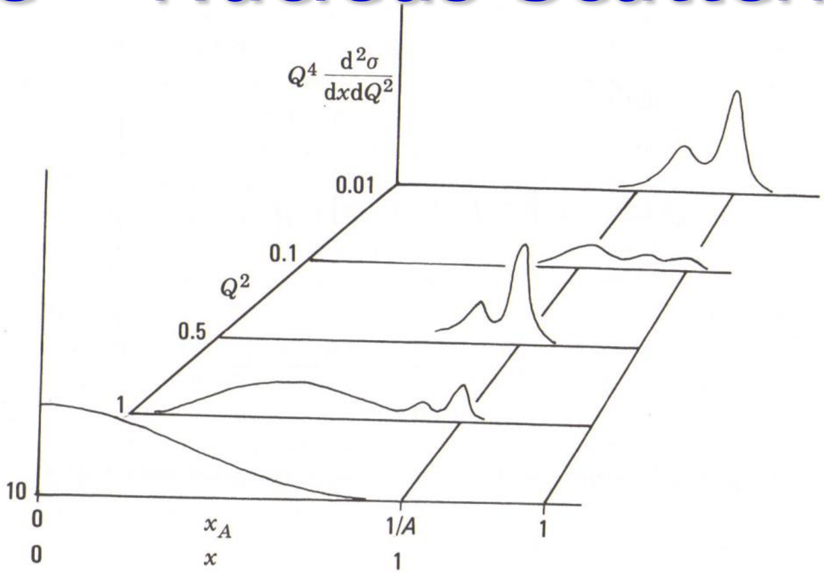
HERA ep collider: H1 and Zeus 1992 - 2007

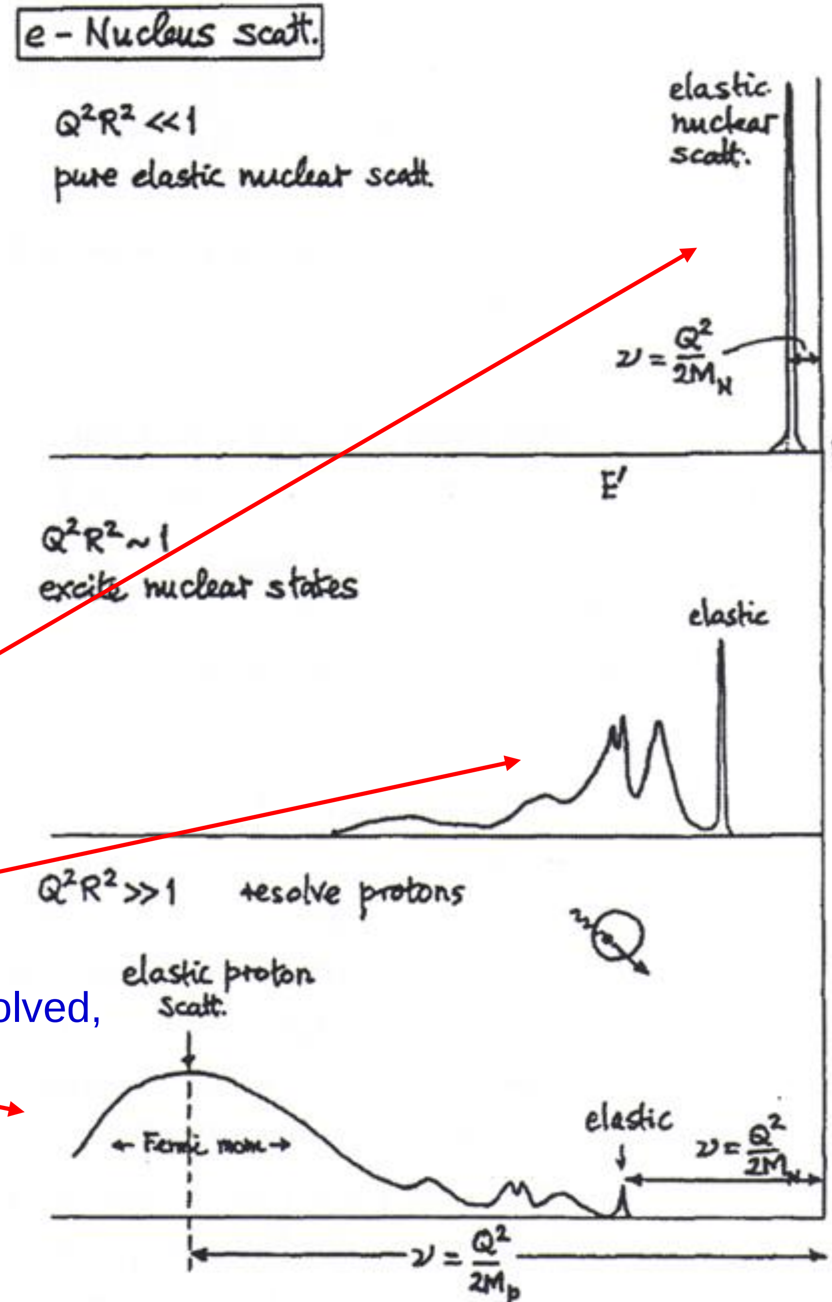
Fixed Target: eN, μN, νN (SLAC, FNAL, CERN)
completed ~ 15 years ago

including p - p colliders



e – Nucleus Scattering





The effective probe is the exchanged virtual photon of wave length $\lambdabar = \hbar c / Q$.

For low Q^2 (~0.01 GeV^2) the nucleus tends to recoil as a whole with $\nu = Q^2 / 2M_A$.

As Q^2 increases nuclear states are excited.

At higher Q^2 (~0.1 GeV^2) the nuclear structure is resolved, and the electron scatters elastically off the constituent nucleons (protons) with $\nu = Q^2/2M_p$ (broad distribution due to Fermi motion).

At even higher Q^2 (> 0.1 GeV^2) the nucleon structure is revealed (first nucleon resonances are excited).

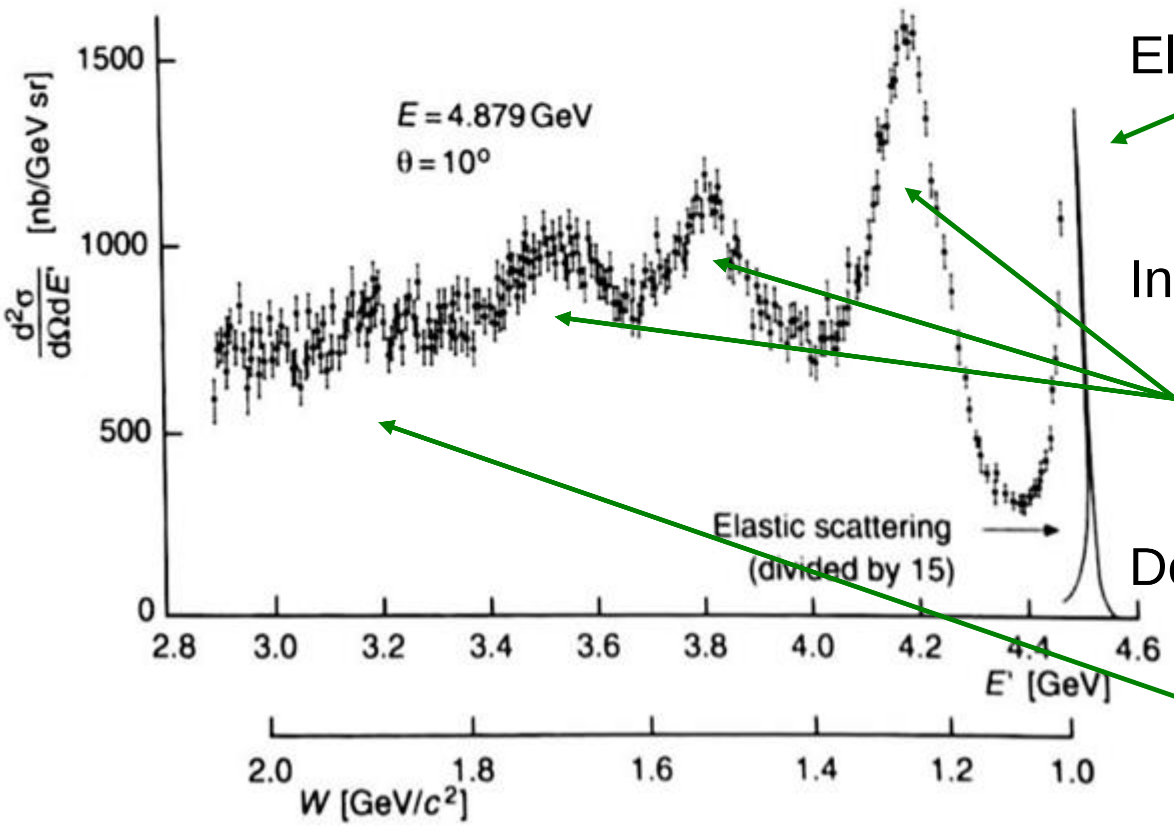
Inelastic Scattering

Scattering of 4.879 GeV electrons from protons at rest

Place detector at 10^o to beam and measure the energy of scattered e^-

Kinematics fully determined from the electron energy and angle!

For this energy and angle determine the invariant mass of the final state hadronic sys.



Elastic scattering

proton remains intact

$W = M$

Inelastic (resonance) scattering

produce "excited states" of the proton e.g. $\Delta^+(1232)$

$W = M_\Delta$

Deep Inelastic Scattering

proton breaks up resulting in a many particle final state

DIS = large W

Due to proton internal structure, elastic scattering at high Q^2 is unlikely and inelastic reactions, where the proton breaks up dominate.

For inelastic scattering the mass of the final state hadronic system $W = M_X > M_p$.

Electron – Nucleon Inclusive Cross Section

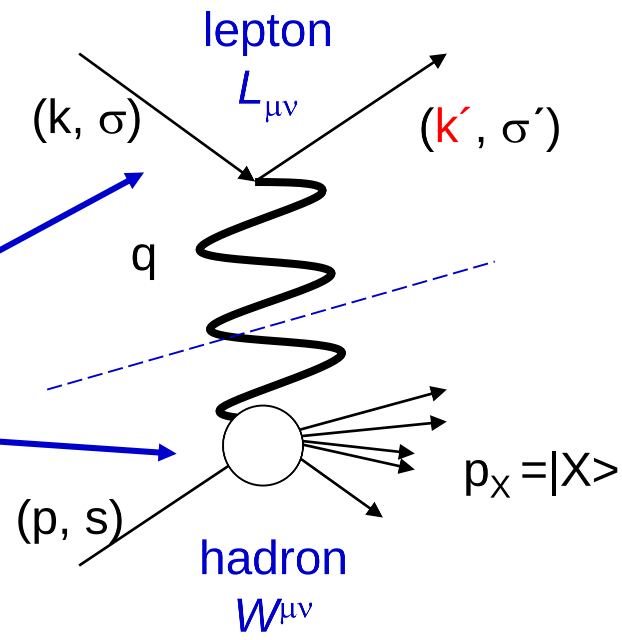
assume a general final hadronic state |X>

e + p $\rightarrow$ e´ + hadrons (unobserved)

work in the one-photon approximation (Born)
the invariant amplitude factorizes in

lepton current

hadron current



follow the procedure used for calculating
e + μ $\rightarrow$ e + μ scattering
(separate sums over lepton and hadron spins)

$$d\sigma \sim \frac{1}{Q^4} L^{el}_{\mu\nu} W^{\mu\nu}_{had}$$

invariant amplitude for e + p $\rightarrow$ e´ + X

$$-iM = \bar{u}(k',\sigma')\left(-ie\gamma^{\mu}\right)u(k,\sigma)\frac{-ig_{\mu\nu}}{q^2}\langle X \,|\, J^{\nu}_{had} \,|\, p,s\rangle$$

evaluate $|M|^2$ and sum over all possible hadronic states
- unpolarized electrons
- no final state polarization observed

$$d\sigma = \frac{e^4}{Q^4}\frac{d^3k'}{(2\pi)^3\, 2E'}\frac{4\pi M}{4ME} L_{\mu\nu} W^{\mu\nu}(q,p)$$

Hadron Tensor

The hadron tensor $W^{\mu\nu}$ parameterizes our ignorance of the hadron structure at the other end of the photon propagator

- average over initial proton spin states (Σ_{spin})
- sum over all hadronic final states |X> (and spins)
- integrate over all final state particle momenta (only particles on mass shell can be observed!)
- Q^2 and ν (or p and q) are independent

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi M}\frac{1}{2}\sum_{spins}\sum_{X}\langle p,s|J^{\dagger}_{\mu}(0)|X\rangle\langle X|J(0)_{\nu}|p,s\rangle(2\pi)^4\delta^{(4)}(p+q-p_X)$$

The lepton tensor $L^{\mu\nu}$ is the same as for eμ scattering (symmetric in $\mu\leftrightarrow\nu$)

$$L_{el}^{\mu\nu} = 2\left[k'^{\mu}k^{\nu} + k^{\mu}k'^{\nu} - (k\cdot k')g^{\mu\nu}\right]$$

The most general form for $W^{\mu\nu}$ constructed out of $g^{\mu\nu}$ and independent momenta p and q (γ^{μ} matrices are not included since we already averaged over spins)

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2}p^{\mu}p^{\nu} - i\frac{W_3}{2M^2}\varepsilon^{\mu\nu\sigma\tau}p_{\sigma}q_{\tau} + \frac{W_4}{M^2}q^{\mu}q^{\nu} + \frac{W_5}{M^2}(p^{\mu}q^{\nu} + q^{\mu}p^{\nu}) + i\frac{W_6}{2M^2}(p^{\mu}q^{\nu} - q^{\mu}p^{\nu})$$

$W_i = W_i(\nu, Q^2)$ or $W_i = W_i(p, q)$ – proton structure functions

(W_3, W_6 reserved for parity-violating structure functions in ν scattering, γ replaced by $W^{\pm}$)

The Inelastic Cross Section

$W^{\mu\nu}$ can be simplified, noting

parity invariance → symmetric form (not for ν scattering) $W_i^{\mu\nu} = W_i^{\nu\mu} \quad (W_3, W_6 \to 0)$

current conservation $q_\mu W_i^{\mu\nu} = q_\nu W_i^{\nu\mu} = 0$ implies

$$W_4 = \left(M^2 / q^2\right) W_1 + \left(p \cdot q / q^2\right)^2 W_2 \qquad W_5 = -\left(p \cdot q / q^2\right) W_2$$

which gives

$$W^{\mu\nu} = -W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(p^\mu - \frac{p \cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p \cdot q}{q^2} q^\nu\right)$$

Unlike for elastic scattering, there are two independent variables, e.g. ν and q^2 .

Contract $L_{\mu\nu}$ and $W^{\mu\nu}$

$$L_{\mu\nu} W^{\mu\nu} = 4W_1 (k \cdot k') + \frac{2W_2}{M^2}\left[2(p \cdot k)(p \cdot k') - M^2 k \cdot k'\right]$$

in lab frame

$$= 4EE'\left\{W_2(\nu, Q^2)\cos^2\frac{\vartheta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\vartheta}{2}\right\}$$

and include the propagator $1/Q^4$, flux and phase space factors for the outgoing electron

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}E'\mathrm{d}(\cos\vartheta)}\right|_{lab} = \frac{8\pi\alpha^2 E'^2}{Q^4}\left\{W_2(\nu, Q^2)\cos^2\frac{\vartheta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\vartheta}{2}\right\}$$

Summary of Cross Section Formulae

and integrate over E´ (in Rosenbluth form)

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{ W_2(\nu,Q^2) + 2W_1(\nu,Q^2)\tan^2\frac{\vartheta}{2} \right\}$$

Mott cross section for spinless and structureless target (integrated in dφ)

$$\frac{d\sigma}{d(\cos\vartheta)} = \frac{8\pi\alpha^2 E'^2}{Q^4}\cdot\frac{E'}{E}\cdot\cos^2\frac{\vartheta}{2} = \frac{2\pi\alpha^2}{4E^2\sin^4\frac{\vartheta}{2}}\cdot\frac{E'}{E}\cdot\cos^2\frac{\vartheta}{2} = \frac{2\pi\alpha^2}{4E^2\sin^4\frac{\vartheta}{2}}\cdot\frac{1}{1+\frac{E}{M}\left(2\sin^2\frac{\vartheta}{2}\right)}\cdot\cos^2\frac{\vartheta}{2}$$

The structure of the target becomes apparent if we summarize the various formulae

$$\frac{d\sigma}{dE'd(\cos\vartheta)} = \frac{8\pi\alpha^2 E'^2}{16E^2E'^2\sin^4\frac{\vartheta}{2}}\frac{E'}{E}\{\ \}$$

"common factor" (E'/E = 1 for static target i.e. no recoil)

eμ → eμ $\quad \{\ \}_{e\mu\to e\mu} = \left(\cos^2\frac{\vartheta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\vartheta}{2}\right)\delta\left(\nu+\frac{q^2}{2m}\right)$

elastic *ep* scattering $\quad \{\ \}_{ep\to ep} = \left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\vartheta}{2} + 2\tau G_M^2\sin^2\frac{\vartheta}{2}\right)\delta\left(\nu+\frac{q^2}{2M}\right)$

inelastic *ep* scattering $\quad \{\ \}_{ep\to eX} = \left(W_2\left(\nu,Q^2\right)\cos^2\frac{\vartheta}{2} + 2W_1\left(\nu,Q^2\right)\sin^2\frac{\vartheta}{2}\right)$

Deep Inelastic Scattering

The most general Lorentz invariant expression for $e^- p \rightarrow e^- X$ inelastic scattering (1-γ exchanged) in terms of Q^2, x, y is:

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{\left(1 - y - \frac{M^2 y^2}{Q^2}\right)\frac{F_2\left(x,Q^2\right)}{x} + y^2 F_1\left(x,Q^2\right)\right\}$$

INELASTIC SCATTERING

cfr.

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{\left(1 - y - \frac{M^2 y^2}{Q^2}\right) f_2\left(Q^2\right) + \frac{1}{2} y^2 f_1\left(Q^2\right)\right\}$$

ELASTIC SCATTERING

The from factors $f_1(Q^2)$ and $f_2(Q^2)$ have been replaced by structure functions

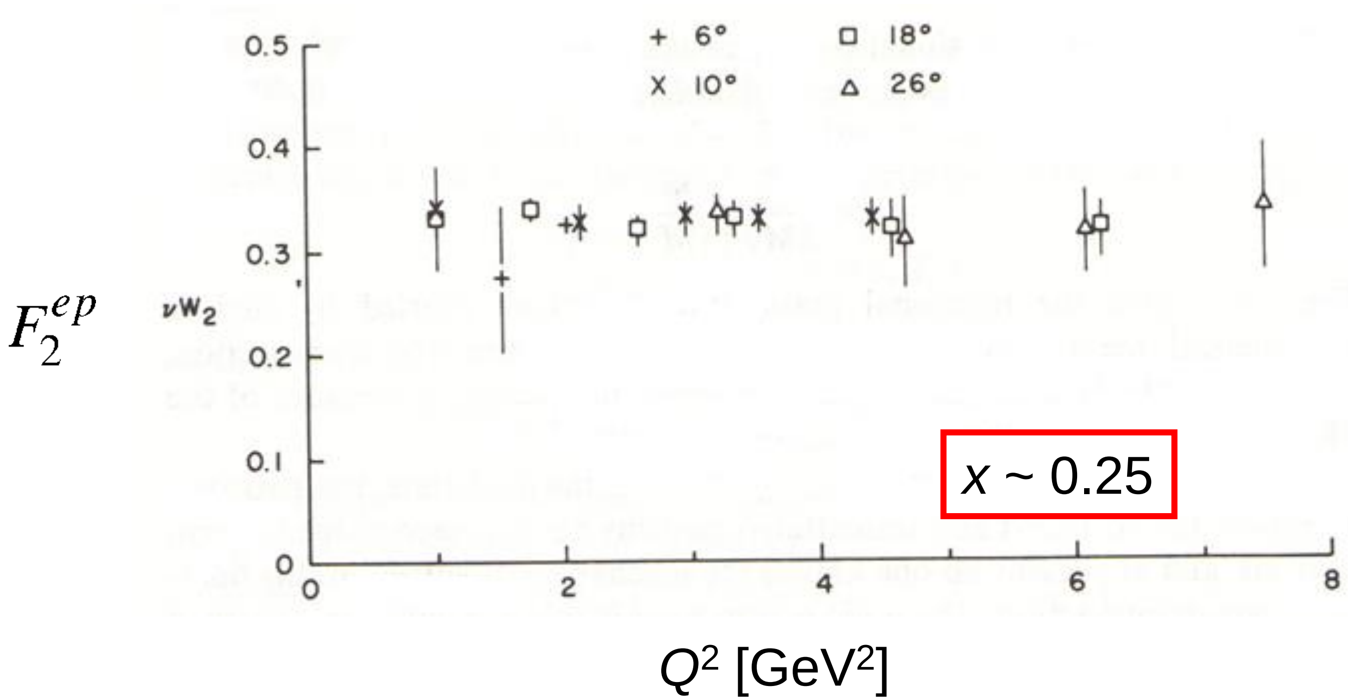
$$F_1\left(x,Q^2\right) \quad \text{and} \quad F_2\left(x,Q^2\right)$$

which are functions of x and Q^2. They cannot be interpreted as the Fourier transforms of the charge and magnetic moment distributions.
They describe the momentum distribution of the partons (quarks) within the proton.
In the Lab. frame, in the limit of high energy $Q^2 >> M^2y^2$, it becomes

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \vartheta/2}\left\{\frac{1}{\nu} F_2(x,Q^2)\cos^2\frac{\vartheta}{2} + \frac{2}{M} F_1(x,Q^2)\sin^2\frac{\vartheta}{2}\right\}$$

Electromagnetic Structure Function — pure Magnetic Structure Function

Q^2 Dependence of F_2



At x ~ 0.25, the data show (almost) no Q^2 dependence!
No obvious scale in $W^2(\nu, Q^2)$ (x ~ 0.25 accidental, but lucky choice)

Interpreted as if electron scatters elastically
off point-like constituents inside the proton,
the so called partons, identified with quarks by Feynman.

$$F_2(x) \equiv x \sum_i e_i^2 f_i(x)$$

Bjorken Scaling

Bjorken (1969) predicted that at very high energy ($Q^2 \to \infty$, $\nu \to \infty$) the dependence of W_1 and W_2 on Q^2 fades away and they become functions of *x* alone

Bjorken limit $Q^2 \to \infty$
$\nu \to \infty$
with $x_{Bj} = Q^2 / 2\nu M$ finite

$$MW_1(\nu, Q^2) \Rightarrow F_1(x)$$
$$\nu W_2(\nu, Q^2) \Rightarrow F_2(x)$$

before the SLAC DIS experiments (1973)
This behavior is know as *scaling*.
Scaling is consequence that the proton is made of free pointlike constitutents:
at high energies the virtual photon interacts with a single essentially free quark

The structure functions for scattering off a quark of flavor *i* are

$$W_1^{\text{i, point}} = e_i^2 \frac{Q^2}{4m_i^2} \delta\left(\nu - \frac{Q^2}{2m_i}\right) = e_i^2 \frac{1}{2m_i} \delta(1 - x_i)$$
$$W_2^{\text{i, point}} = e_i^2 \frac{2m_i}{Q^2} \delta\left(\nu - \frac{Q^2}{2m}\right) = e_i^2 \frac{2m_i}{Q^2} \delta(1 - x_i)$$

meaning of the δ function:
the final parton is on the mass shell (free) as the initial one

note
e_i fractional charge of quark *i*
$e_i e$ charge of quark *i*

with $x_i = Q^2 / 2q \cdot p_i$ and p_i the quark's momentum.

Although p is the proton's four-momentum, we do not know the quark's momentum p_i; let's set $p_i = \xi_i p$ ($x_i = x / \xi_i$):

$$W_1^{\text{i, point}} = e_i^2 \frac{Q^2}{4m_i^2}\delta\left(\nu - \frac{Q^2}{2m_i}\right) = e_i^2 \frac{1}{2m_i}\delta\left(1 - x_i\right) = e_i^2 \frac{1}{2M}\delta\left(\xi_i - x\right)$$

$$W_2^{\text{i, point}} = e_i^2 \frac{2m_i}{Q^2}\delta\left(\nu - \frac{Q^2}{2m}\right) = e_i^2 \frac{2m_i}{Q^2}\delta\left(1 - x_i\right) = e_i^2 \frac{2M\ x^2}{Q^2}\delta\left(\xi_i - x\right)$$

Let $f_i(\xi_i)$ be the probability that the quark i carries momentum fraction ξ_i

Integrating over ξ_i

$$W_1 = \sum_i \int_0^1 d\xi_i f_i\left(\xi_i\right)\frac{e_i^2}{2M}\delta\left(\xi_i - x\right) = \frac{1}{2M}\sum_i e_i^2 f_i\left(x\right)$$

$\Rightarrow \xi_i = x_{Bj}$

$$W_2 = \sum_i \int_0^1 d\xi_i f_i\left(\xi_i\right) q_i^2 \frac{2M\ x^2}{Q^2}\delta\left(\xi_i - x\right) = \frac{2M}{Q^2}x^2\sum_i e_i^2 f_i\left(x\right)$$

and finally

$$MW_1 = \frac{1}{2}\sum_i e_i^2 f_i\left(x\right) \equiv F_1\left(x\right)$$

$$\frac{Q^2}{2M\ x}W_2 = x\sum_i e_i^2 f_i\left(x\right) \equiv F_2\left(x\right)$$

$$F_1\left(x\right) \equiv \frac{1}{2}\sum_i e_i^2 f_i\left(x\right)$$

$$F_2\left(x\right) \equiv x\sum_i e_i^2 f_i\left(x\right)$$

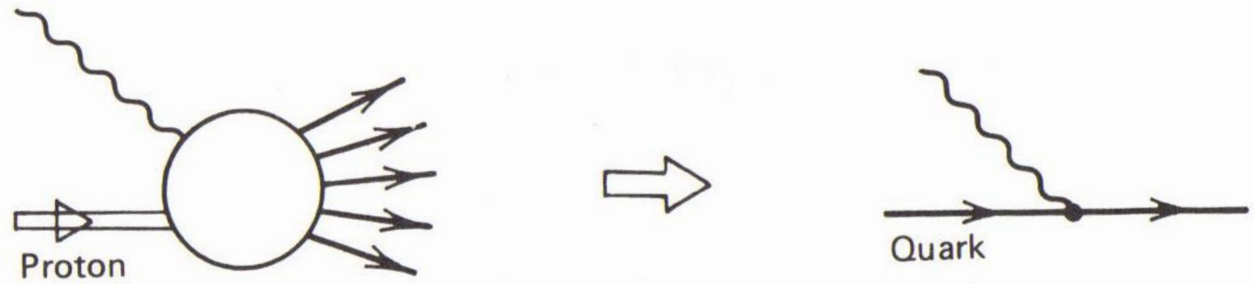
which implies also the Callan-Gross relation (DIS master formula)

$$F_2\left(x\right) = 2x\,F_1\left(x\right)$$

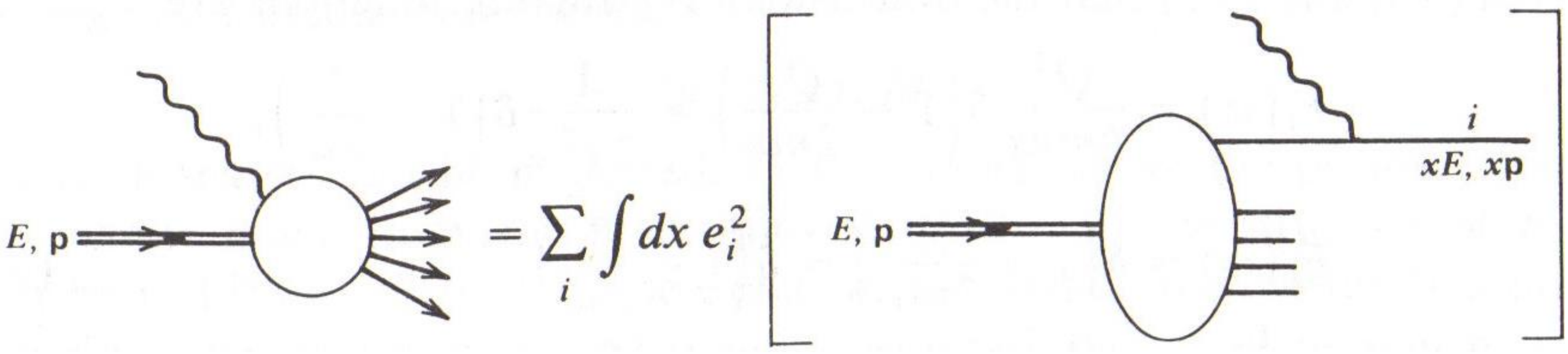
Quarks have spin ½!

Ideas Behind the QPM

1. The nucelon consists of dynamic point-like scattering centers – partons (Feynman)



2. The cross section is the incoherent sum of the scattering cross sections off the individual partons (two types: charged, spin ½ ≡ quarks and
0 charge, spin 1 ≡ gluons



3. The hadronization process occurs on a much longer time scale compared to the hard scattering ($\tau_{had} >> \tau_{int}$) and can be neglected in determining the scattering cross section.

Deep inelastic: $Q^2 >> M^2$ short distance $r \sim 1/Q << r_P$ [$Q^2 > 1$ GeV^2]

(r_P radius of the nucleon)

$\nu >> M$ EM interaction $\tau \sim 1/\nu << \tau_P$ [$\nu >$ few GeV]

(τ_P characteristic time of internal nucleon dynamics)

Before the interaction: the proton constituents are strongly bound and evolve from one configuration to another with a time τ_P, and do not see arriving the probe (photon).

The interaction: the EM interaction happens on a time scale $\tau << \tau_P$, it depends on the instantaneous configuration of the proton; during this time the constituents can be considered as free.

After the interaction: the constituents recombine to form the final hadrons with a time τ_P

Large Q^2: the interaction happens on the single constituents incoherently and measures the probability to find the constituent in a particular configuration inside the nucleon.
Incoherent: we sum the $|\text{amplitudes}|^2$

Inclusive: we do not observe the hadronic final state, we observe only the *probability* of various possible parton configurations.
inclusive cross section ≡ incoherent sum of cross sections off single elementary constituents, which are free during the interaction

In the proton rest frame **P** = 0 the constituents are strongly bounded and τ_P can be short.
In the frame where **P** is very large ($\rightarrow \infty$), τ_P can be very long (Lorenz time dilatation).
The cross section can be calculated in any frame, provided it is relativistically invariant.

QPM Cross Section

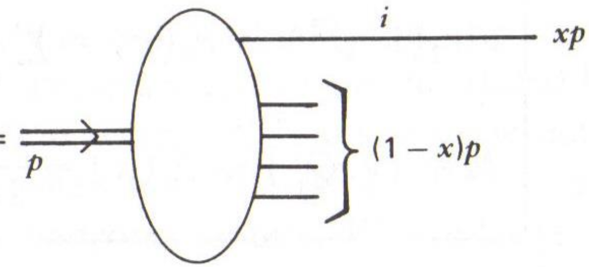
The partons (quarks) carry a fraction *x* of the nucleon's momentum and energy:

proton $\begin{cases} E \\ p_L \\ p_T = 0 \\ M \end{cases}$ parton $\begin{cases} xE \\ xp_L \\ p_T = 0 \\ m = xM \end{cases}$

assume p_T = 0 (collinear)
note that the mass of a parton is a not well defined concept

$f_i(x)$ parton momentum distribution function
i.e. probability to find a quark of flavor *i* carrying a momentum *x* of the proton

$$f_i(x) = \frac{dP_i}{dx} =$$



Then the cross section can be written as

$$\frac{d^2\sigma}{dQ^2} = \sum_i \int \mathrm{d}x \; e_i^2 f_i(x) \left(\frac{d\sigma}{dQ^2}\right)_{\text{point}} \quad \Rightarrow \quad \frac{d^2\sigma}{dx dQ^2} = \sum_i e_i^2 f_i(x) \left(\frac{d\sigma}{dQ^2}\right)_{\text{point}}$$

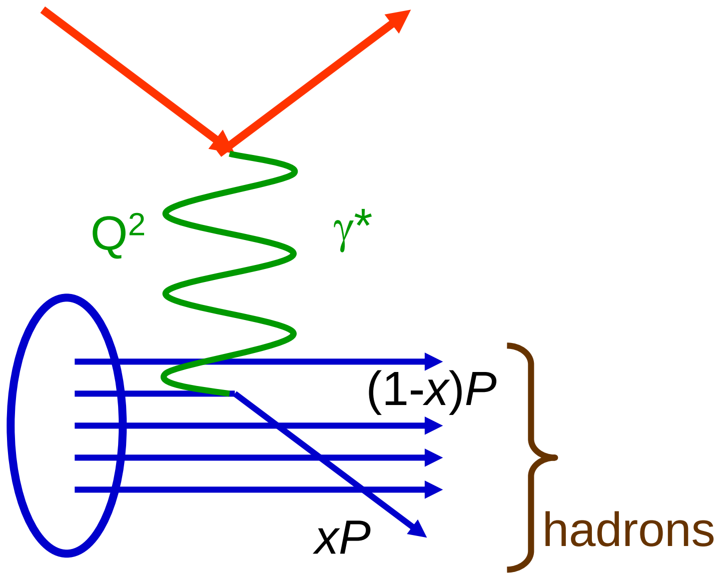
and inserting the $e\mu \rightarrow e\mu$ cross section (point-like) we obtain

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4 x} \frac{E'}{E} \left[\sum_i e_i^2 x f_i(x) \cos^2\frac{\vartheta}{2} + \frac{\nu}{M} \sum_i e_i^2 f_i(x) \sin^2\frac{\vartheta}{2} \right]$$

$\nu W_2 = F_2$ $\quad$ $2MW_1 = 2F_1$

Quark – Parton Model

$$d\sigma(x,Q^2) \sim \frac{1}{Q^4} L_{\mu\nu} W^{\mu\nu} \rightarrow \quad d\sigma(x,Q^2) \sim q(x,Q^2) \times d\hat{\sigma}^{lq\rightarrow lq}$$



quark densities

electron – quark scattering
(point-like, calculable in QED !)

we factorize the cross section in a calculable part and in a non calculable one

- interior structure of nucleon
  - valence and sea quark distributions
- quantitative QCD tests
  - scaling violation
  - gluon density
  - running $\alpha_S(Q^2)$
  - renormalization group

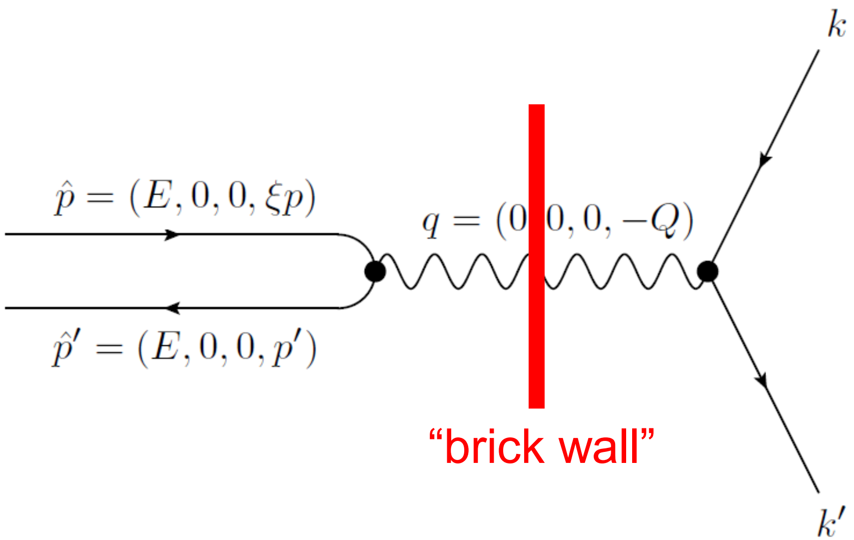
Important limit (Bjorken scaling limit):

$Q^2 \rightarrow \infty$

$\nu \rightarrow \infty$ with Q^2 / ν finite

$x_{Bj} \equiv x \equiv P_q / P_N$ $\quad x \equiv$ proton's momentum fraction carried by the struck quark

The Breit Frame



$\vec{k} = +\vec{q}/2 \qquad \vec{k}' = -\vec{q}/2$ electron

$\vec{p} = -\vec{q}/2 \qquad \vec{p}' = +\vec{q}/2$ proton

$q^\mu = (0, \vec{q}) \quad q^0 = \nu = 0$ photon

Since $q^2 < 0$ we can boost the photon along its direction of propagation such that q^0 vanish, i.e. $\nu = 0$: the photon carries momentum **q** but no energy

$\Rightarrow$ no energy transfer to the proton: $\nu = 0$

the partons are collinear, no transverse momentum w.r.t. to the proton momentum

Breit frame now also as infinite momentum frame,

since the proton moves with very high momentum toward the photon

Strictly speaking, the parton model works only in this frame:

(for a proton at rest, what is the momentum fraction x_i carried by the quark?)

Spin of Quarks

In the QPM, the F_1 and F_2 structure functions are related by the Callan-Gross relation

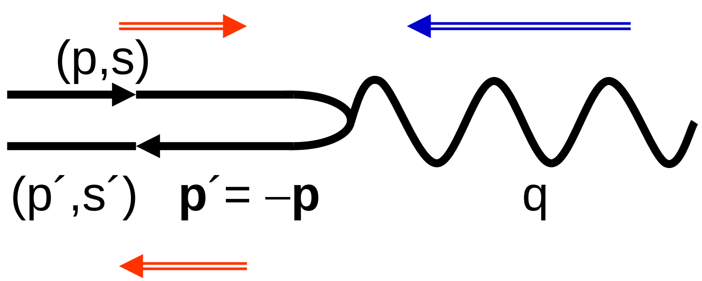
$$F_2(x) = 2xF_1(x) = \sum_i e_i^2 x f_i(x) = \sum_i e_i^2 x\left[q_i + \bar{q}_i\right]$$

The Callan-Gross relation reflects the spin ½ nature of quarks. ($F_L \to 0$)

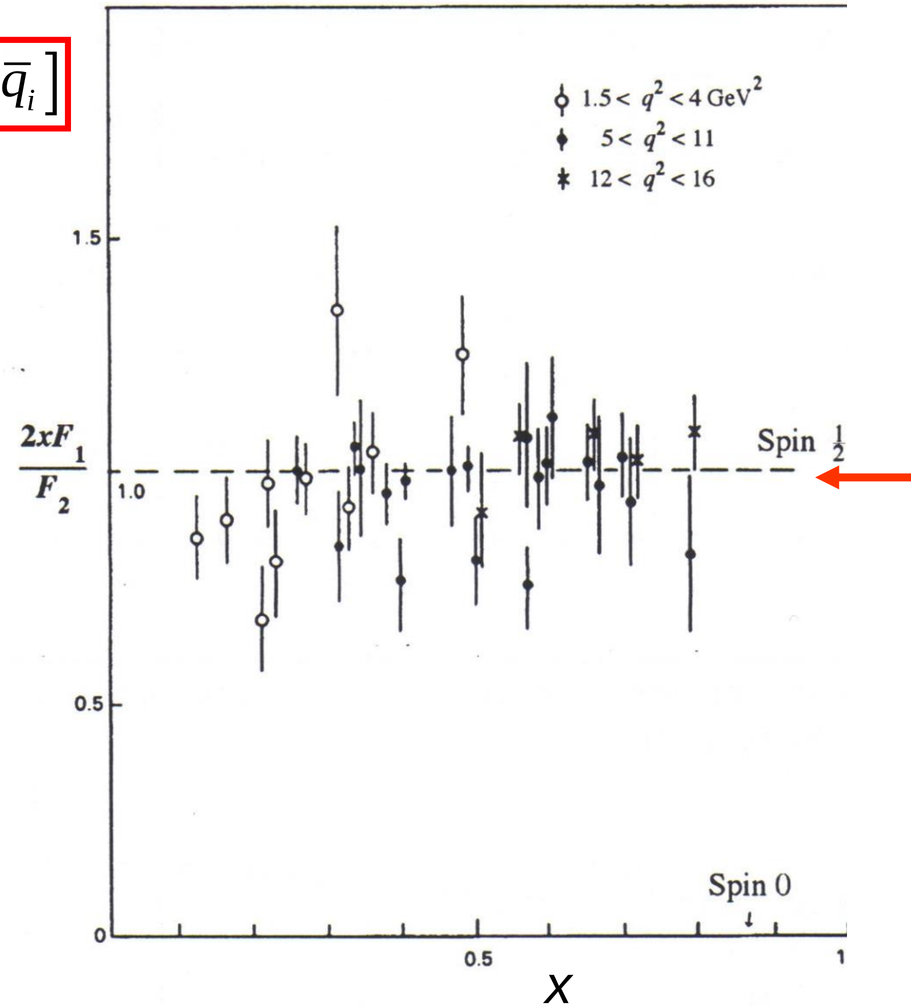
For spin 0 quarks, one would expect that the transverse (purely magnetic) structure function F_1 (x) vanishes (i.e. = 0)

spin ½	$2xF_1 / F_2$	$\to 1$
spin 0	$2xF_1 / F_2$	$\to 0$

$J_z = -½$



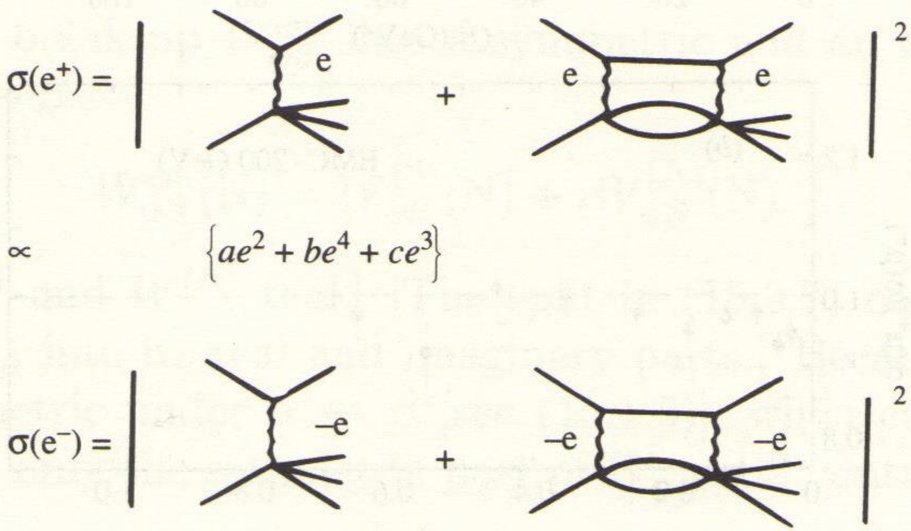
$J_z = -½$



a spin ½ quark can absorb a transverse photon $\sigma_L/\sigma_T \to 0$
a spin 0 quark cannot absorb a transverse photon $\sigma_L/\sigma_T \to \infty$

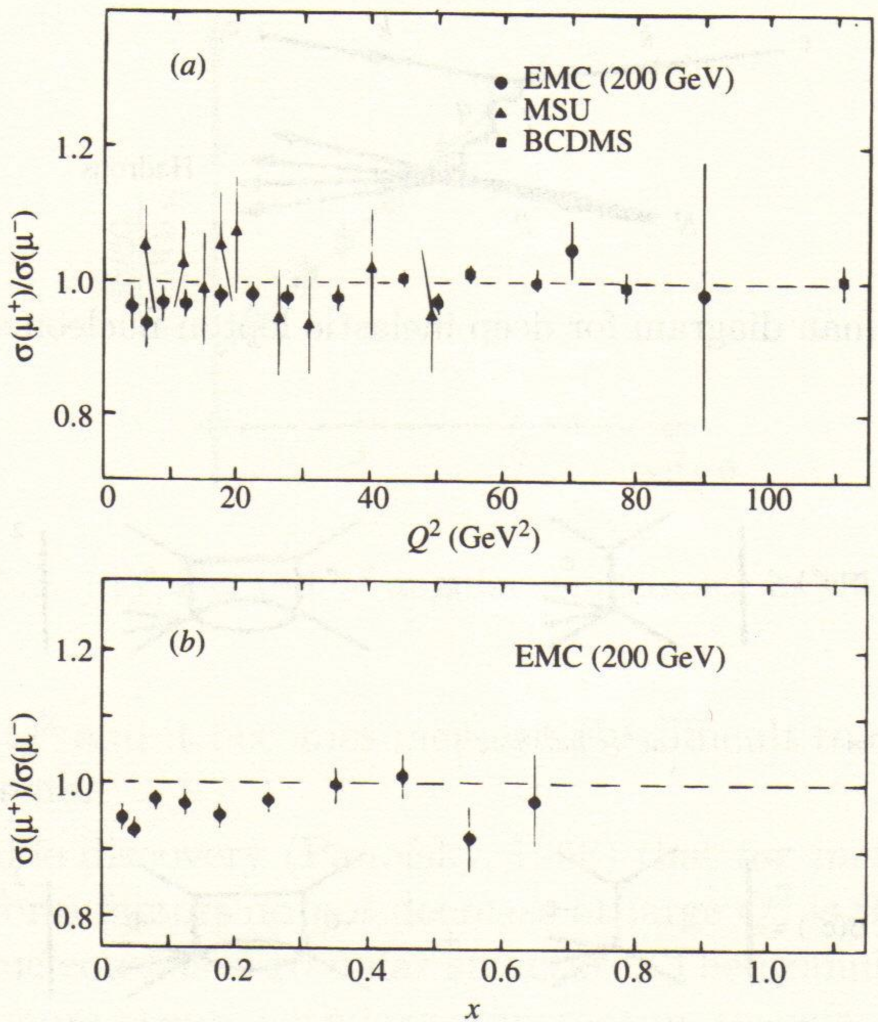
Is the 1-γ Approximation Good Enough ?

Consider e^+p and e^-p scattering



$$\sigma(e^+) \propto \{ae^2 + be^4 + ce^3\}$$

$$\sigma(e^-) \propto \{ae^2 + be^4 - ce^3\}$$



In the 2–γ exchange the interference term has opposite sign

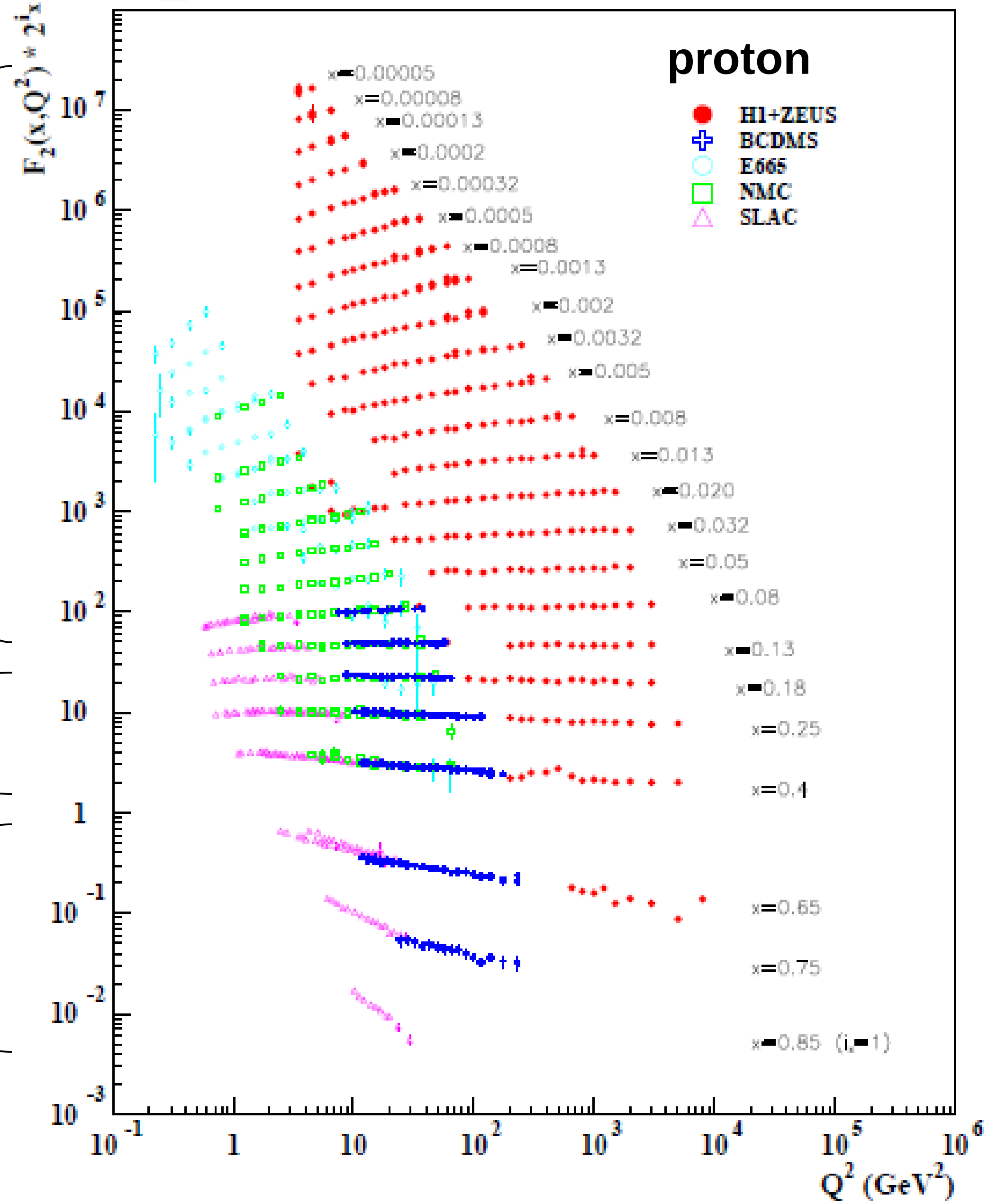
⇒ if 2–γ exchange important, expect large difference between $\sigma(e^+)$ and $\sigma(e^-)$

Q^2 Dependence of F_2

Strong Q^2 (⇑) dependence

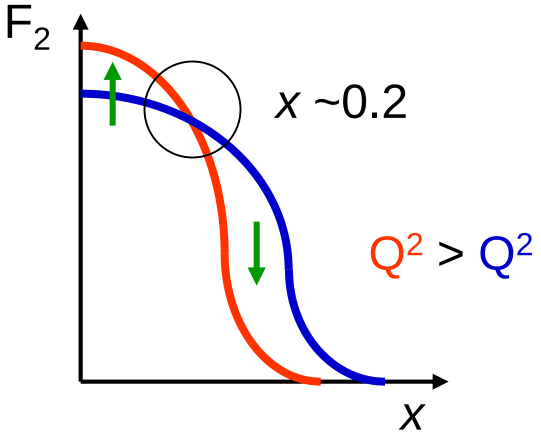
Almost no Q^2 dependence

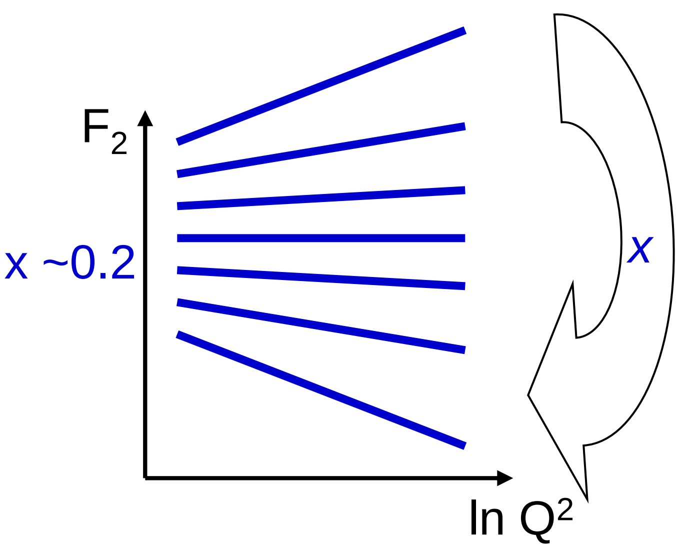
Strong Q^2 (⇓) dependence

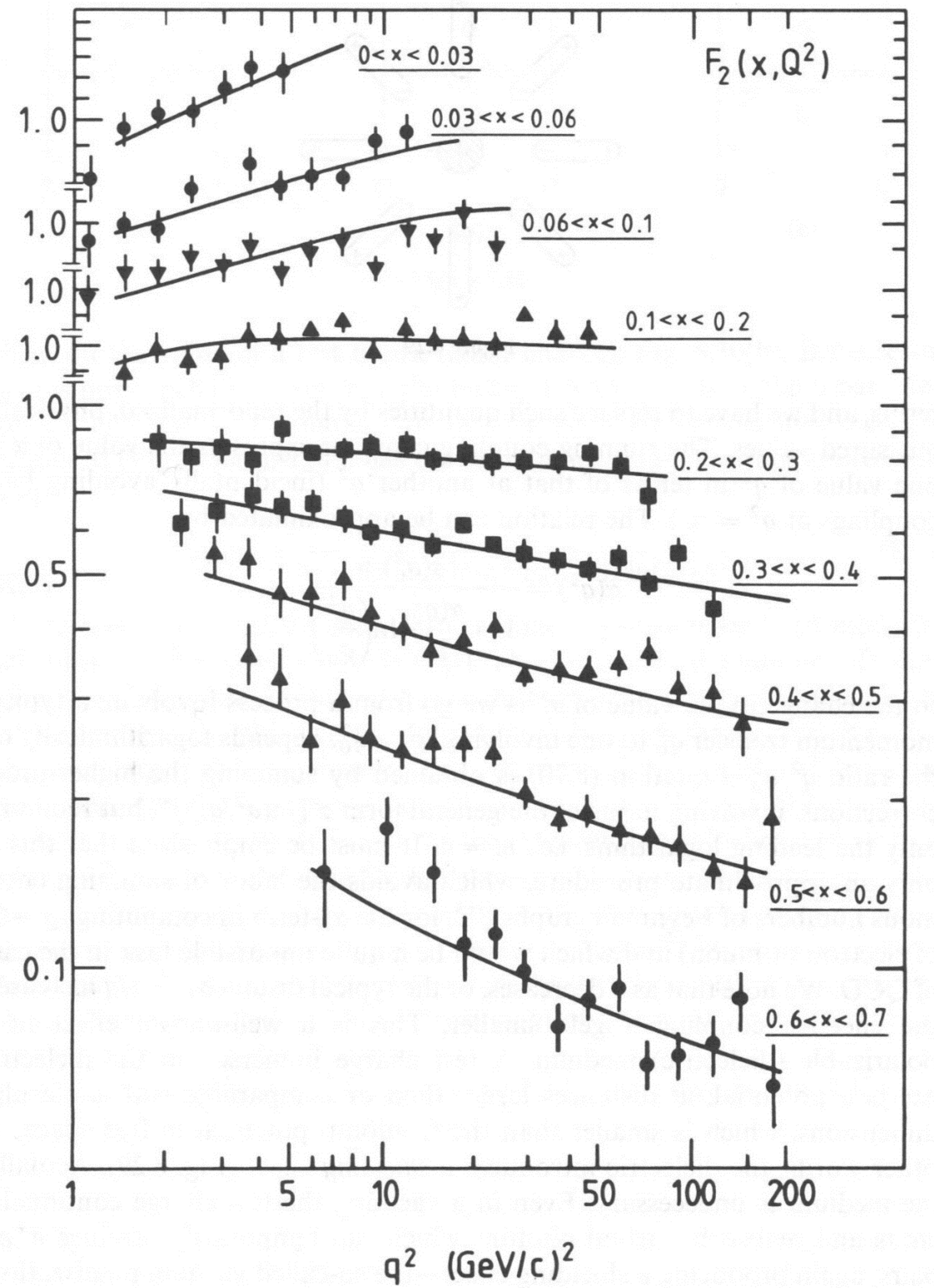


[~ 40 years of research !]

Scaling Violations







something is missing ⇒ gluons

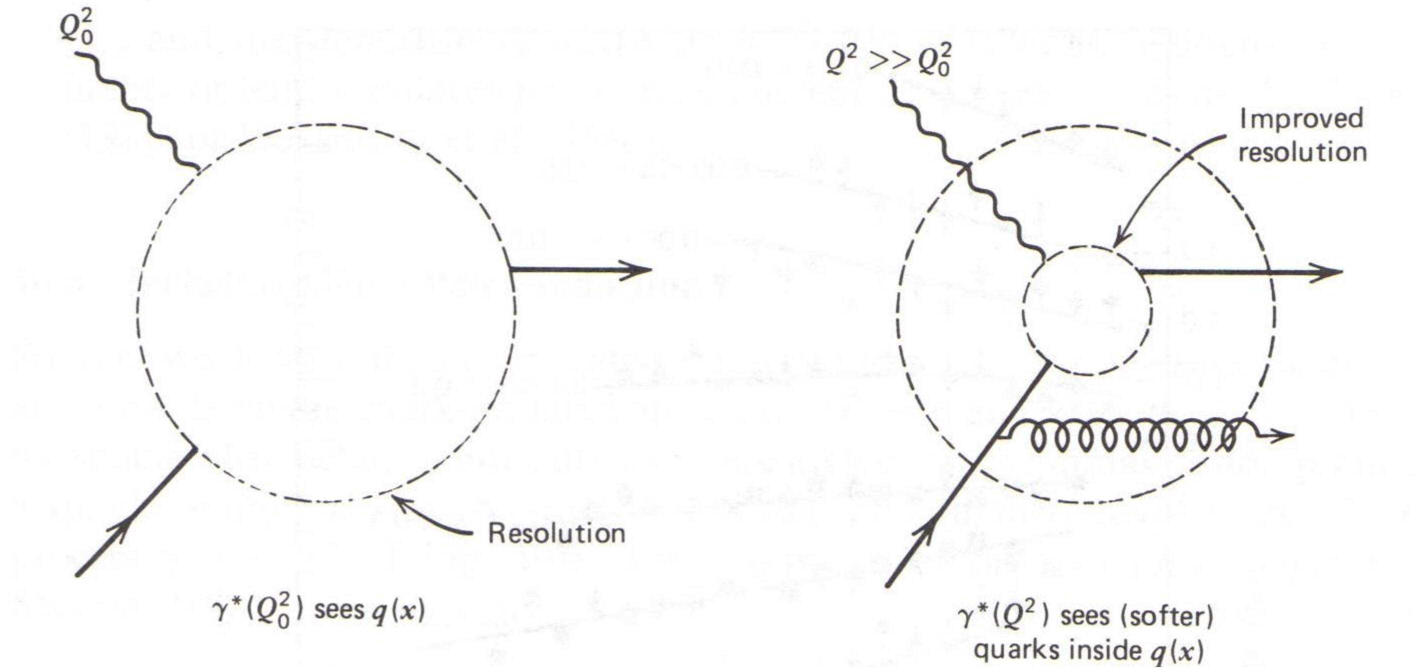
Scaling Violation

F_2 is Q^2 dependent.

Is the Quark – Parton model picture consistent with pQCD ?

YES
with modifications

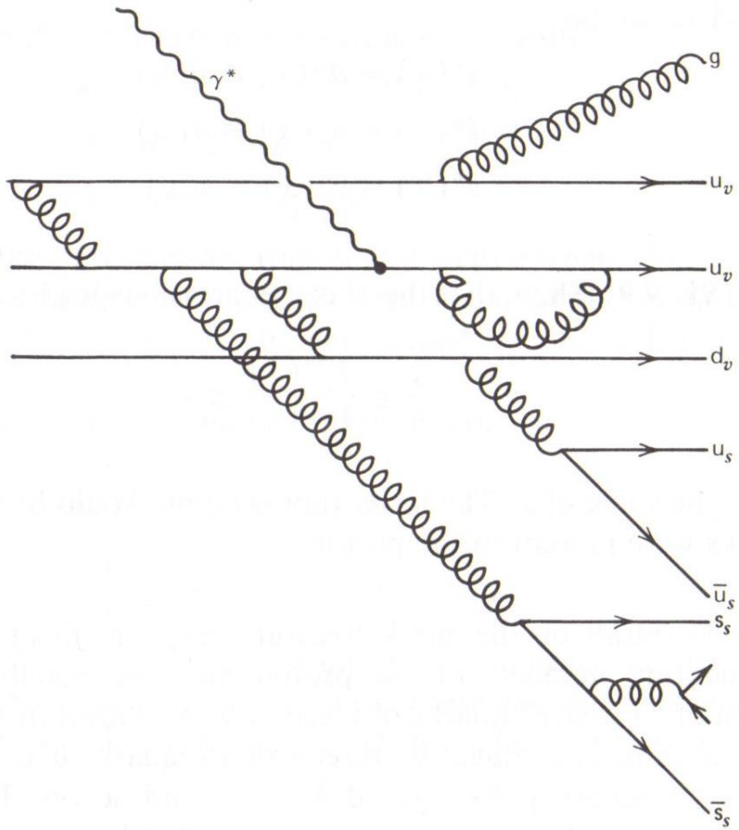
asymptotic freedom
(logarithmic) scaling violations
⇒ very powerful test of QCD

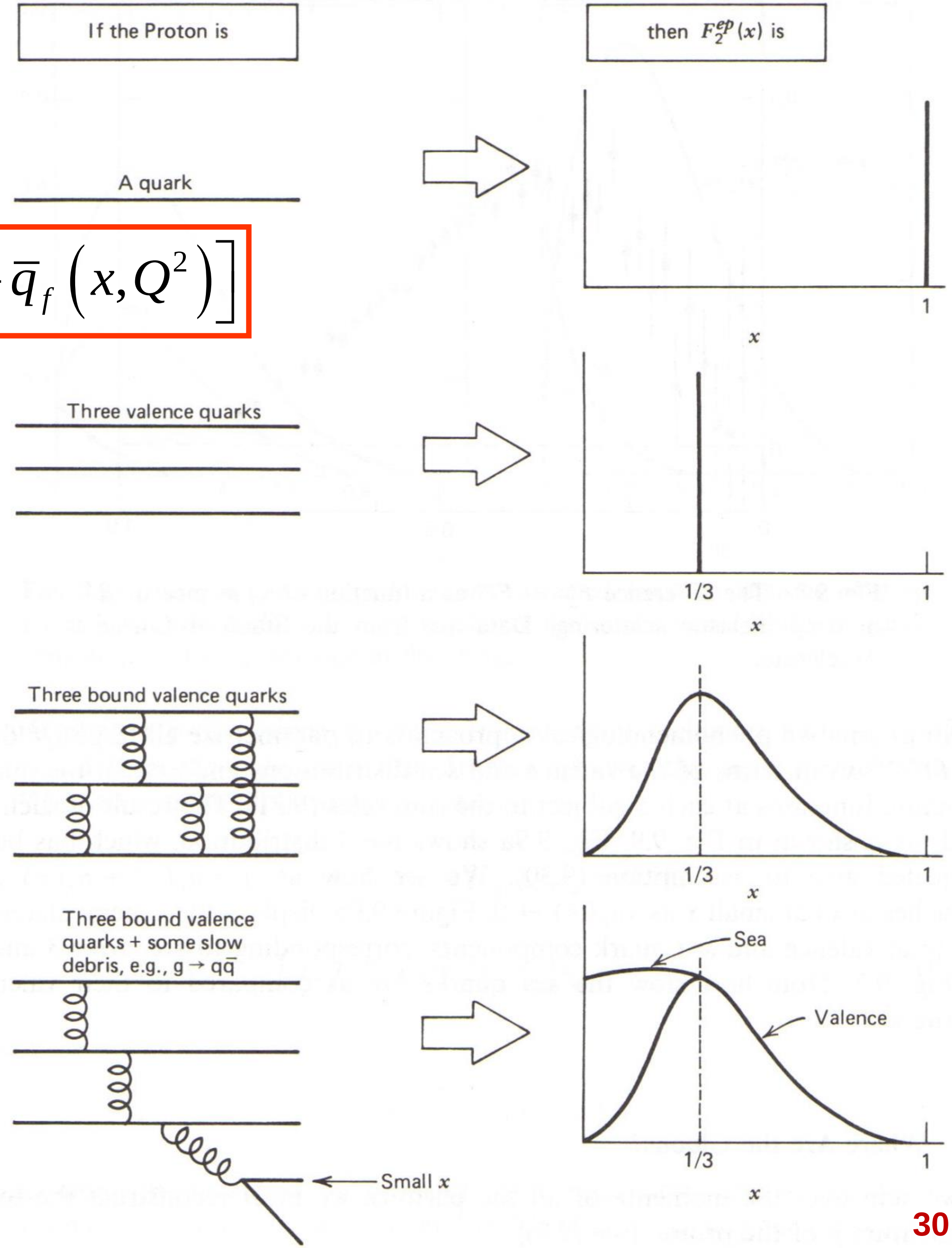


QCD cannot predict the parton distribution functions $f(x)$.
However once they are measured at a given Q_0^2,
QCD can predict their evolution to a different Q^2 ($Q^2 > Q_0^2$ or $Q^2 < Q_0^2$).

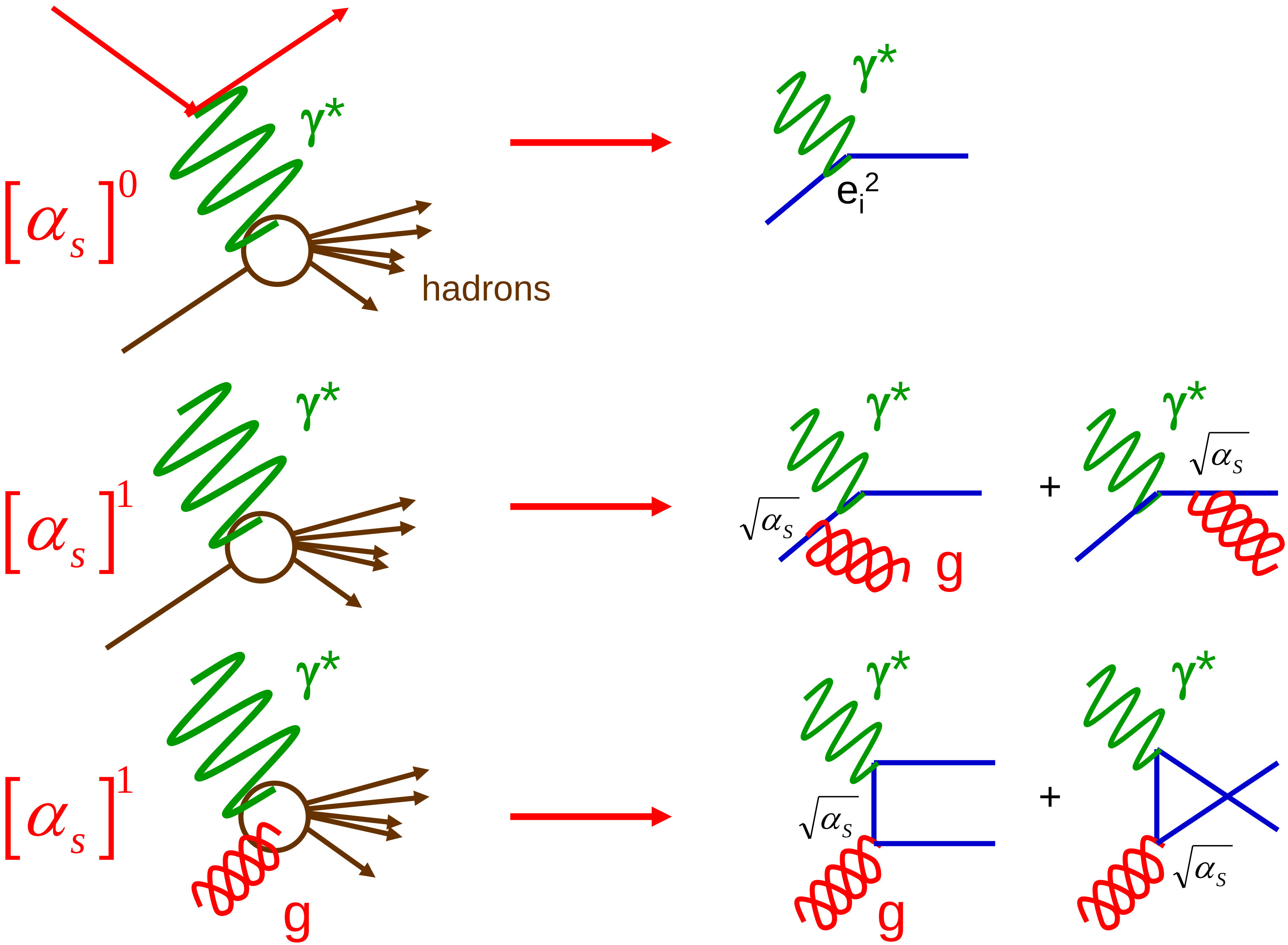
Evolving Nucleon

$$F_2^{ep}\left(x,Q^2\right)=x\sum_f e_f^2\left[q_f\left(x,Q^2\right)+\bar{q}_f\left(x,Q^2\right)\right]$$



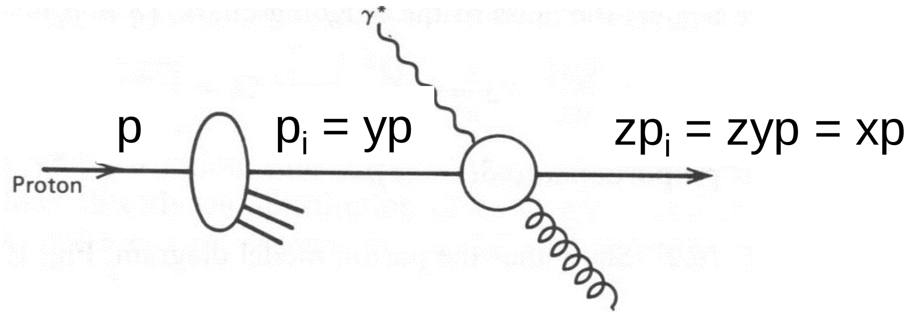


Including α_S Contributions



These are the lowest order α_S corrections to the DIS cross section. So far α_S corrections have been calculated up to $\alpha_S{}^3$ (NNLO – next-to-next-to-leading order).

The γ^*-Proton and γ^*-Parton Frame



we work in the
infinite momentum frame
Breit frame

γ^* - proton frame

p

$x = Q^2 / 2p \cdot q$

γ^* - parton frame

$p_i = yp$

$z = Q^2 / 2p_i \cdot q = x / y$

$x = zy$

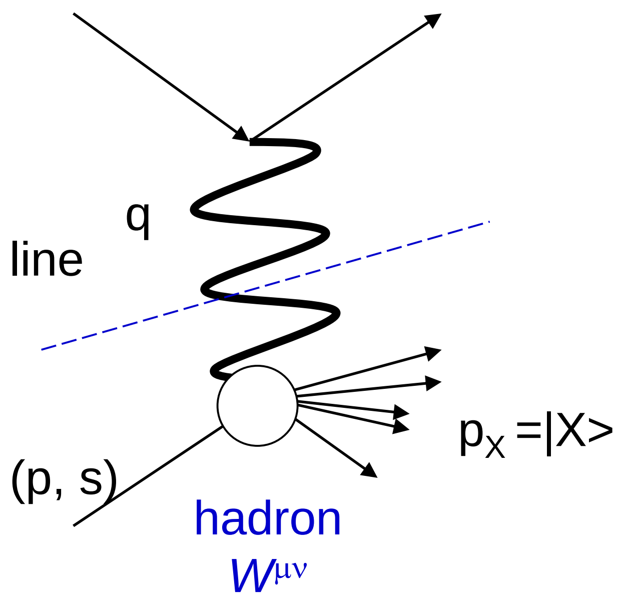
y – momentum fraction of the parton i before gluon emission

z – momentum fraction of the parton i after gluon emission w.r.t. initial parton momentum *yp* with the conditions $x = zy$ and $y > x$.

The virtual photon sees a parton with momentum fraction *x*.
However, if we look more closely, the parton with momentum fraction *x* originated from a parton of momentum fraction $y > x$, which emits a gluon with momentum fraction $1 - z$ (the momentum fraction of the parton after gluon emission is z w.r.t. y!).

The parton interacting with the γ^* then has momentum $yzp = xp$.

Embedding γ^*-Parton Processes in DIS



In order to embed the higher order α_S diagrams (QCD) in the structure functions W_1 and W_2 ,
we have to express $W_{1,2}$ in terms of $\gamma^* - p$ cross sections.

The important question is what happens below the dashed line with the electron the source of virtual photons.

$\gamma^* - q$ invariant amplitude $$-iM = -ie_i e\varepsilon^\mu \; \bar{u}(p')\gamma_\mu u(p)$$

$\gamma^* - p$ invariant amplitude $$-iM = -i\varepsilon^\nu e_i e \langle X | J_\nu^{had} | p,s \rangle$$

$\gamma^* - p$ cross sections (sum / integrate over all possible final states X)

$$\sigma^{tot}(\gamma p \to X) = \frac{1}{2K2M}\frac{1}{2}\sum_{spins}\sum_X \varepsilon^{*\mu}\varepsilon^\nu e^2 \langle p,s|J^\dagger{}_\mu|X\rangle\langle X|J_\nu|p,s\rangle(2\pi)^4\delta^{(4)}(p+q-p_X)$$

recall the hadron tensor $W_{\mu\nu}(p,q) = \frac{1}{4\pi M}\frac{1}{2}\sum_{spins}\sum_X \langle p,s|J^\dagger{}_\mu|X\rangle\langle X|J_\nu|p,s\rangle(2\pi)^4\delta^{(4)}(p+q-p_X)$

The $\gamma^* - p$ cross sections is finally given by with K (ν = K for real photons) the flux factor

$$\sigma_\lambda^{tot} = \frac{4\pi^2\alpha}{K}\left(\varepsilon_\lambda^{*\mu}\varepsilon_\lambda^\nu W_{\mu\nu}\right)$$

Photon Polarization

Real photons ($q^2 = 0$): left / right circularly polarized light

helicity $\lambda = \pm 1$

$$\lambda = 1 \qquad \varepsilon_+ = \frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\lambda = -1 \qquad \varepsilon_- = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$q^\mu = \begin{pmatrix} \nu \\ 0 \\ 0 \\ \nu \end{pmatrix} \qquad q^2 = 0 \qquad q_\mu \varepsilon^\mu = 0$$

$$\sum_\lambda \varepsilon^{\mu,*} \varepsilon^\nu = -g^{\mu\nu}$$

Virtual photons ($q^2 < 0$)

helicity $\lambda = \pm 1, 0$

$$\lambda = 1 \qquad \varepsilon_+ = \frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\lambda = 0 \qquad \varepsilon_0 = \frac{1}{\sqrt{Q^2}}\left(\sqrt{Q^2+\nu^2},0,0,\nu\right)$$

$$\lambda = -1 \qquad \varepsilon_- = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$q^\mu = \begin{pmatrix} \nu \\ 0 \\ 0 \\ \sqrt{Q^2+\nu^2} \end{pmatrix} \qquad q^2 < 0 \qquad q_\mu \varepsilon^\mu = 0$$

$$\sum_\lambda \varepsilon^{\mu,*} \varepsilon^\nu = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}$$

The flux of virtual photons is not a well defined concept (it diverges for $Q^2 \to 0$)

Hand convention: require $W^2 = M^2 + 2MK$ to hold (energy of the hadronic system)

$\Rightarrow K = (W^2 - M^2) / 2M = \nu - Q^2 / 2M$

Recall the hadron tensor in terms of W_1 and W_2

$$W^{\mu\nu} = -W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

Then

$$\sigma_+ = \frac{4\pi^2\alpha}{K}\left(+\varepsilon_+^{*\mu}\varepsilon_+^\nu g_{\mu\nu}W_1 + \varepsilon_+^{*\mu}\varepsilon_+^\nu p_\mu p_\nu \frac{W_2}{M^2}\right)$$

and

$$\sigma_L = \frac{4\pi^2\alpha}{K}\left(-\varepsilon_0^{\mu}\varepsilon_0^{*\nu} g_{\mu\nu}W_1 + \varepsilon_0^{\mu}\varepsilon_0^{*\nu} p_\mu p_\nu \frac{W_2}{M^2}\right)$$

Contract the tensor products

$$\varepsilon_\pm^\mu\varepsilon_\pm^{*\nu}T_{\mu\nu}^1 = 1 \qquad \varepsilon_\pm^\mu\varepsilon_\pm^{*\nu}T_{\mu\nu}^2 = 0 \qquad \varepsilon_L^\mu\varepsilon_L^{*\nu}T_{\mu\nu}^1 = -1 \qquad \varepsilon_L^\mu\varepsilon_L^{*\nu}T_{\mu\nu}^2 = \frac{Q^2+\nu^2}{Q^2}$$

The cross section for transversely polarized photons is then

$$\sigma_T = \frac{\sigma_+ + \sigma_-}{2} = \frac{4\pi^2\alpha}{K}W_1\left(\nu, Q^2\right)$$

and for longitudinally polarized photons

$$\sigma_L = \frac{4\pi^2\alpha}{K}\left[\left(\frac{Q^2+\nu^2}{Q^2}\right)W_2\left(\nu,Q^2\right) - W_1\left(\nu,Q^2\right)\right]$$

$Q^2 \to 0$ limit
(real photons)

$$\sigma_T = \sigma^{\text{tot}}\left(\gamma N\right) = \frac{4\pi^2\alpha}{K}W_1 \qquad \sigma_L \to 0$$

Back to the Parton Model

In the DIS limit this reduces to

$$2F_1 = \frac{\sigma_T}{\sigma_0}$$

$$\frac{F_2}{x} = \frac{\sigma_T + \sigma_L}{\sigma_0}$$

with σ_0 a *normalization* factor

$$\sigma_0 = \frac{4\pi^2\alpha}{2MK} \approx \frac{4\pi^2\alpha}{s}$$

Finally we can express F_2 in terms of the $\gamma^* - q$ interaction

$$\frac{F_2(x,Q^2)}{x} = \left(\frac{\sigma_T(x,Q^2)}{\sigma_0}\right)_{\gamma^* p} = \sum_i \int_0^1 dz \int_0^1 dy\, f_i(y)\delta(x - zy)\left(\frac{\hat{\sigma}_T(z,Q^2)}{\hat{\sigma}_0}\right)_{\gamma^* i}$$

where σ_T, σ_0 are the cross sections describing $\gamma^* - p$ and $\gamma^* - q$ interactions (photo-absorption)

After integration over z (y > x!)

$$\frac{F_2(x,Q^2)}{x} = \left(\frac{\sigma_T(x,Q^2)}{\sigma_0}\right)_{\gamma^* p} = \sum_i \int_x^1 \frac{dy}{y} f_i(y)\left(\frac{\hat{\sigma}_T(x/y,Q^2)}{\hat{\sigma}_0}\right)_{\gamma^* i}$$

If all QCD effects were absent (i.e. no gluon emission) we should recover the parton model result for $\gamma^* q \to q$:
in this case $z = 1$ since the parton carries the full momentum fraction of the initial parton and y = x:

$$\frac{\hat{\sigma}_T\left(z,Q^2\right)}{\hat{\sigma}_0} = e_i^2\delta(1-z) \qquad \text{and} \qquad \hat{\sigma}_L\left(z,Q^2\right) = 0$$

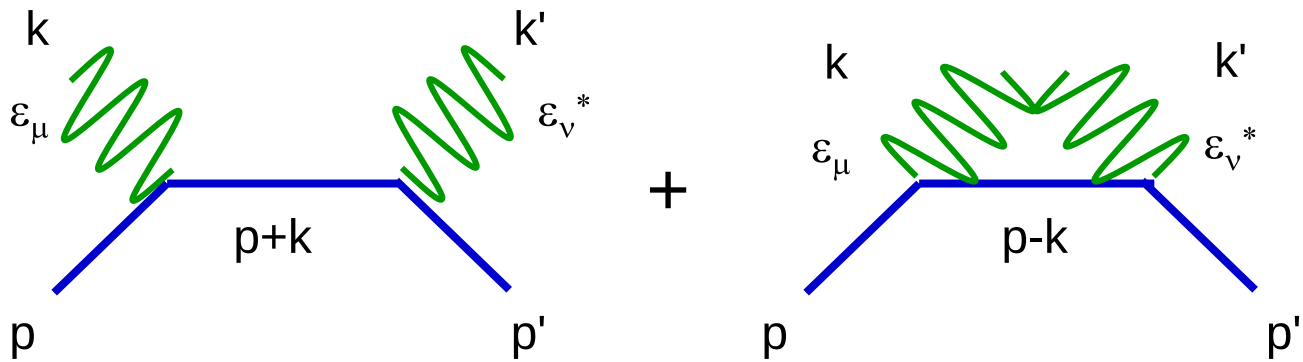
(i.e. the $\gamma^* q \to q$ cross section)

This gives

$$\frac{F_2\left(x,Q^2\right)}{x} = \sum_i e_i^2 \int_x^1 \frac{dy}{y} f_i(y)\delta\left(1-x/y\right) = \sum_i e_i^2 f_i(x)$$

which is the parton model expression for F_2 with no Q^2 dependence.
In summary, we can express the structure functions in terms of partonic cross sections.

Compton Scattering



$$\sum_T \varepsilon_\mu^{T*}\varepsilon_{\mu'}^{T*} = -g_{\mu\mu'}$$

$$\left\langle |M_1|^2 \right\rangle = 32\pi^2 e_i^2 \alpha^2 \left(-\frac{u}{s}\right) \qquad \left\langle |M_2|^2 \right\rangle = 32\pi^2 e_i^2 \alpha^2 \left(-\frac{s}{u}\right) \qquad \left\langle M_1 M_2^* \right\rangle = 0$$

$$\left\langle |M|^2 \right\rangle = 32\pi^2 e_i^2 \alpha^2 \left(-\frac{u}{s} - \frac{s}{u}\right)$$

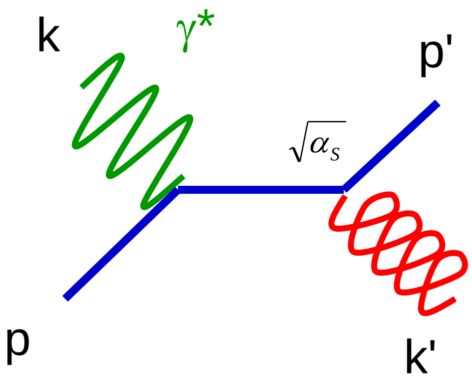
there is no interference term for real photons

For a virtual photon of mass $k^2 = -Q^2$ we have to take into account also the longitudinal polarization of the γ^* and the interference term $M_1 M_2^*$ does not vanish

$$\left\langle |M|^2 \right\rangle = 32\pi^2 e_i^2 \alpha^2 \left(-\frac{u}{s} - \frac{s}{u} + \frac{2Q^2 t}{su}\right)$$

with $s = (k+p)^2 = M \ + 2k\cdot p - Q^2 = 2k'\cdot p'$

The Gluon Emission Cross Section



To calculate the $\gamma^* q \rightarrow qg$ scattering cross section recall the QED Compton scattering $\gamma^* e \rightarrow \gamma e$.

The incoming photon is a virtual photon γ^*
and the ordering of the outgoing particles is different
(t – channel instead of the u – channel, so u → t, t → u)

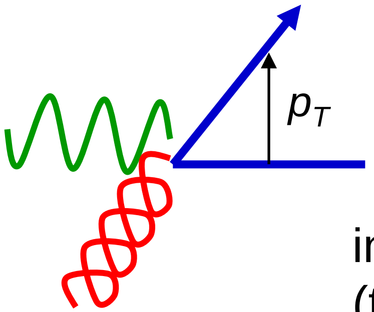
$$\left\langle |M|^2 \right\rangle = 32\pi^2 \left(e_i^2 \alpha \alpha_S\right) \frac{4}{3} \left(-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}} \right)$$

We replaced the photon emission vertex with the gluon emission vertex by substituting $\alpha \rightarrow \alpha_S$ and inserting the color factor $C_F = 4/3$ (and $\alpha \rightarrow e_i^2\alpha$ for the quark *i*).
The interference term appears because the photon is virtual (absent for a real γ).
The incoming and outgoing quark and emitted gluon are on the mass shell (real).

After inserting the phase space and flux factors the cross section is given by

$$\frac{d\hat{\sigma}(\gamma^* q \rightarrow qg)}{dt} = \frac{1}{16\pi\,\hat{s}^2} \left\langle |M|^2 \right\rangle = \frac{8\pi\, e_i^2 \alpha\, \alpha_S}{3\hat{s}^2} \left(\frac{1}{-\hat{t}} \right) \left(\hat{s} + \frac{2(\hat{s} + Q^2)Q^2}{\hat{s}} \right)$$

The interesting quantity is the transverse momentum p_T of the outgoing quark with respect to the incoming virtual photon (p_T can be measured from the outgoing hadron / jet direction w.r.t. the γ^*, and is observable)



$$p_T^2 = \frac{\hat{s}\hat{t}\hat{u}}{\left(\hat{s}+Q^2\right)^2} \xrightarrow[-\hat{t} << \hat{s}]{} \frac{-\hat{s}\hat{t}}{\hat{s}+Q^2} \qquad \text{and} \qquad d\Omega = \frac{4\pi}{\hat{s}} dp_T^2$$

in a sense p_T^2 measures the *hardness* of the interaction ($p_T^2 \sim -t$)
(the above expression is valid for small angle scattering and ($-t << s$))

For small angle scattering ($d\sigma / d\Omega$) the cross section as a function of $p_T^2 = -t$ is given by

$$\frac{d\hat{\sigma}\left(\gamma^* q \to qg\right)}{dp_T^2} \approx \frac{8\pi\, e_i^2 \alpha\, \alpha_S}{3\hat{s}^2}\left(\frac{1}{p_T^2}\right)\left(\hat{s} + \frac{2(\hat{s}+Q^2)Q^2}{\hat{s}}\right)$$

Recalling $z = \frac{Q^2}{2p_i \cdot q} = \frac{Q^2}{\hat{s}+Q^2}$ and the expression for p_T^2, we can rewrite the above cross section as

$$\frac{d\hat{\sigma}\left(\gamma^* q \to qg\right)}{dp_T^2} \approx e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} \frac{4}{3}\left(\frac{1+z^2}{1-z}\right) = e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{q\leftarrow q}\left(z\right)$$

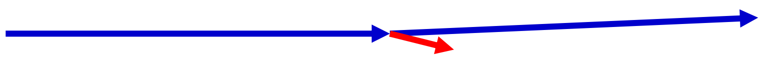
with $\sigma_0 = 4\pi^2\alpha / s$ and

$$P_{q\leftarrow q}\left(z\right) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

Splitting Functions

$$P_{q\leftarrow q}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

$P_{q\leftarrow q}$ is known as the quark splitting function.
It gives the probability of a quark emitting a gluon with momentum fraction $1 - z$.
The quark momentum fraction is $x = yz$.
The singularity for $z \to 1$ is associated with a soft massless gluon emission.



It is an example of an infrared divergence, which cancels when taking into account also the virtual QCD corrections for the γ^*q vertex (lines and vertex).

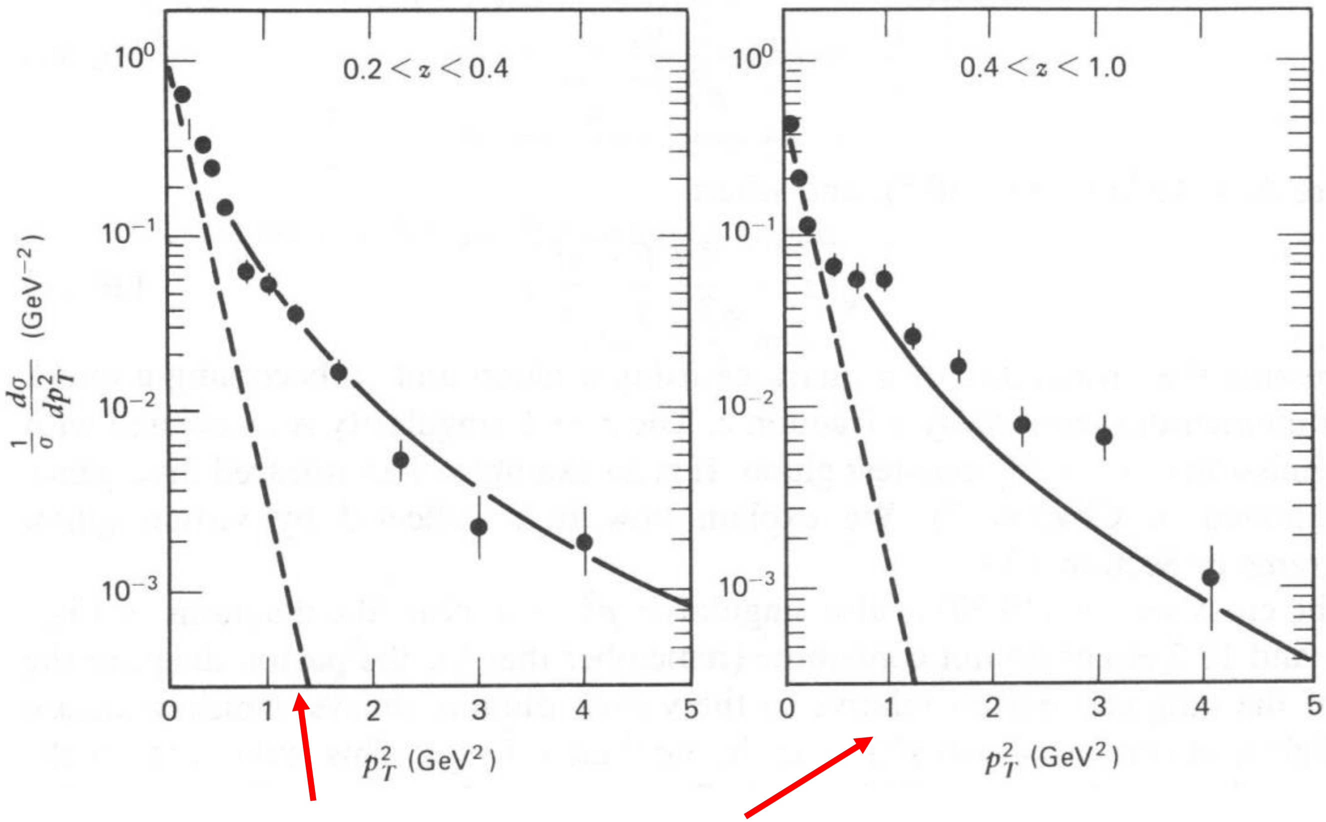
The visible effect of gluon emission (radiation) is that the emitted hadron / jet will have a transverse momentum p_T w.r.t. the incident γ^* direction.

p_T Distributions: Evidence for Gluons

Evidence in the cross section!

$\mu + p \rightarrow \mu' + h + X$

p_T distribution of produced hadrons



expectations with no gluon radiation

(not evidence of sub-structure !)

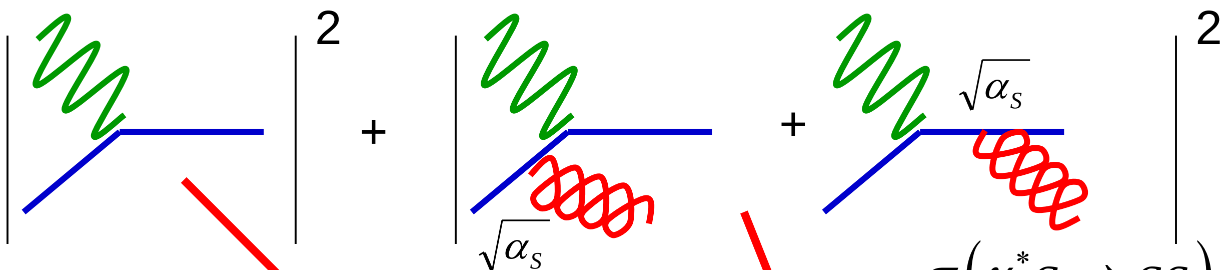
$F_2(x,Q^2)$

To evaluate the overall contribution of the gluon emission cross section to F_2 we integrate the partonic cross section over p_T^2 over the whole allowed p_T range (we have to introduce a cutoff – min $p_T^2 = \mu^2$ to regularize the infrared divergence):

$$\hat{\sigma}(\gamma^* q \to qg) = \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2} \approx e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{q\leftarrow q}(z) = e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_S}{2\pi} P_{q\leftarrow q}(z) \log \frac{Q^2}{\mu^2} \right)$$

Adding this contribution to the parton model cross section, we arrive at the following α_S QCD effect

$$\frac{F_2(x,Q^2)}{x} \sim$$



$$\sigma(\gamma^* q \to qg) \sim \alpha_S \log Q^2/\mu^2$$

$$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left[q(y) \left(\delta(1 - x/y) + \frac{\alpha_S}{2\pi} P_{q\leftarrow q}(x/y) \log \frac{Q^2}{\mu^2} \right) \right]$$

The presence of the log Q^2 term means that the scaling prediction (no Q^2 dependence) is not anymore valid (it is violated).

In QCD, F_2 is a function of x and Q^2. The variation of F_2 with Q^2 is logarithmic (weak).

We can rewrite F_2 in the parton-like form:

$$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \int_x^1 \frac{\mathrm{d}y}{y}\left(q(y) + \Delta q(y,Q^2)\right)\delta(1 - x/y) = \sum_q e_q^2\left(q(x) + \Delta q(x,Q^2)\right)$$

with $\Delta q(x,Q^2)$ the α_S QCD *correction* to F_2

$$\Delta q(x,Q^2) = \frac{\alpha_S}{2\pi}\log\frac{Q^2}{\mu^2}\int_x^1 \frac{dy}{y} q(y) P_{q\leftarrow q}\left(x/y\right)$$

Since $\Delta q(x,Q^2)$ varies only logarithmically
consider the logarithmic change of Δq (i.e. for and interval of $\Delta(\ln Q^2)$).
We obtain the following integro-differential equation for $q(x,Q^2)$,
known as Altarelli - Parisi evolution equation (DGLAP)

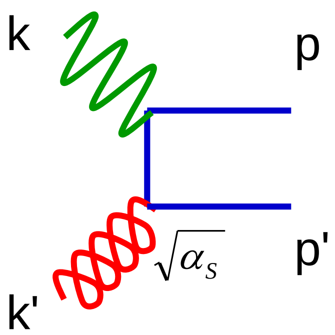
$$\frac{\Delta q(x,Q^2)}{\Delta \log Q^2} \quad\rightarrow\quad \frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y} q(y,Q^2) P_{q\leftarrow q}\left(x/y\right)$$

In reality also α_S depends on Q^2! The μ^2 cut-off drops out from the differential.

In QCD we cannot predict the shape of the parton distribution functions.
However, once they are measured at a fixed Q_0^2, using the QCD evolution equations we can predict their form (evolution) at any Q^2.

Gluon Pair Production

At order α_S, the process $\gamma^*q \to qg$ is not the only one contributing to the cross section. We have to consider also the gluon pair production (photon-gluon fusion) diagram.



To calculate the process $\gamma^*g \to q\bar{q}$
recall again the QED Compton scattering $\gamma^*\, e \to \gamma\, e$.
We obtain the PGF process by exchanging the u – channel with the s – channel: $u \to s$, $s \to u$ (crossing).
We will encounter again an interference term since the incoming photon is virtual.

$$\langle |M|^2 \rangle = 32\pi^2 \left(e_i^2 \alpha \alpha_S\right) \frac{1}{2} \left(\frac{\hat{u}}{\hat{t}} - \frac{\hat{t}}{\hat{s}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} \right)$$

Repeating the same calculations as for $\gamma^*q \to qg$
($\alpha^2 \to e_i^2\alpha\alpha_S$ and inserting the color factor $C_F = 1/2$ in this case) we obtain

$$\hat{\sigma}(\gamma^* g \to q\bar{q}) \approx e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_S}{2\pi} P_{q\leftarrow g}(z) \log \frac{Q^2}{\mu^2} \right)$$

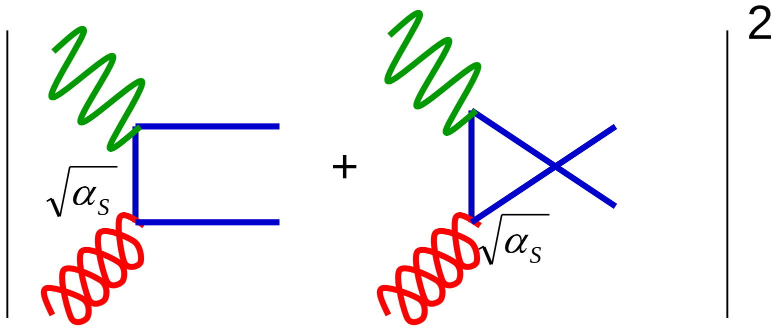
with

$$P_{q\leftarrow g}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

$P_{q\leftarrow g}$ is the gluon splitting function and gives the probability that the quark carries a fraction z of the initial gluon momentum (no singularities in $P_{q\leftarrow g}$.

Symbolically:

$$\left.\frac{F_2\left(x,Q^2\right)}{x}\right|_{\gamma^* g\to q\bar{q}} \sim$$



$$\sigma\left(\gamma^* g \to q\bar{q}\right) \sim \alpha_S \log Q^2/\mu^2$$

$$\left.\frac{F_2\left(x,Q^2\right)}{x}\right|_{\gamma^* g\to q\bar{q}} = \sum_q e_q^2 \int_x^1 \frac{dy}{y}\left[g(y)\frac{\alpha_S}{2\pi}P_{q\leftarrow g}\left(x/y\right)\log\frac{Q^2}{\mu^2}\right]$$

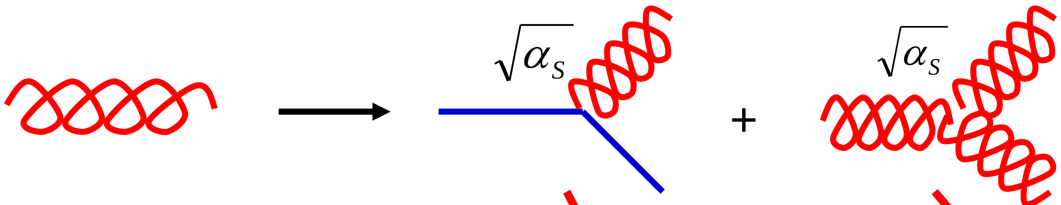
We have introduced a new distribution, the gluon density *g(x)*, which gives the probability of finding a gluon in the proton carrying a momentum fraction *x*.
Note the logarithmic dependence also for this correction. μ^2 is (again) a cut-off parameter to regularize the infrared divergences.

Including the $\gamma^* g \to q\bar{q}$ pair production process, the complete quark density evolution Altarelli-Parisi evolution equation becomes

$$\frac{d}{d\log Q^2}q(x,Q^2) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{q\leftarrow q}\left(x/y\right) + g(y,Q^2)P_{q\leftarrow g}\left(x/y\right)\right]$$

Gluon Evolution

To close the system, we have to consider also the evolution of gluons.



$\sim \alpha_S \log Q^2/\mu^2$

Proceeding in a similar way as for the quark evolution (more complicated calculations are involved) we arrive at the following expression for the gluon evolution Altarelli-Parisi evolution equation:

$$\frac{d}{d\log Q^2} g(x,Q^2) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left(\sum_i q_i(y,Q^2)P_{g\leftarrow q}\left(x/y\right) + g(y,Q^2)P_{g\leftarrow g}\left(x/y\right)\right)$$

with

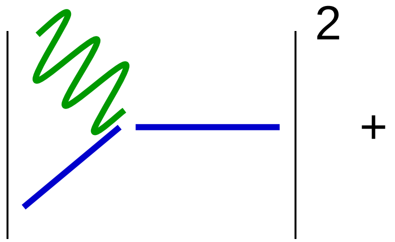
$$P_{g\leftarrow q}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{g\leftarrow g}(z) = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

$P_{g\leftarrow q}$ and $P_{g\leftarrow g}$ are the quark and gluon splitting functions, respectively.

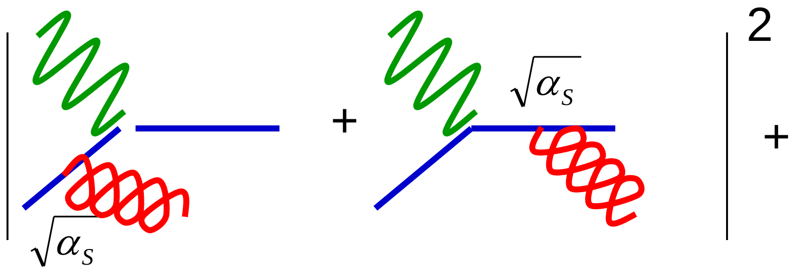
pQCD α_S Expansion of F_2

$$\frac{F_2\left(x,Q^2\right)}{x} \sim$$



$\Rightarrow$ scaling violations $\sim \alpha_S \log Q^2/\mu^2$

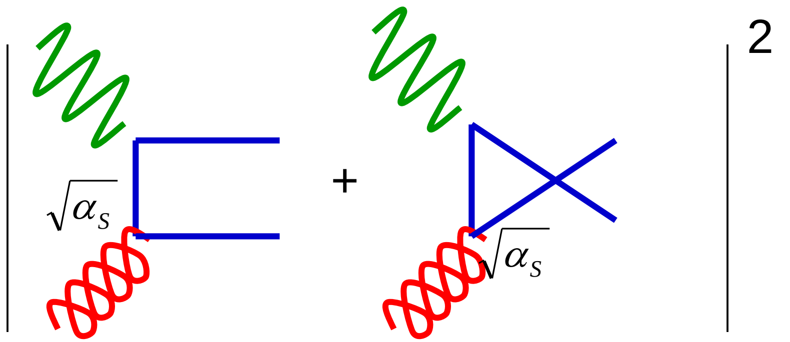
reduces
high-x partons



cross sections

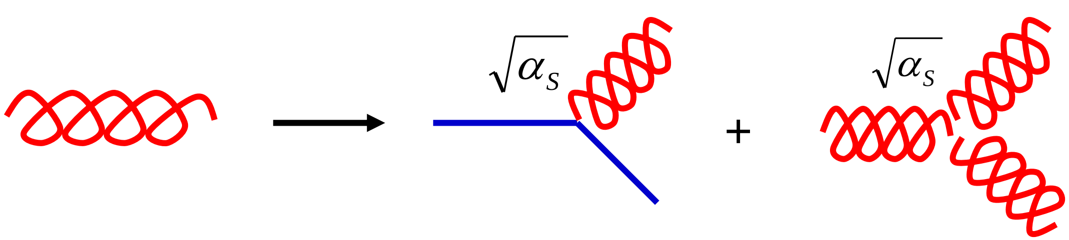
$$\sigma\left(\gamma^* q \to qg\right) \sim \alpha_S \log Q^2/\mu^2$$

creates
low-x partons



$$\sigma\left(\gamma^* g \to q\bar{q}\right) \sim \alpha_S \log Q^2/\mu^2$$

to close the system, we also need the gluon evolution



$$\sim \alpha_S \log Q^2/\mu^2$$

Summarizing

Altarelli-Parisi evolution equation for quark densities

$$\frac{d}{d\log Q^2} q_i(x,Q^2) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{q\leftarrow q}(x/y) + g(y,Q^2)P_{q\leftarrow g}(x/y)\right]$$

Altarelli-Parisi evolution equation for gluon density

$$\frac{d}{d\log Q^2} g(x,Q^2) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left(\sum_i q_i(y,Q^2)P_{g\leftarrow q}(x/y) + g(y,Q^2)P_{g\leftarrow g}(x/y)\right)$$

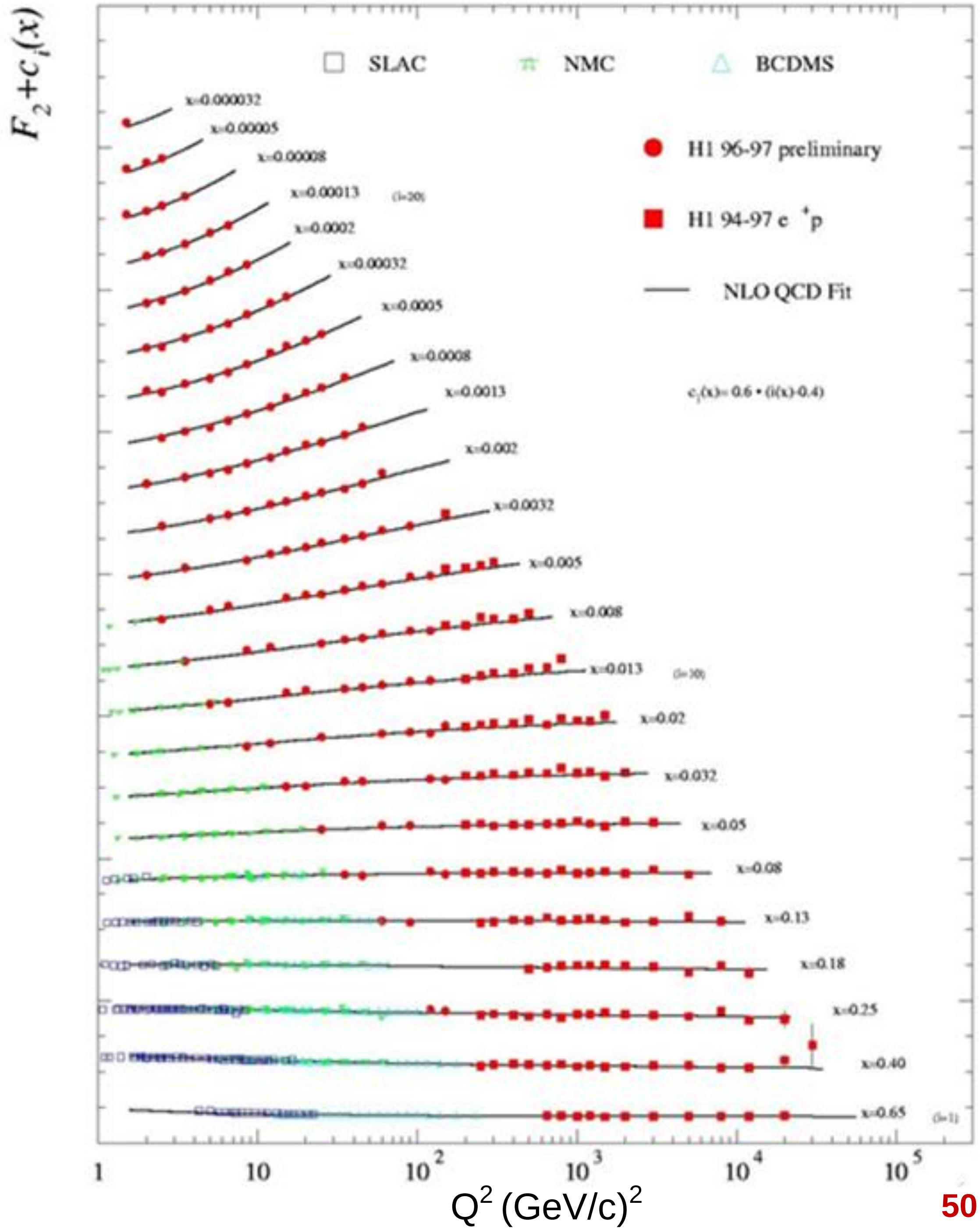
Splitting functions

$$P_{g\leftarrow q}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z} \qquad P_{q\leftarrow q}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

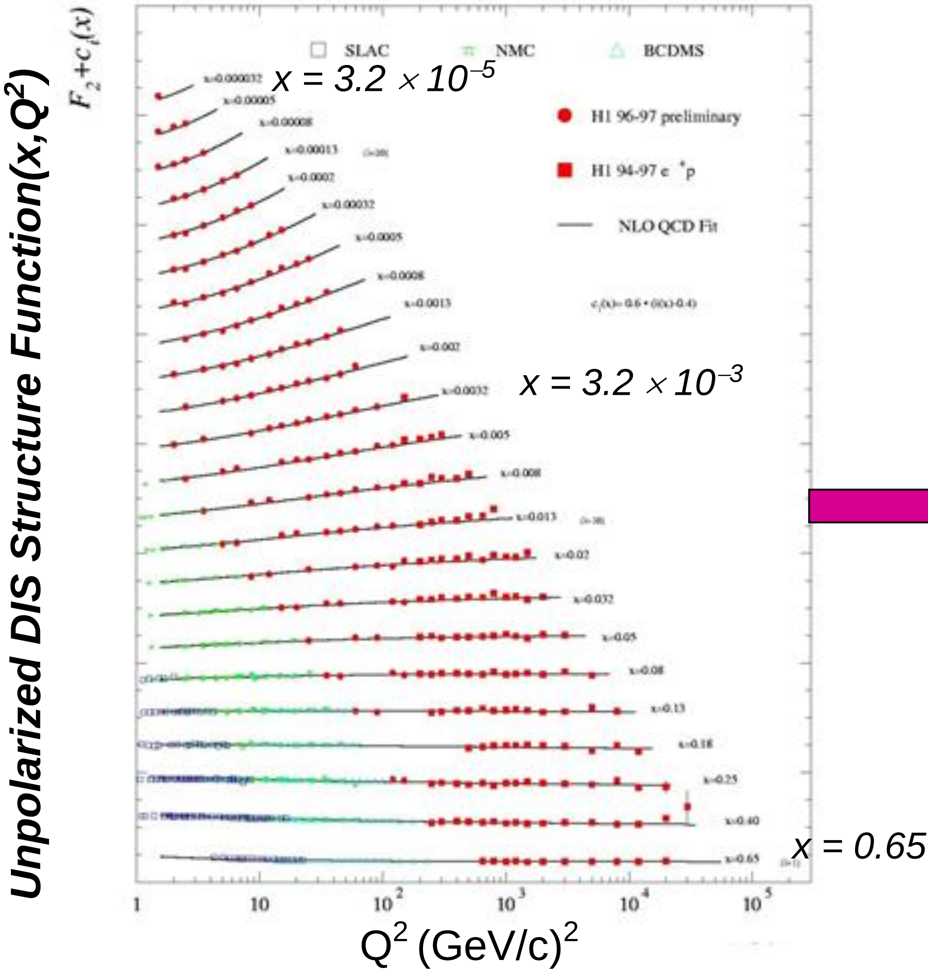
$$P_{g\leftarrow g}(z) = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) \qquad P_{q\leftarrow g}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

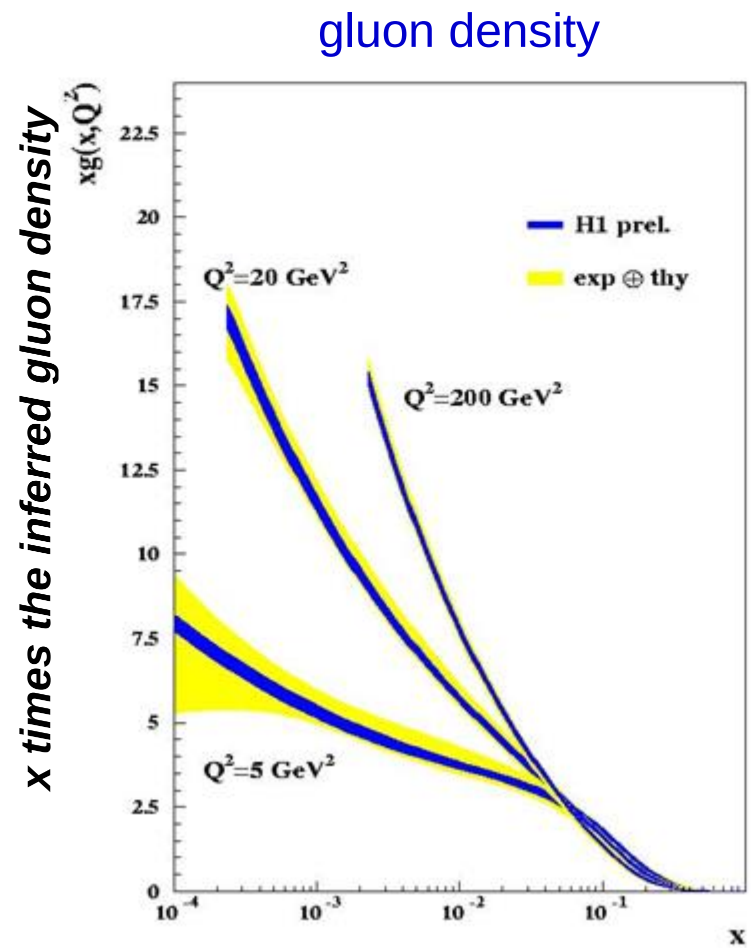
In QCD we cannot predict the shape of the parton distribution functions.
However, once they are measured at a fixed Q_0^2, using the QCD evolution equations we can predict their "evolution" to any Q^2.

F_2: Comparison data - pQCD



F_2: Comparison data - pQCD





gluon distribution determined indirectly from $dF_2/d\log Q^2$ scaling violations

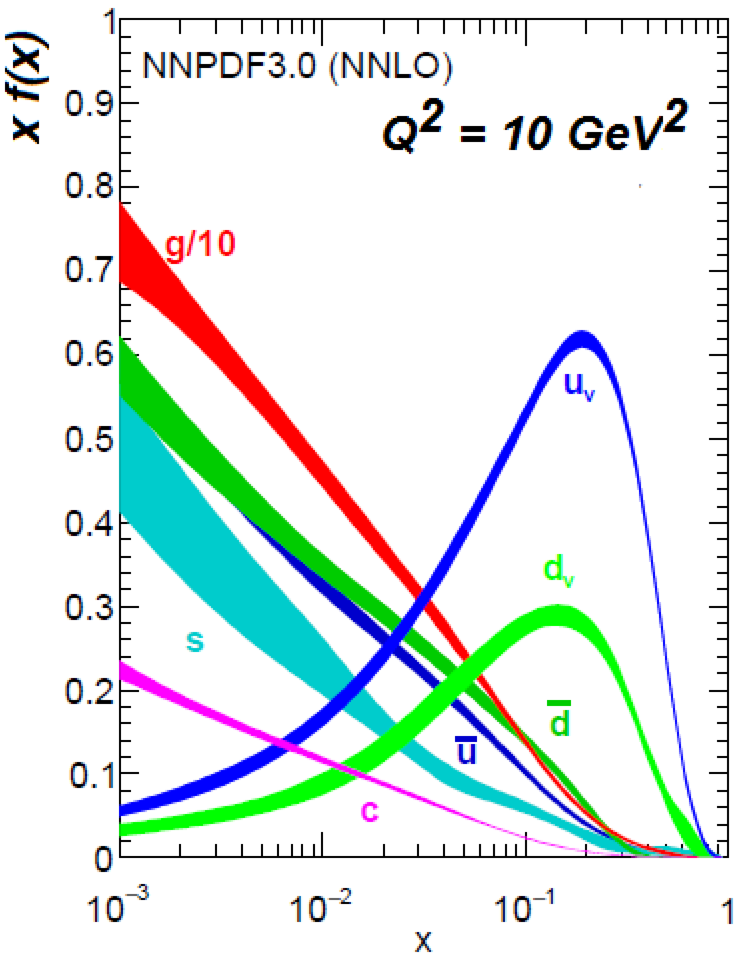
QCD fits at NLO and NNLO (incomplete calculations) available

at LO, scaling violations $\sim \alpha_S \log Q^2/\mu^2$ and $\alpha_S\left(Q^2\right) = \dfrac{4\pi}{\beta_0 \log\left(Q^2/\Lambda^2_{QCD}\right)}$

$\Rightarrow \alpha_S g \sim$ const

Parton Distribution Functions

Parton distribution functions are obtained from a fit to all experimental data.
Neutrino scattering allows for the flavor decomposition.
Hadron-hadron collisions give access to gluon distributions + scaling violations in DIS.



Apart from at large x

$$u_V(x) \approx 2d_V(x)$$

For x < 0.2 gluons dominate

In fits to data assume

$$u_s(x) = \overline{u}(x)$$

Small strange quark component

$$s(x)$$

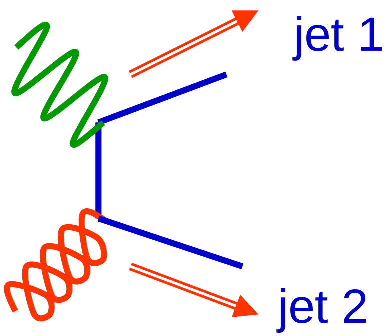
Not fully understood

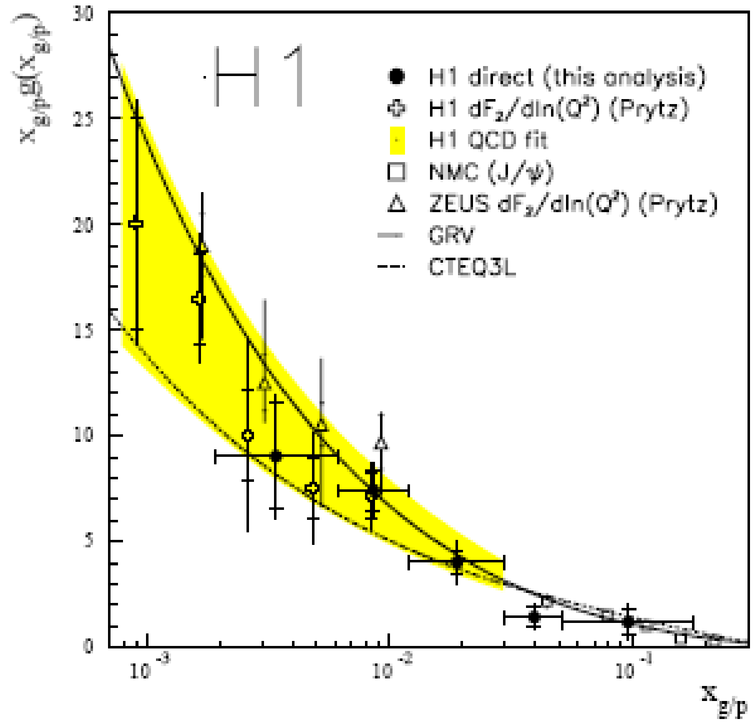
$$\overline{d}(x) > \overline{u}(x)$$

(Gottfried sum rule violation)

Direct Measurement of Gluons

di-jet production

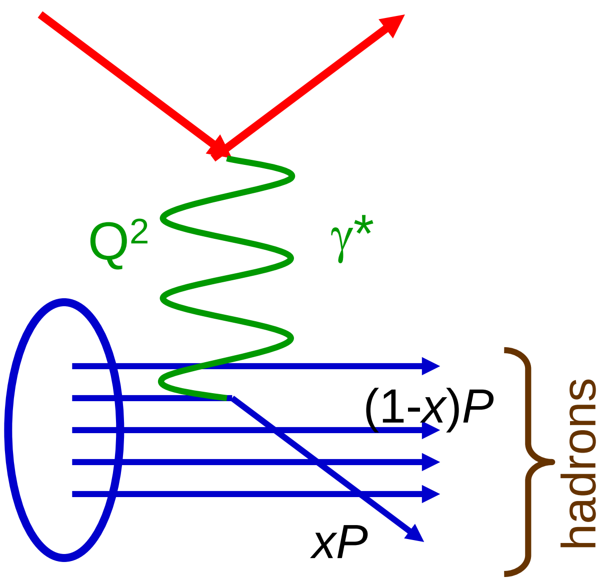




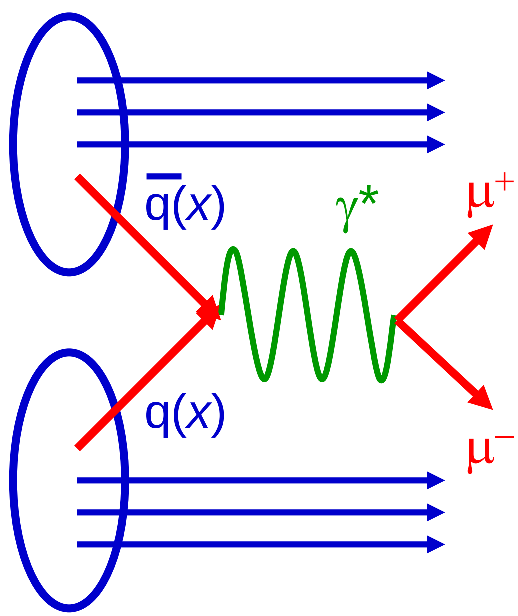
Universality

The parton distributions are universal:
once experimentally known (measured in a certain process)
they can be used to predict others

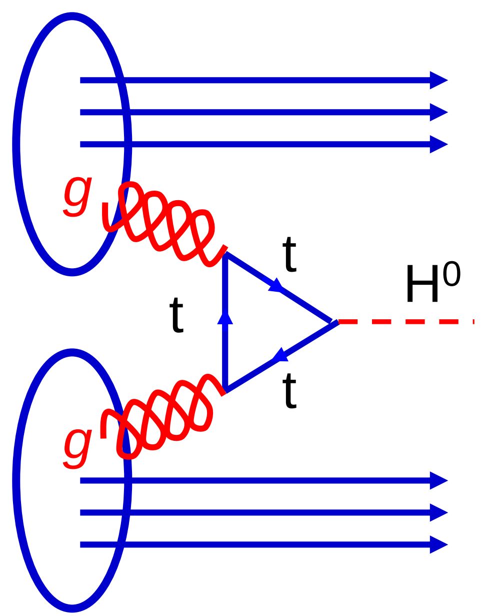
deep inelastic ep → eX



Drell–Yan pp → $\mu^+\mu^-$ + X



Higgs production



For Next Week

Study the material and prepare / ask questions
Study ch. 8 (sec. 4, 5) and ch. 10 (sec. 1 to) in Halzen & Martin
and / or ch. 8 in Thomson

Do the homeworks

Next week we will study the $e^+ e^- \rightarrow q \bar{q}$ process
have a first look at the lecture notes, you can already have questions
read ch. 11 (sec. 1 to 7) in Halzen & Martin and / or ch. 10 (sec. 6) in Thomson