Advanced Particle Physics 2

Strong Interactions and Weak Interactions

L6 – *p* – *p* Interactions

(http://dpnc.unige.ch/~bravar/PPA2/L6)

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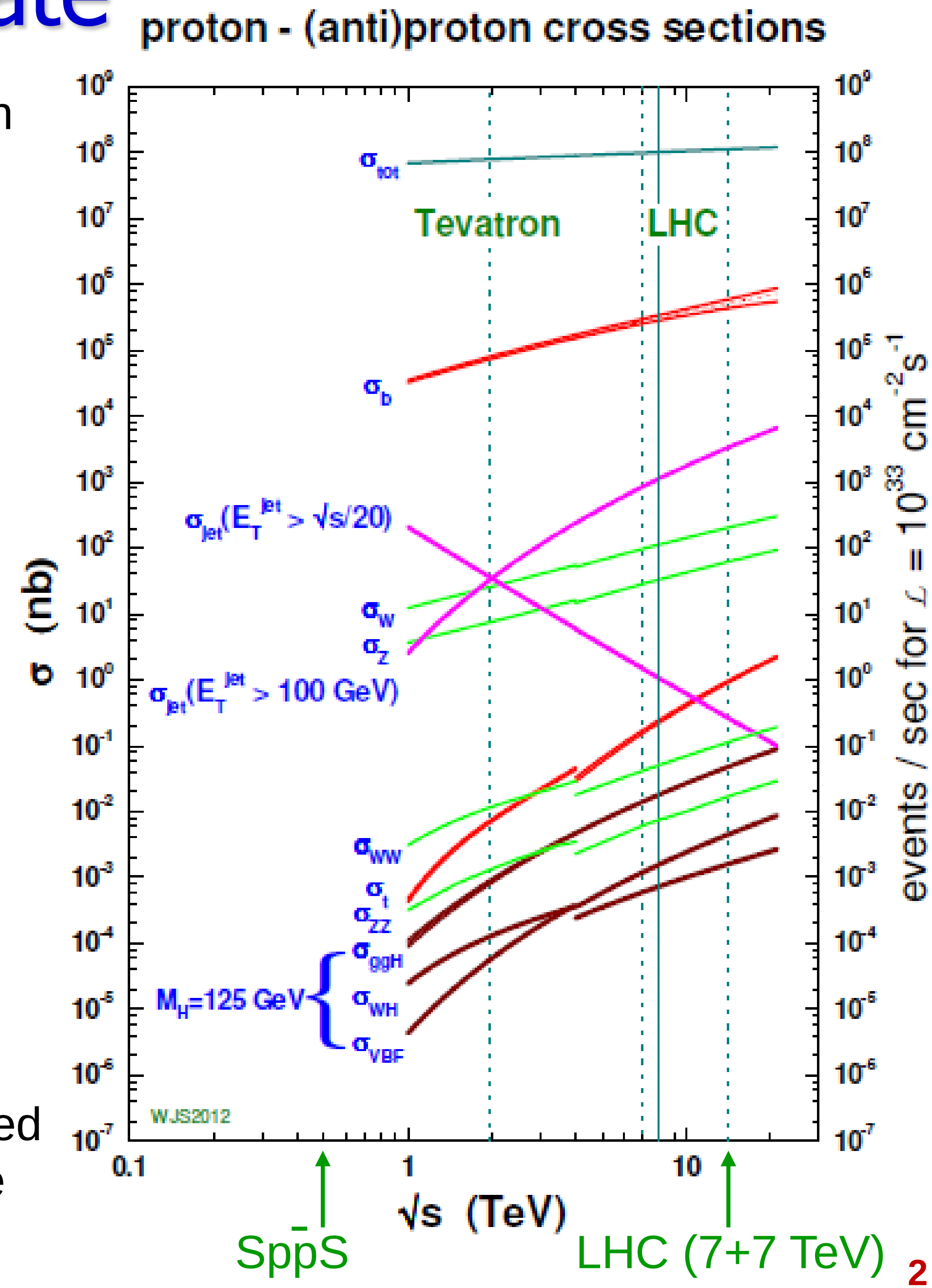
What Can We Calculate

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**.

Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases.

For **HARD** processes, e.g. W boson or high-E_T jet production, the rates and event properties can be predicted with high precision using QCD perturbation theory.

For **SOFT** processes, e.g. total cross sections or diffractive processes the rates and event properties are dominated by non-perturbative QCD effects, which are much less understood and not calculable.



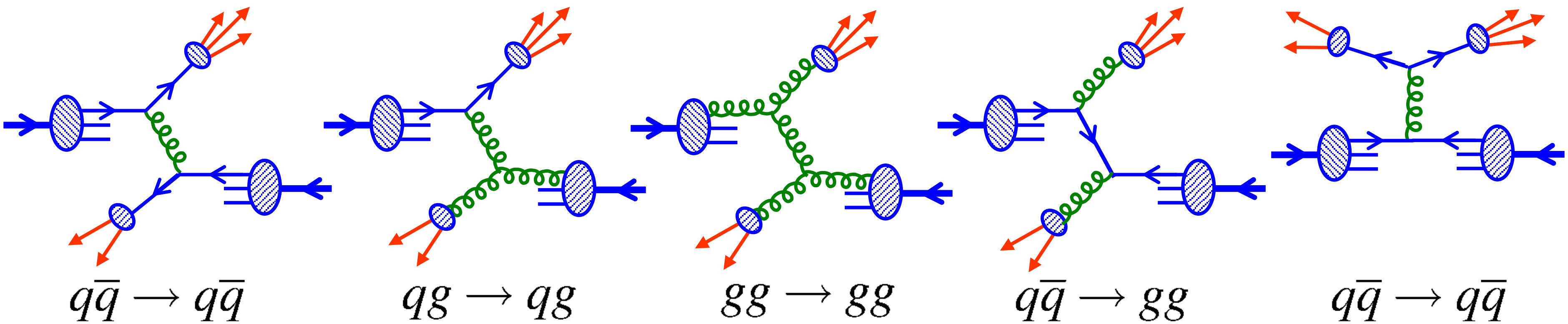
Hadron – Hadron Collisions

much higher c.o.m. energies than achievable with e^+e^- machines

new particle searches at high mass scales

the colliding protons can be seen as two beams of free partons with a braod spectrum

underlying process: interaction of two partons (quarks, antiquarks, or gluons)



3 variables to describe the interaction:

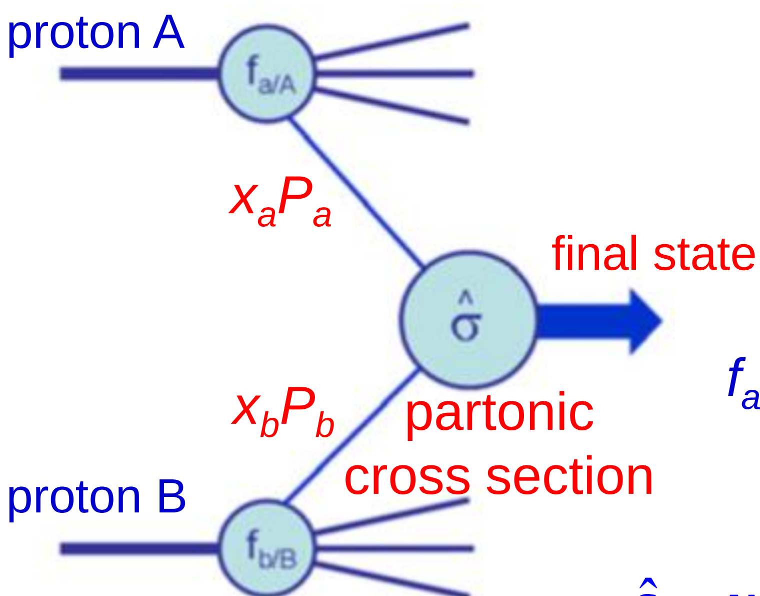
the parton momenta x_1 and x_2, and Q^2, which measures the hardness of the interaction
[Q^2 is not well defined, it depends on the process under study ☹]

cfr. e-p scattering (DIS): 2 variables x_{Bj} and Q^2 , and

e^+e^- annihilation (fixed beam energies): 1 variable the scattering angle θ , s is fixed

Understanding p – p Cross-sections

In QCD, hadron – hadron collisions are described as an incoherent sum of elementary interactions between the hadron constituents – the partons (quarks, the antiquarks, and the gluons).



For a specific final state,
we must add all possible amplitudes
(i.e. Feynman diagrams)
leading to that particular final state
(2 jets, top quark, W boson, Higgs, …)

$f_{a/A}(x_a)$ = probability to find parton a in proton A, carrying a momentum fraction x_a

$\hat{\sigma}$ = elementary cross-section at parton level $a + b \rightarrow c + d$

$\hat{s} = x_1 x_2 s$ = partonic c.o.m. energy

$$\sigma_{AB\rightarrow X} = \iint_{x_a x_b} dx_a dx_b \, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2) \times \hat{\sigma}_{ab\rightarrow X}(\alpha_S(Q^2))$$

The interacting partons carry only a fraction of the incident protons' momenta.
The incident protons (of fixed energy) can be viewed as beams of free partons with a large spectrum of energies.
In the scattering between partons, therefore, the energy is not fixed with $\hat{s} < s$.

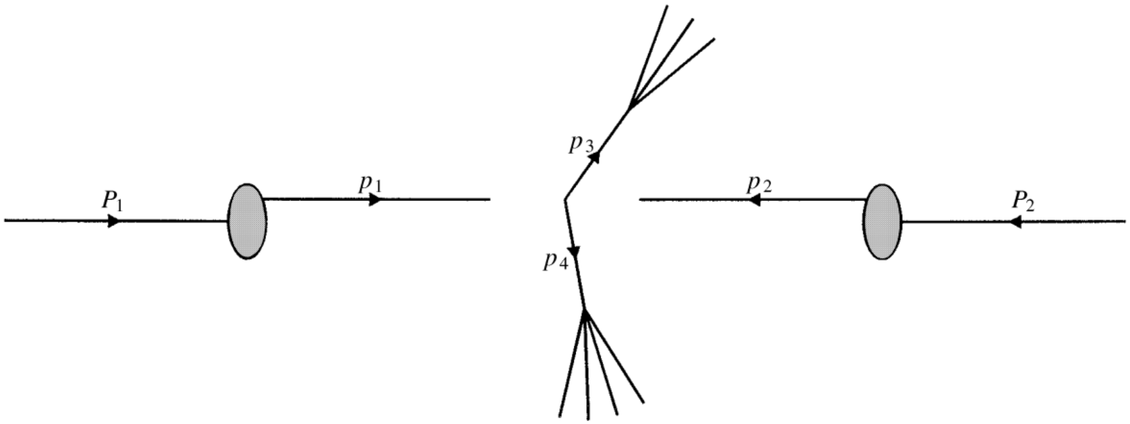
The hadron – hadron interactions has the form

$$A + B \rightarrow \mathrm{jet}_1 + \mathrm{jet}_2 + \mathrm{jet}_3 + \mathrm{jet}_4$$

The generated hadrons will be collimated in jets around the direction of the outgoing partons emerging from the elementary reaction (sub–process)

$$a + b \rightarrow c + d$$

jet_1 and jet_2 are the fragments of partons c and d
jet_3 and jet_4 are the fragments of the spectators, which did not participate in the interaction
Since the initial state (pp system) is colorless, there must be some soft gluon exchange between the jets to assure that also the final state is colorless.

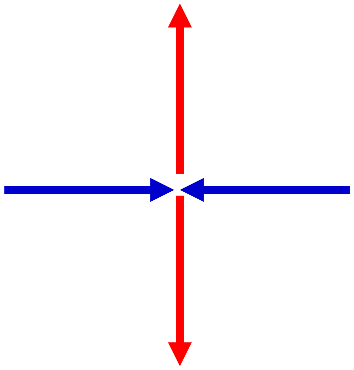


At high energies even more high p_T jets are expected to be produced. They originate from the radiation of hard gluons from the quarks (higher order QCD diagrams), or Z^0 and $W^{+/-}$ decays, top and bottom decays, …

QCD 1 + 2 → 3 + 4 Process

$$\frac{d\hat{\sigma}}{d\hat{t}}(1+2\to 3+4) = \frac{1}{16\pi\ \hat{s}^2}\left\langle \left|M\left(1+2\to 3+4\right)\right|^2\right\rangle = \frac{\pi\alpha_S^2}{\hat{s}^2}|A|^2$$

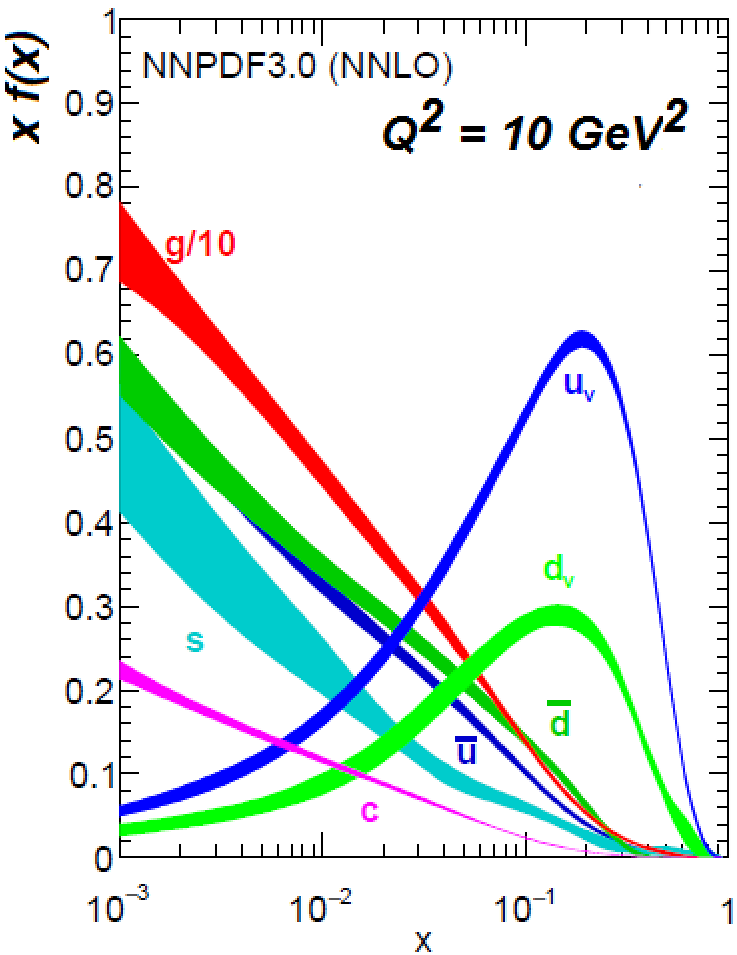
Process	$\lvert A\rvert^2$	*strength* at 90^0 in c.o.m.
$q_1q_2\to q_1q_2,\ q_1\bar{q}_2\to q_1\bar{q}_2$	$\frac{4}{9}\frac{s^2+u^2}{t^2}$	2.22
$q_1q_1\to q_1q_1$	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}\right)-\frac{8}{27}\frac{s^2}{ut}$	3.26
$q_1\bar{q}_1\to q_2\bar{q}_2$	$\frac{4}{9}\frac{t^2+u^2}{s^2}$	0.22
$q_1\bar{q}_1\to q_1\bar{q}_1$	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2}+\frac{t^2+u^2}{s^2}\right)-\frac{8}{27}\frac{u^2}{st}$	2.59
$q\bar{q}\to gg$	$\frac{32}{27}\frac{u^2+t^2}{ut}-\frac{8}{3}\frac{u^2+t^2}{s^2}$	1.04
$gg\to q\bar{q}$	$\frac{1}{6}\frac{u^2+t^2}{ut}-\frac{3}{8}\frac{u^2+t^2}{s^2}$	0.15
$qg\to qg$	$-\frac{4}{9}\frac{u^2+s^2}{us}+\frac{u^2+s^2}{t^2}$	6.11
$gg\to gg$	$\frac{9}{2}\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{st}{u^2}\right)$	30.4



The coefficient 1 / (16 π s^2) is the phase space and flux factor.

Parton Distribution Functions

Parton distribution functions are obtained from a fit to all experimental data.
Neutrino scattering allows for the flavor decomposition.
Hadron-hadron collisions give access to gluon distributions + scaling violations in DIS.



$$F_1(x) \equiv \frac{1}{2}\sum_i q_i^2 f_i(x) \qquad F_2(x) \equiv x\sum_i q_i^2 f_i(x)$$

Apart from at large x

$$u_V(x) \approx 2d_V(x)$$

For x < 0.2 gluons dominate

In fits to data assume

$$u_s(x) = \overline{u}(x)$$

Small strange quark component

$$s(x)$$

Not fully understood

$$\overline{d}(x) > \overline{u}(x)$$

(Gottfried sum rule violation)

Factorization

Key point: FACTORIZATION ! In general, the hard hadron reaction

$$A + B \to C + D$$

can be factorized in terms of:

$$\sigma_{AB\to X} = \int dx_a dx_b \, f_{a/A}(x_a, \mu_F^2) \, f_{b/B}(x_b, \mu_F^2) \times \left[\hat{\sigma}_0 + \alpha_S\left(\mu_R^2\right) \hat{\sigma}_1 + \cdots \right]_{ab\to X}$$

1. the elementary (elastic) parton – parton cross section $d\sigma / dt$ for the reaction

$$a + b \to c + d$$

2. the parton density functions $f_a^A(x_a)$ which give the probability of finding a parton of momentum fraction x_a and flavor a in the hadron A (same for hadron B),

3. the fragmentation function $D_c^C(z)$ for parton c to fragment into the hadron C with a momentum fraction z, or to evolve to particular hadronic final state

μ_F – factorization scale (separates long and short distance physics, i.e. the Q^2 of PDFs)

μ_R –renormalization scale of α_S (i.e. the Q^2 at which of α_S is calculated)

Finite corrections left behind are not universal and must be calculated for each process giving rise to order α_S^2 (and higher) perturbative corrections.

An all-orders cross section has no dependence on μ_F and μ_R.
A residual dependence remains (to order α_S^{n+1}) for a finite order α_S^n calculation.

At first order, in the perturbative expansion, we have several 2-body partonic processes that can contribute to the interaction: $qq \to qq$, $qg \to qg$, $gg \to gg$, etc.
These processes are calculable from the corresponding Feynman diagrams using the QCD coupling constant

$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(33-2n_f\right)} \frac{1}{\log\left(Q^2 / \Lambda_{QCD}\right) + C}$$

Λ_{QCD} ~ 200 MeV
the additional constant *C* includes the contributions of higher order diagrams;
the calculation of higher order terms is usually very lengthy and difficult (many Feynman diagrams involved).

Q^2 is a momentum scale that characterizes the hardness of the reaction (probed distance)
in general we cannot define exactly the scale for each process (as long as contributions of higher order diagrams are not determined).
In DIS Q^2 is given by the virtuality $Q^2 < 0$ of the exchanged photon,
in e^+e^- annihilation by the total c.o.m. energy $Q^2 > 0$).
Usually we employ one of the following definitions for Q^2:

$$Q^2 = -\hat{t} \qquad \sim p_T^2$$

$$Q^2 = \hat{s} \qquad \sim 4p_T^2$$

$$Q^2 = \frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2} \qquad \sim \frac{4}{3}p_T^2$$

for $\cos\theta_{cm} = 0$

with $\mu_R^2 = \mu_F^2 = Q^2$ and vary μ^2 between 1/2 Q^2 and $2 \times Q^2$ ($\to$ scale uncertainty)

The Role of Structure Functions

Given that quarks and gluons are not free inside the hadrons, we have to consider also
1. the probability of finding a parton of given momentum inside the interacting hadrons (the parton distribution function *f(x)*) and
2. the probability that the outgoing parton produced fragments into a hadron h (fragmentation function *D(z)*), or to evolve to a particular final state (i.e. jets).

These probabilities cannot be obtained from a (perturbative) QCD calculation. They are measured / extracted in different experiments and QCD can predict their evolution (Q^2 dependence).

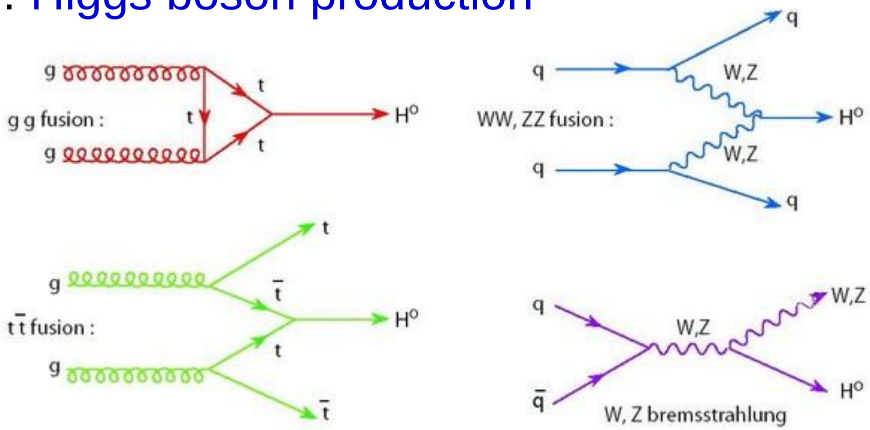
The factorization hypothesis allows us to use the same structure and fragmentation functions measured in DIS, e^+e^- annihilation, and Drell-Yan process to predict the cross sections in different processes. The validity of this hypothesis has to be verified each time by an exact calculation of the perturbative series and confront with experiment.

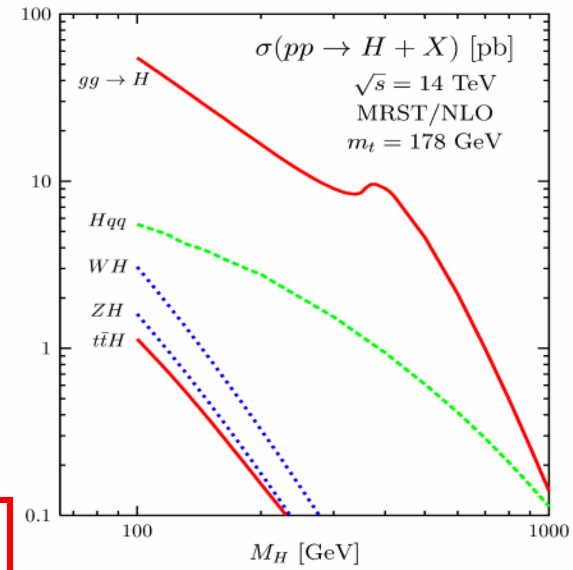
The separation between short- and long-distance physics is controlled by the factorization scale μ_F, which can be chosen in several ways.

A parton emitted at large p_T is part of the short-distance sub–process.
A parton emitted with a small $p_T < \mu_F$ is considered to be part of the hadron structure and is absorbed into the parton distribution function.

How We Do the Calculation

Example: Higgs boson production





$$\sigma(pp \to HX) \sim \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \; g(x_1) g(x_2) \; \hat{\sigma}(gg \to H)$$

To calculate the production cross section (i.e. for Higgs production) we have to sum over all possible sub-processes and have to integrate over x_1 and x_2 (and let the Higgs boson decay).

For this we use Monte Carlo event generators (i.e. use MC techniques to integrate):

1. initialization (calculate the partonic cross section and store them in some tables)
2. generate randomly x_1 and x_2 using the parton distribution functions
3. calculate the cross-section for a specific kinematics (x_1 and x_2)
4. generate the final state (stochastic process, here let he Higgs boson decay)
5. repeat 2., 3., and 4. many times and add the cross-sections (with appropriate weights)
6. end the ”simulation”, i.e. print or plot the results
7. compare the “calculations” to measurements

Hard vs. Soft Processes

In order for the parturbative QCD calculation to be valid, it is important that the process is a short-distance interaction, so that $\alpha_S(Q^2)$ is small enough for the perturbative expansion to be valid; in other words p_T^2 (or Q^2) or M_X must be large.
uncertainty principle:

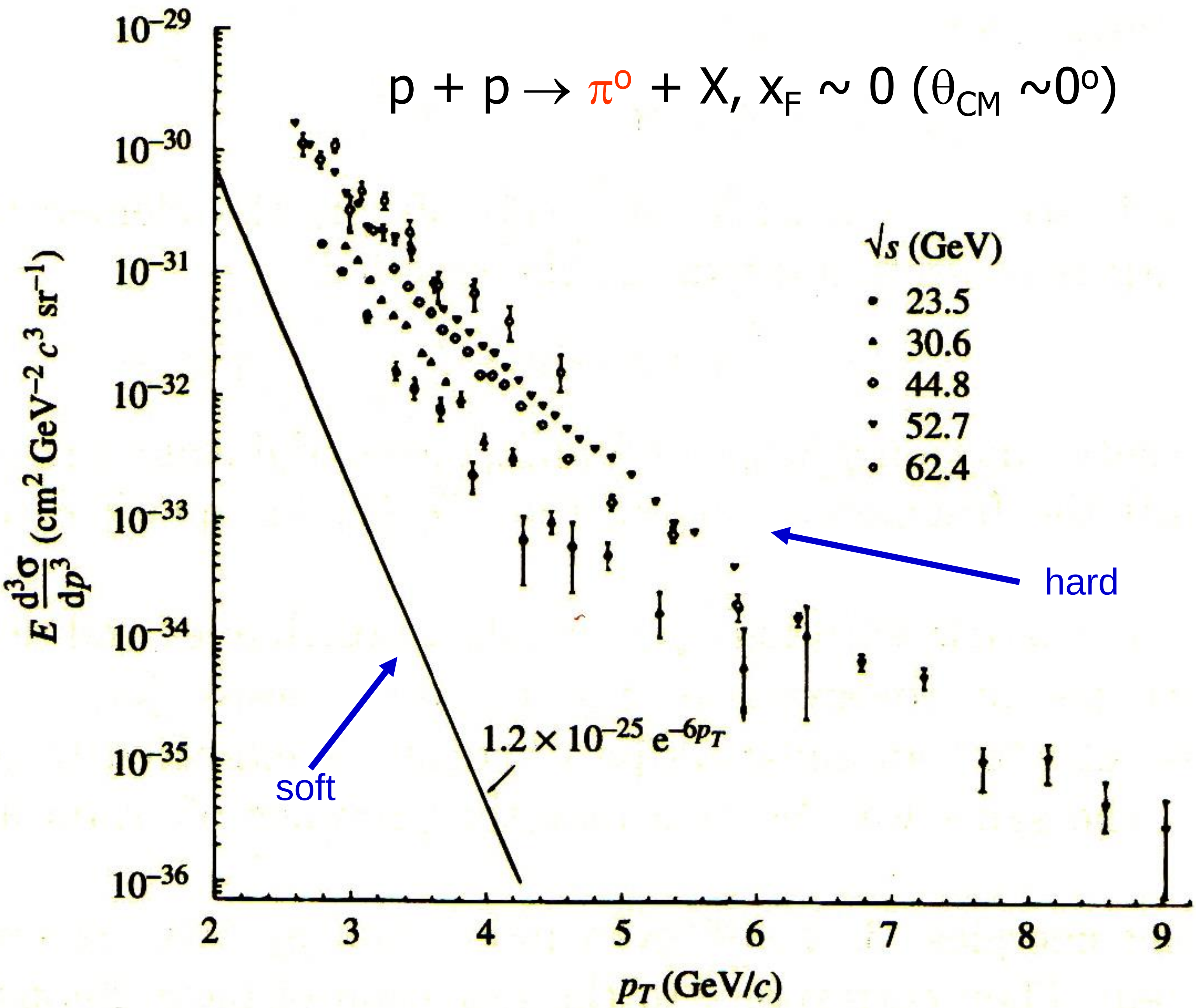
large energy → short time scale → ignore interactions with spectator partons

The average p_T of hadrons produced in hadronic collisions is $\langle p_T \rangle \sim 0.4$ GeV !
Perturbative QCD calculations, however, work for $Q^2 > 10$ GeV^2.
Large p_T processes are rare processes; the cross sections is smaller by several orders of magnitude w.r.t. the dominant production.
Low p_T physics – referred to as soft physics, is not calculable within QCD, and only phenomenological approaches are possible. Typical cross sections are of the order of millibarns, while large p_T cross sections are of the order of nanobarns.

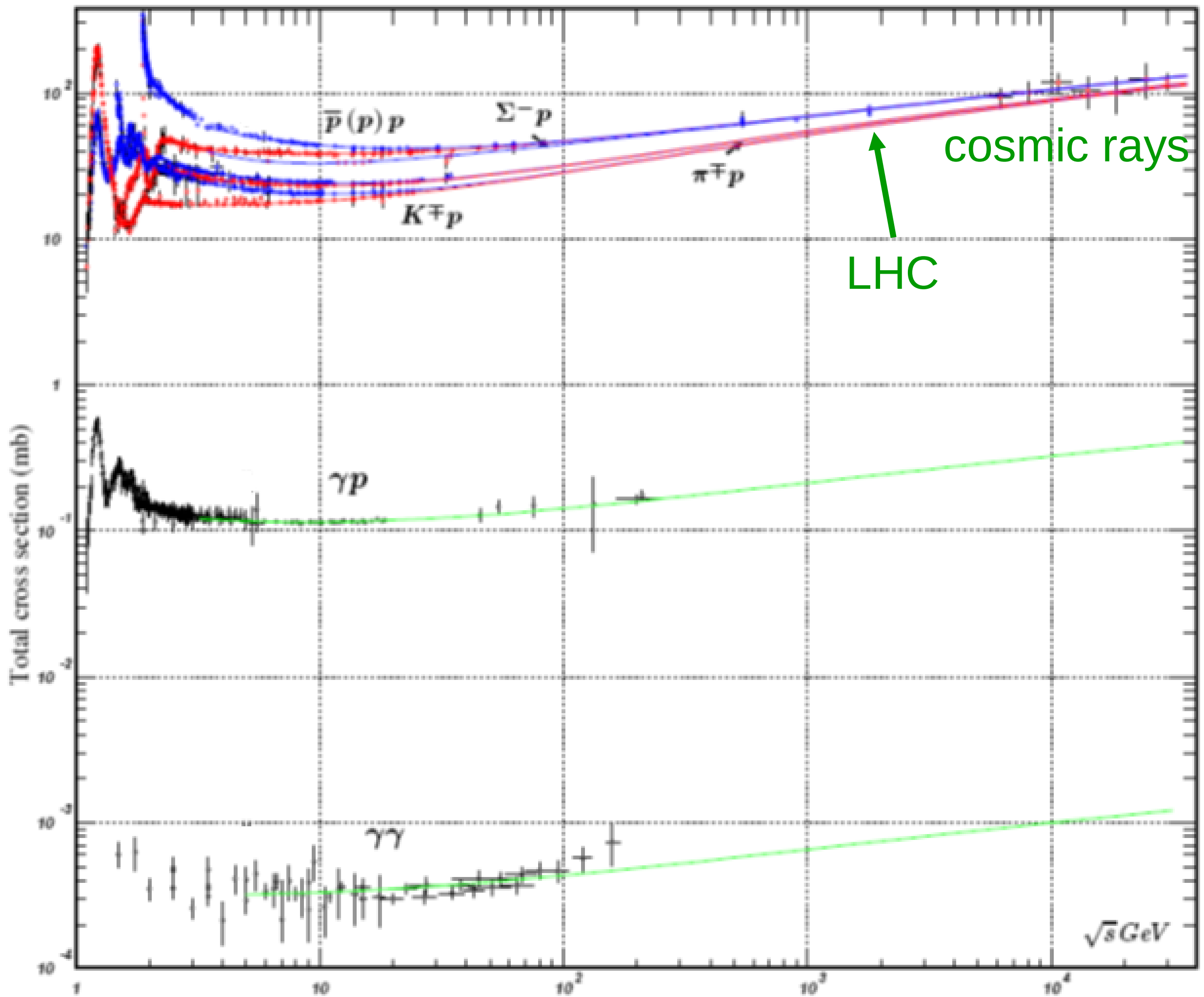
Soft interaction cross sections are usually described by the phenomenological expression like

$$\frac{d\sigma^{pp\to h}}{dx_F\, dp_T} \sim (1-x_F)^{\beta(p_T^2)} \exp(-4p_T)$$

$x_F = 2p_L^* / \sqrt{s}$ is the Feynman variable and describes the longitudinal phase space,
p_T is the transverse momentum of the outgoing hadron and describes the transverse ph. s


p + p → π° + X, x_F ~ 0 (θ_CM ~0°)
√s (GeV)
23.5
30.6
44.8
52.7
62.4
hard
soft
1.2 × 10^-25 e^-6p_T
E d³σ/dp³ (cm² GeV⁻² c³ sr⁻¹)
p_T (GeV/c)
10^-29
10^-30
10^-31
10^-32
10^-33
10^-34
10^-35
10^-36
2
3
4
5
6
7
8
9

Typical Hadronic Cross-sections

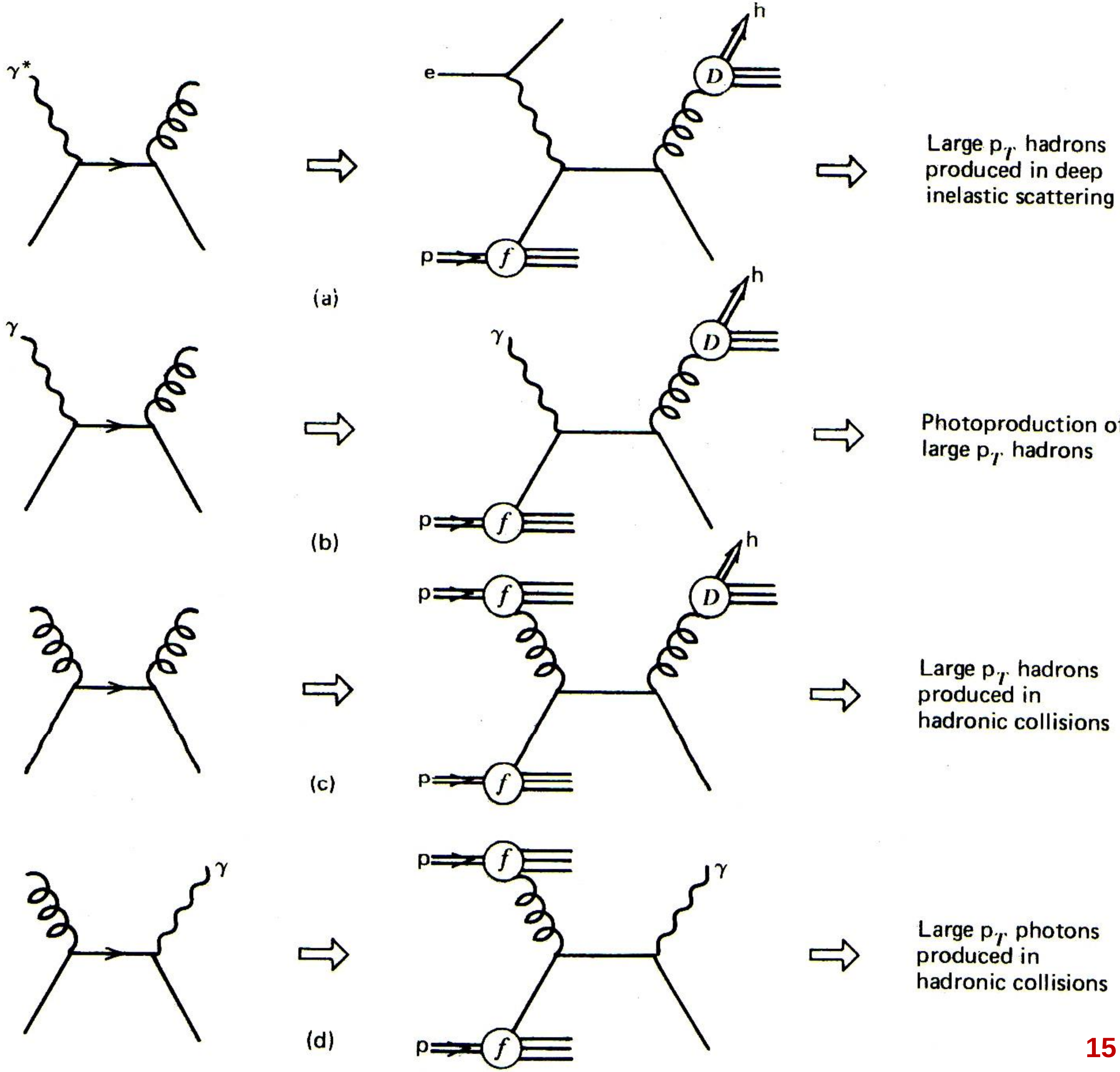


All hadronic cross sections as function of *s* can be described by a universal expression

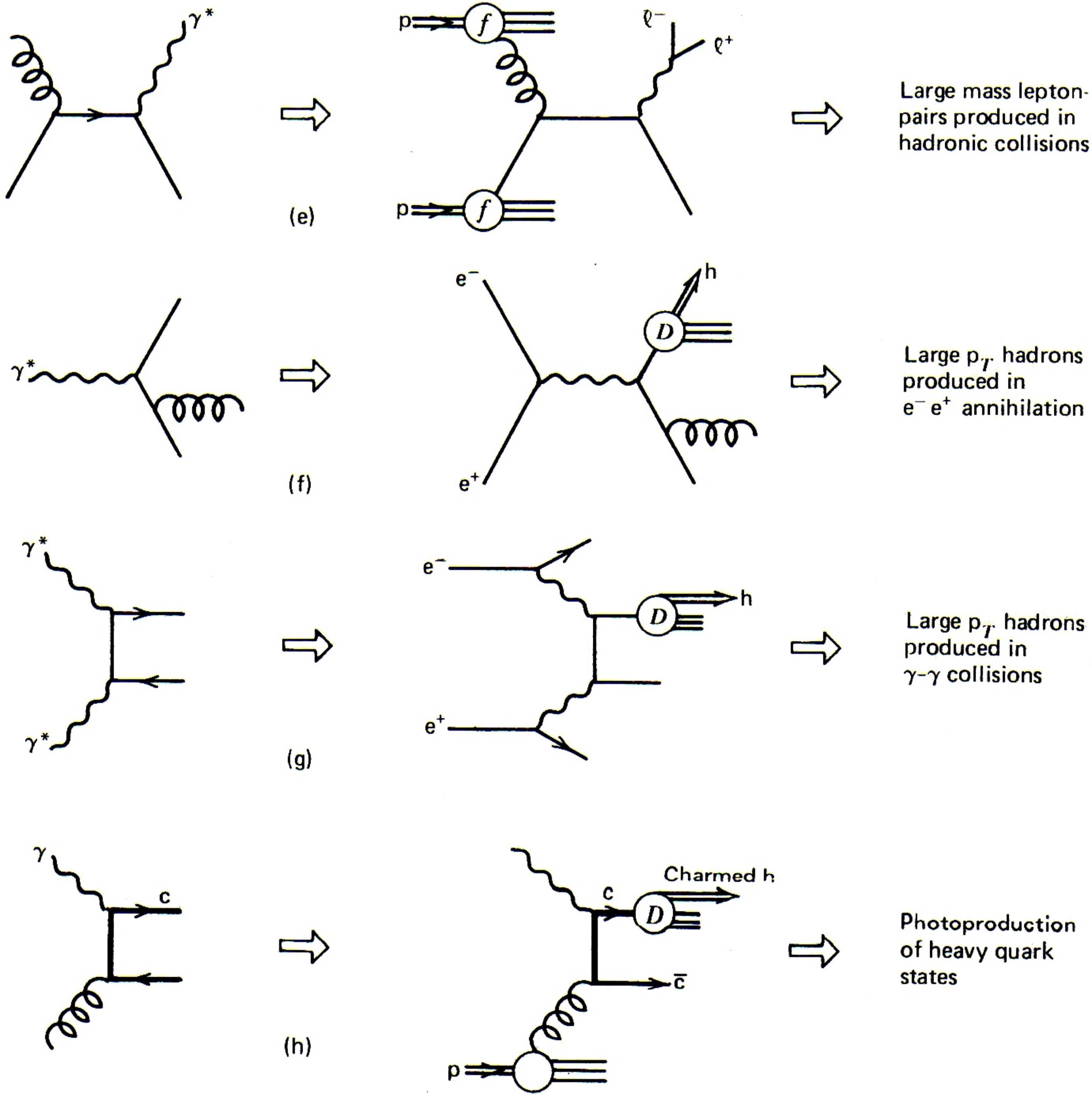
$$\sigma(s) = X\ s^{-0.4525} + Y\ s^{0.0808}$$

with same exponents but different coefficients *X* and *Y*.

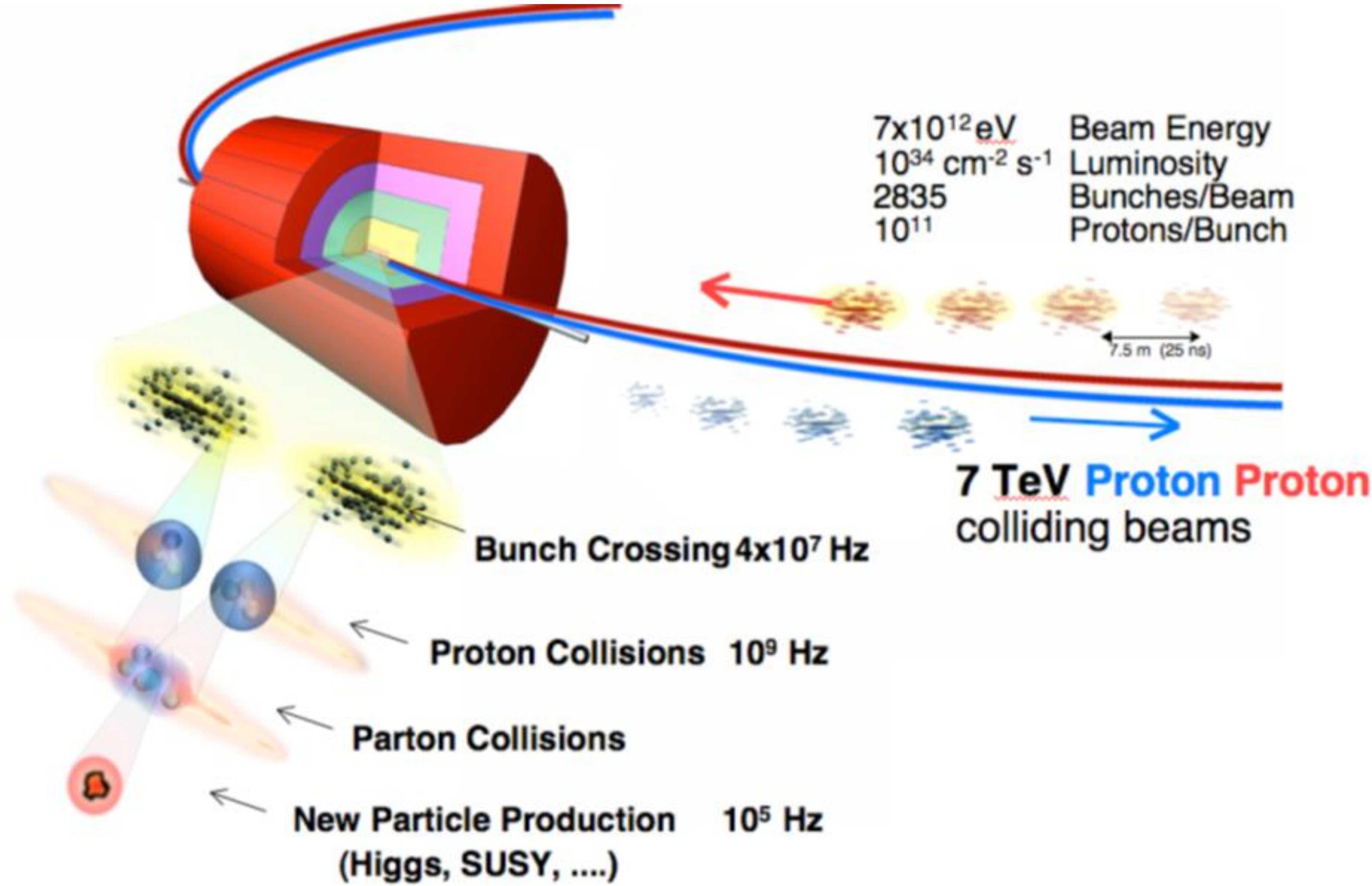
Some Typical QCD Processes (1)



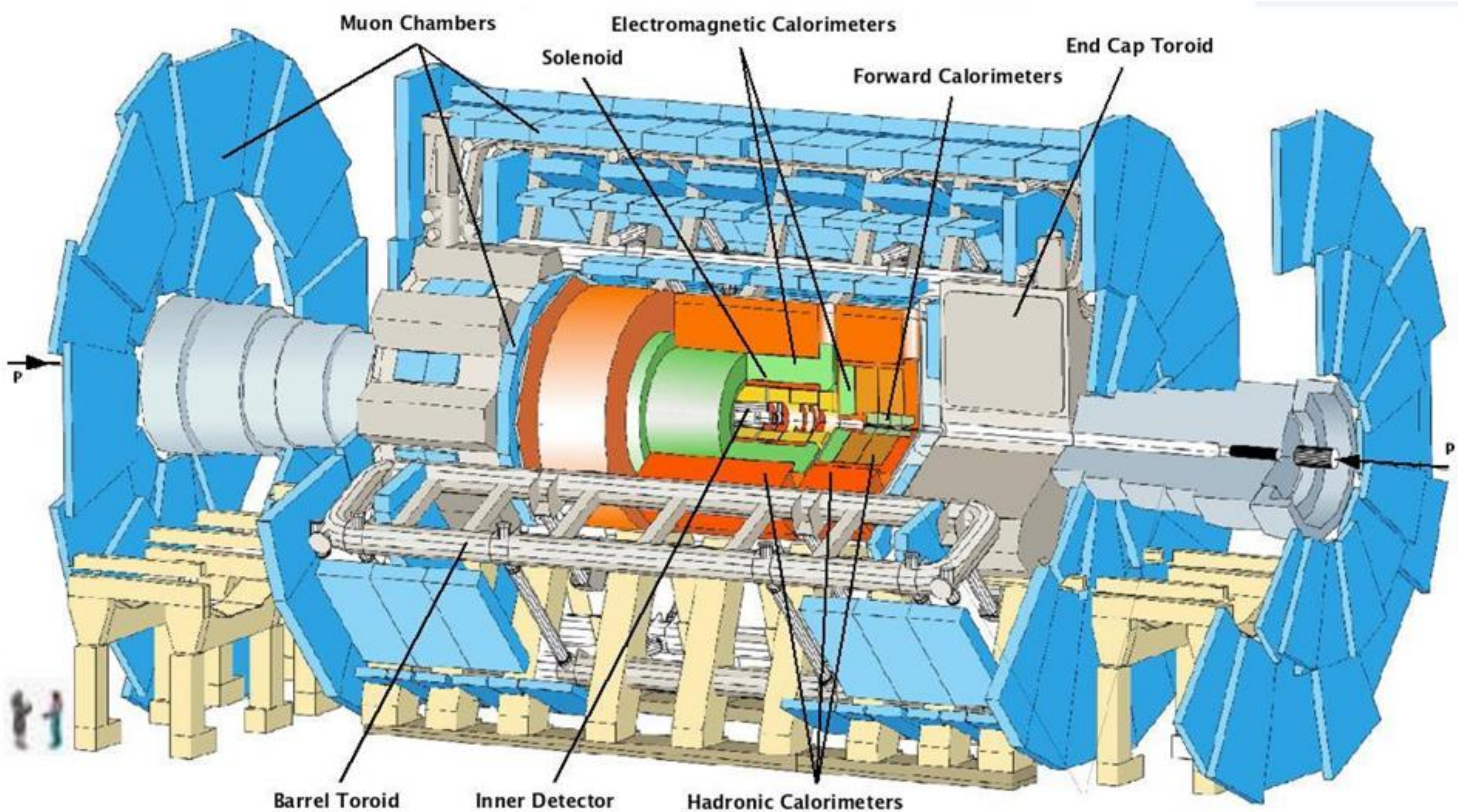
Some Typical QCD Processes (2)



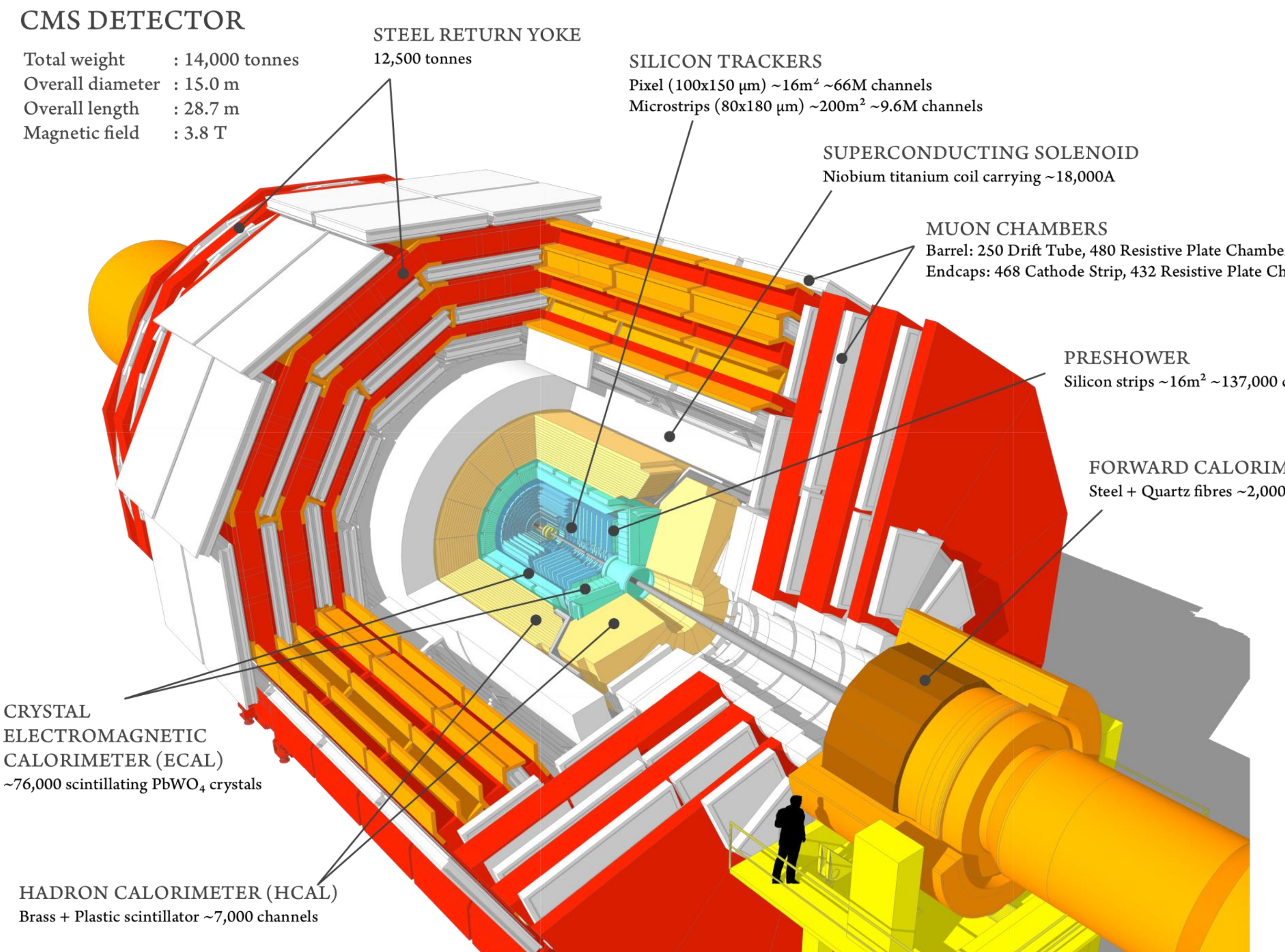
The Large Hadron Collider LHC



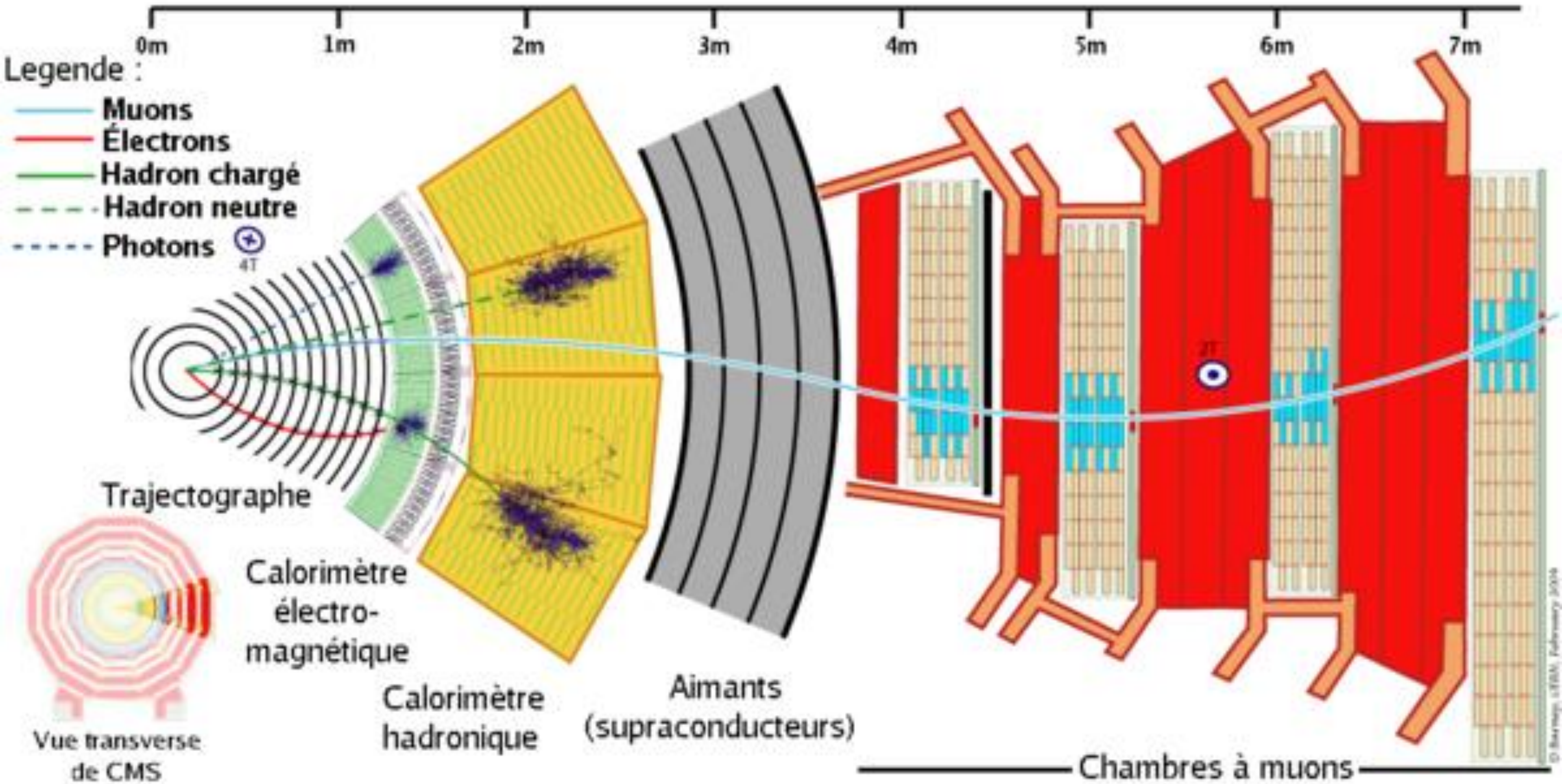
The ATALS Experiment @ LHC



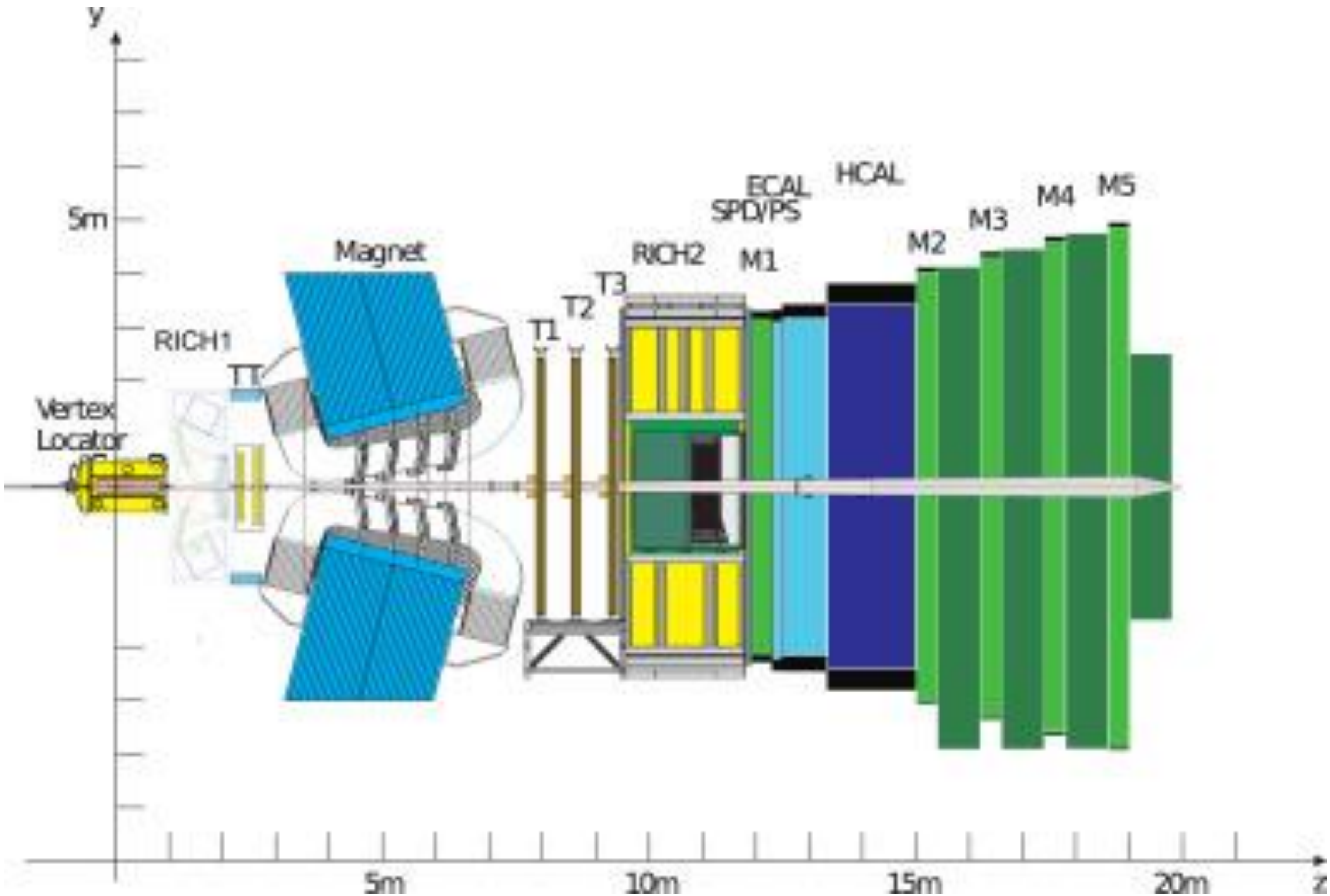
The CMS Experiment @ LHC



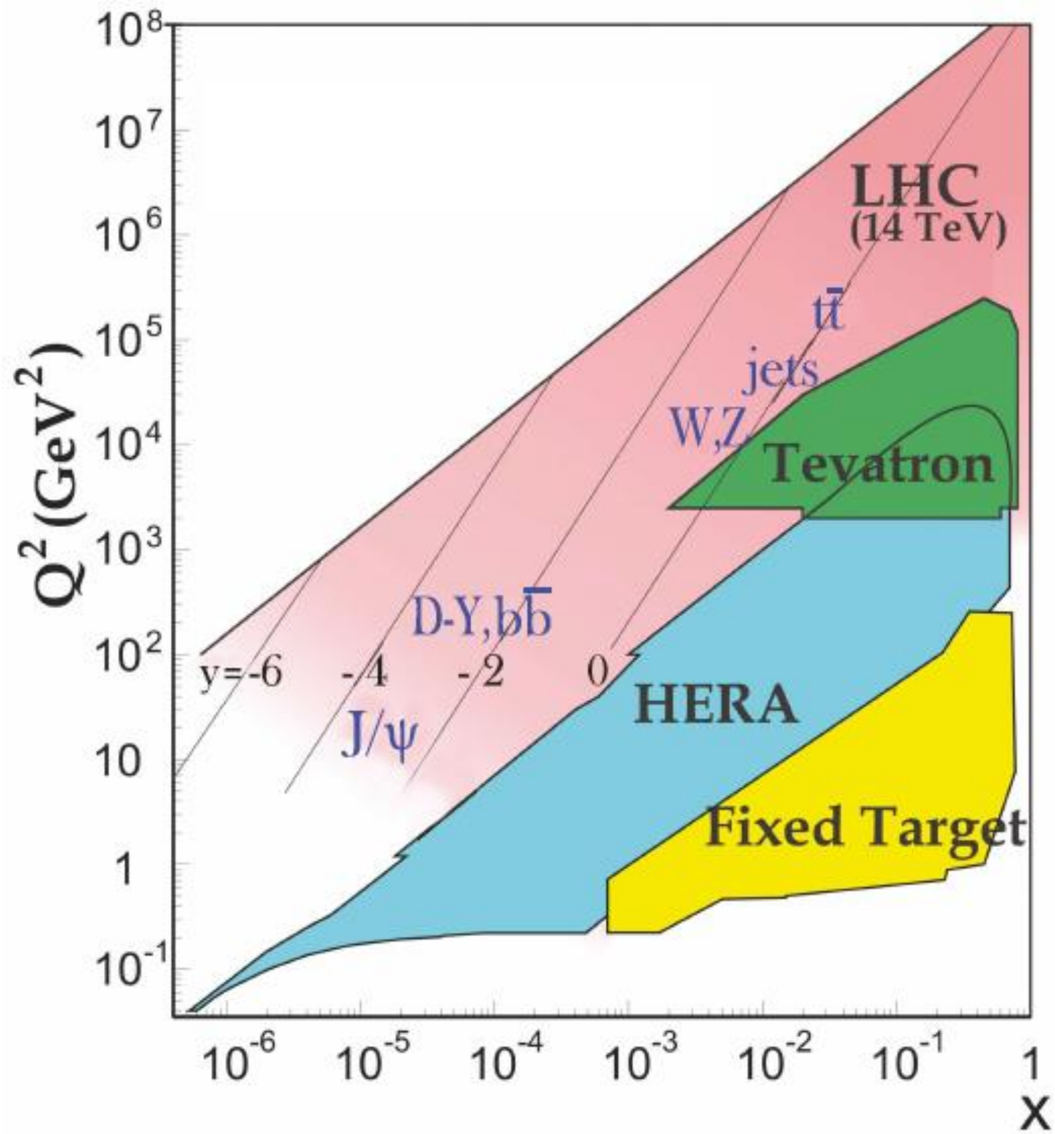
The CMS Experiment @ LHC



The LHCb Experiment @ LHC



Kinematical Reach of LHC



Proton – Proton Collisions at LHC

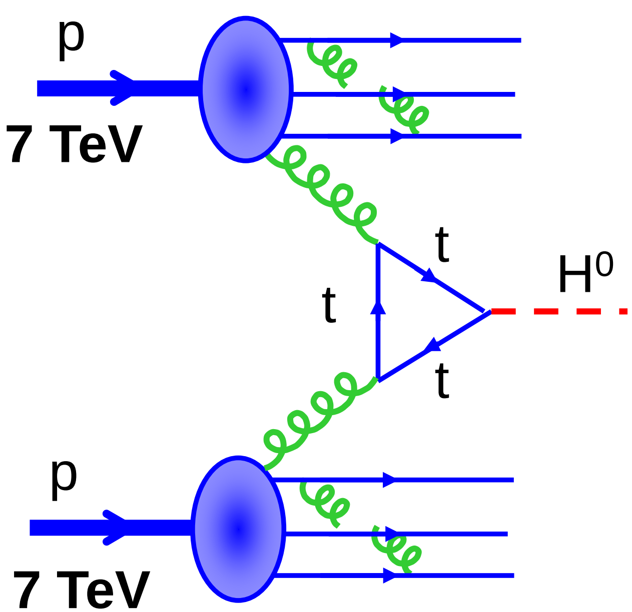
Measurements of structure functions not only provide a powerful test of QCD, the parton distribution functions are essential for the calculation of cross sections at *pp* and $p\bar{p}$ colliders.

Example: Higgs production at the Large Hadron Collider LHC

The LHC collides 7 TeV protons on 7 TeV protons

However underlying collisions are between partons

Higgs production the LHC dominated by "gluon-gluon fusion"

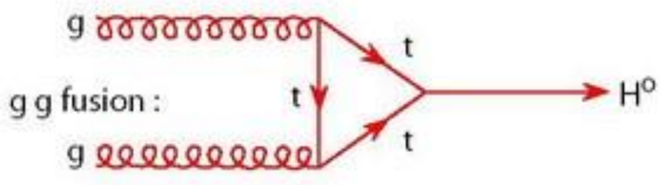


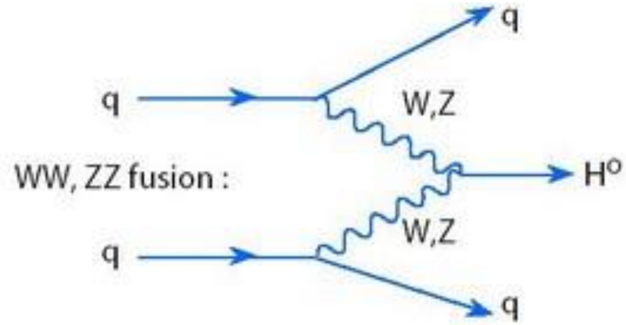
Cross section depends on gluon PDFs

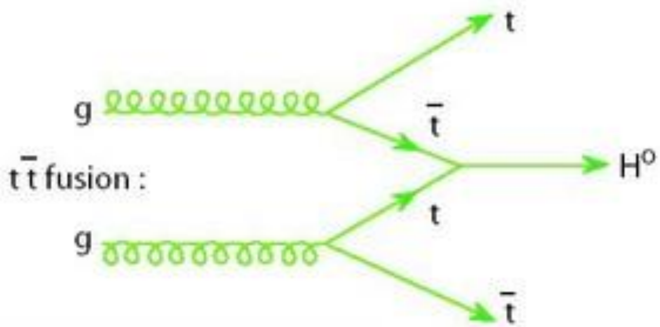
$$\sigma(pp \rightarrow HX) \sim \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\ g(x_1)g(x_2)\ \hat{\sigma}(gg \rightarrow H)$$

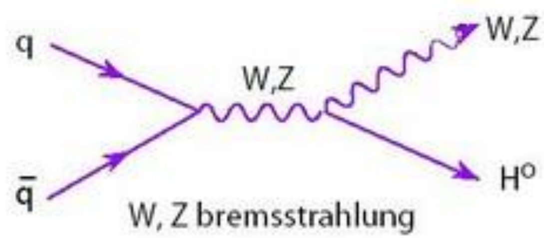
Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section

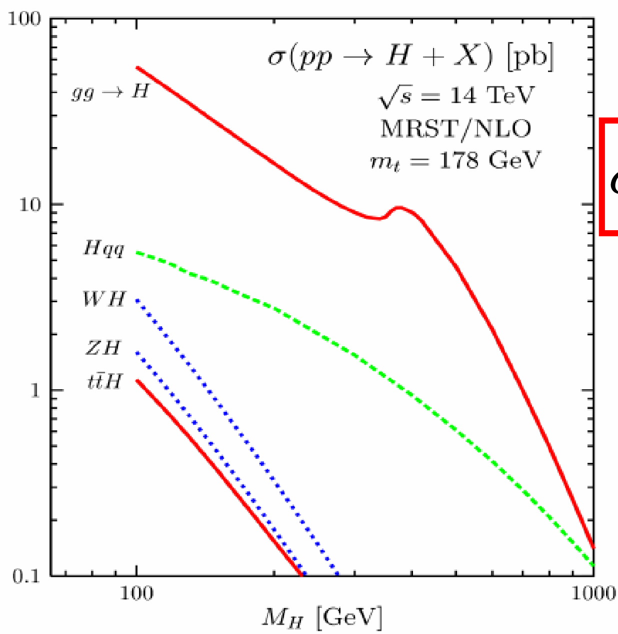
Higgs Production Mechanisms











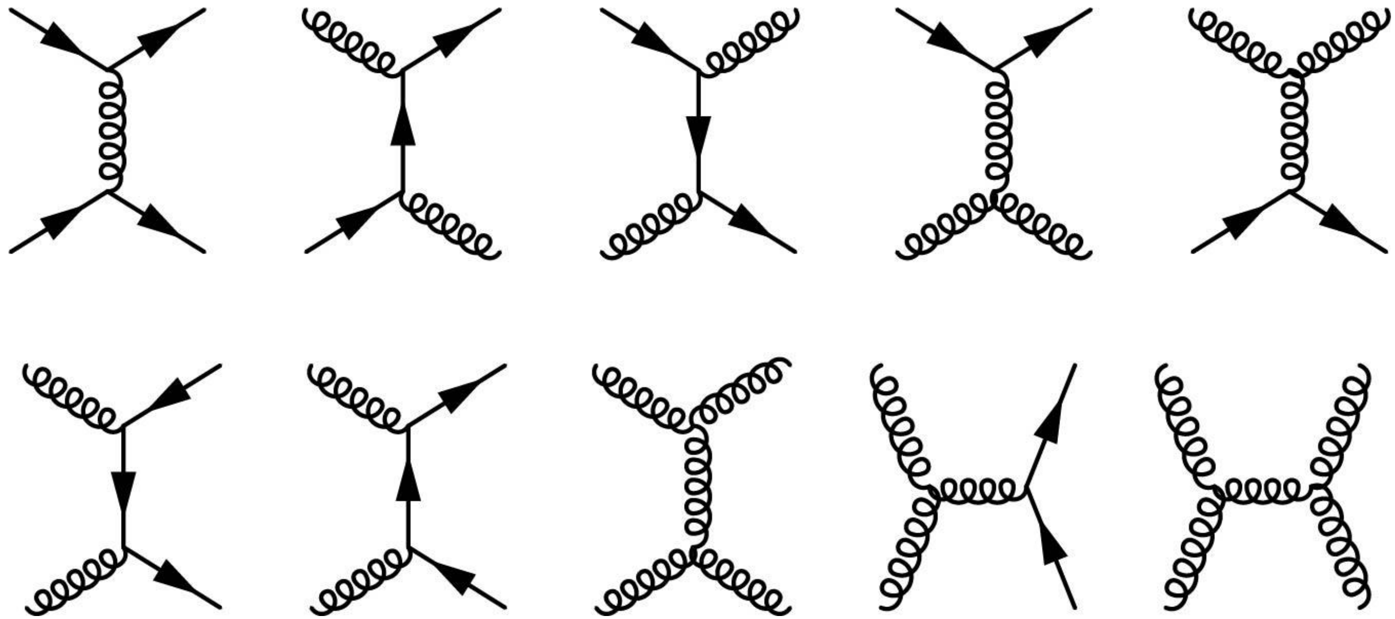
$$\sigma(pp \to HX) \sim \sum_i \int_0^1 dx_1 \int_0^1 dx_2 \;\; f_1^{(i)}(x_1)\, f_2^{(i)}(x_2)\;\; \hat{\sigma}_i(gg \to H)$$

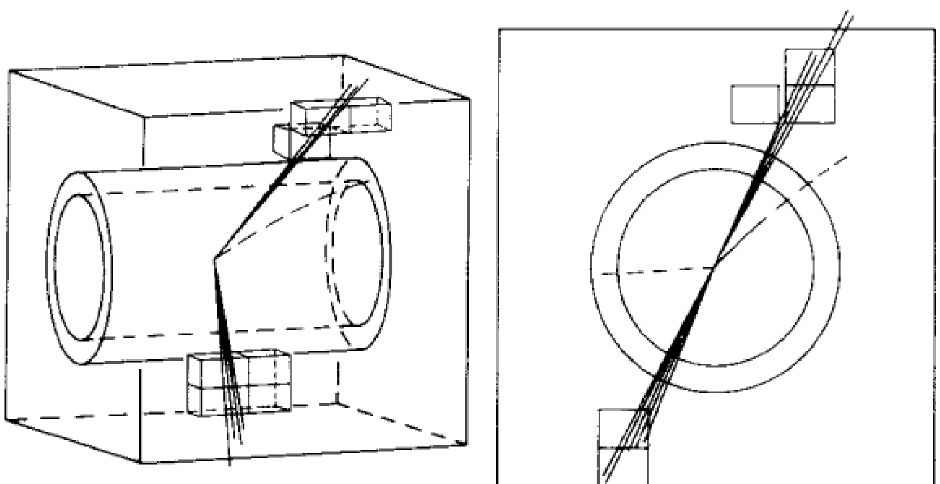
add incoherently all possible sub-processes
(in principle can distinguish on the base of the final state)

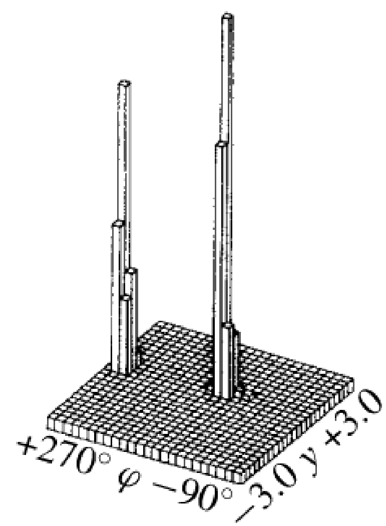
as scale we can take the mass of the Higgs boson

$$Q^2 \simeq M_H^2 \sim 10^4\,\mathrm{GeV}^2$$

Feynman Diagrams for Di-Jet Production







not back-to-back because $x_1 \neq x_2$

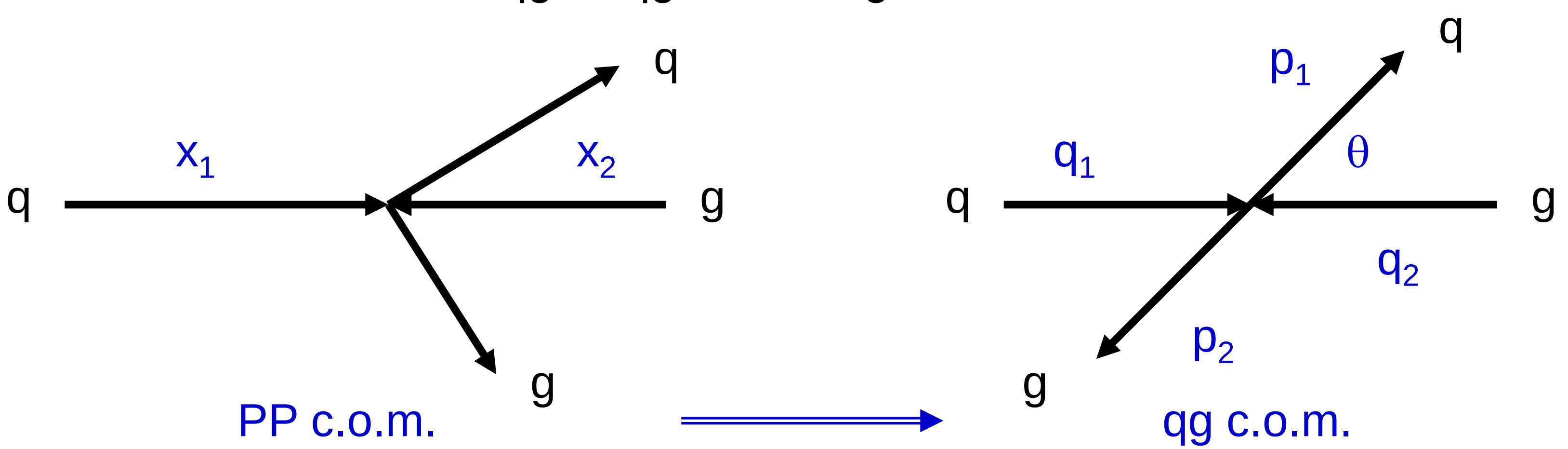
Di-Jet Kinematics

parton + parton → parton + parton

q_1 q_2 p_1 p_2

In the c.o.m. of the interacting partons, there are only two independent variables: the c.o.m. energy $\hat{s}$ and the polar angle θ or $p_T = |p| \cos\theta$.
(this is the usual 2-body process encountered already several times)

Consider for instance the qg → qg scattering:



$$\left|\vec{P}_A\right| = \left|\vec{P}_B\right| = \frac{E_{cm}}{2} = \frac{\sqrt{s}}{2}$$

$$q_1 = x_1 P_A = x_1 \frac{\sqrt{s}}{2} \qquad q_2 = x_2 P_B = x_2 \frac{\sqrt{s}}{2}$$

q – initial state

p – final state

Consider the difference between parton momenta

$$q_1 - q_2 = (x_1 - x_2)\frac{\sqrt{s}}{2} = P_{1z} + P_{2z}$$

where P_{1z}, P_{2z} are the longitudinal momenta of the outgoing jets
A first measurable quantity is

$$x_F = x_1 - x_2 = \frac{P_{1z} + P_{2z}}{E_{cm}}$$

x_F can be determined from P_{1z}, P_{2z} .

A second measurable quantity can be derived from the parton c.o.m. energy ŝ

$$\hat{s} = (q_1 + q_2)^2 = 4q_1q_2 = 4x_1x_2E_{cm}^2 = x_1x_2s$$

$$\hat{s} = (p_1 + p_2)^2 = M_{12}^2$$

$$\Rightarrow \tau \equiv \frac{M_{12}^2}{s} = \frac{\hat{s}}{s} = x_1x_2$$

only a fraction of the c.o.m. energy is available for the partonic sub – process

where M_{12}^2 is the (invariant mass)^2 of the di-jet.

Finally, from the kinematics of the outgoing jets

$$\left.\begin{aligned} x_F &= x_1 - x_2 \\ \tau &= x_1x_2 \end{aligned}\right\} \qquad x_{1,2} = \frac{1}{2}\left(x_F \pm \sqrt{x_F^2 + 4\tau}\right)$$

we can determine experimentally x_1 and x_2.

The Differential Cross Section

The di-jet production invariant cross section

$$h_A + h_B \rightarrow jet_1 + jet_2 + X$$

can be factorized as

$$E_1 E_2 \frac{d^6\sigma}{d^3 p_1\, d^3 p_1} = \sum_{Q_1 Q_2} \int_0^1 dx_1 \int_0^1 dx_2\, f_A(x_1, \mu_F)\, f_B(x_2, \mu_F)\, \frac{\hat{s}}{2\pi} \frac{d\hat{\sigma}\left(\alpha_s(\mu_R)\right)^{AB\rightarrow J_1+J_2+X}}{d\hat{t}} \delta^4(q_1 + q_2 - p_1 - p_2)$$

The two kinematical variables $d^3p_1 d^3p_2$ will disappear after integration (δ function).
x_1 and x_2 can be determined from the jet kinematics.

We have to add all possible 2-body parton interactions $Q_1Q_2 \rightarrow P_1P_2$ and convolute with the corresponding parton distribution functions.
Since we are not detecting a specific hadron in the final state, there are no fragmentation functions involved.
The interaction dynamics is contained in the sub–process cross section

$$\frac{d\hat{\sigma}}{d\hat{t}} \propto \sum |M|^2$$

$\sum |M|^2$ if the sub–processes are distinguishable, like $q\bar{q} \rightarrow q\bar{q}$ and $q\bar{q} \rightarrow gg$

$\left|\sum M\right|^2$ if the sub–processes are not distinguishable ($\rightarrow$ interference terms!) .

To obtain the cross section, the sub–process cross section must be convoluted with the probability of finding a parton Q_1 and Q_2 inside the hadrons h_A and h_B, respectively: $f_A(x_1)dx_1$ and $f_B(x_2)dx_2$.

This convolution can change completely the relative weight of various sub–processes. At low x, gluons dominate; at large x valence quarks dominate.

The factorization and renormalization scales usually are taken equal to the p_T of the outgoing jets:

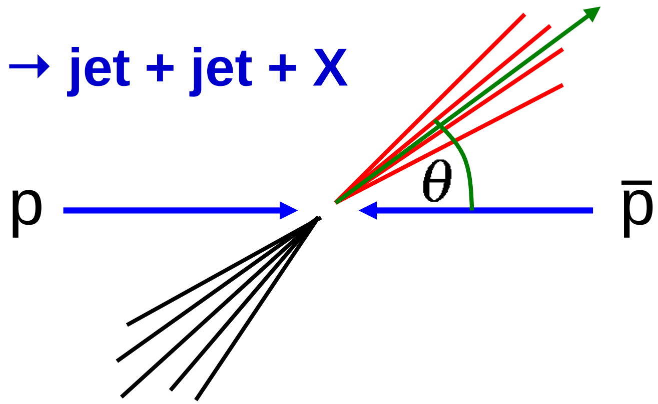
$$\mu_F = \mu_R = p_T$$

At leading order, changing α_S (i.e. μ_R) by 10% can vary the cross section by 50%!
In general, leading order calculations are not satisfactory, but we have to start somewhere
Most processes have been calculated beyond the first order in α_S.
Considering all diagrams, real or virtual, for the production of 3 or 4 partons the cross sections are written as

$$\hat{\sigma} = C_{LO}\alpha_S^2 + C_{NLO}\alpha_S^3 + C_{NNLO}\alpha_S^4 + \cdots$$

Test QCD predictions by looking at production of pairs of high energy jets

$p\bar{p} \rightarrow$ jet + jet + X

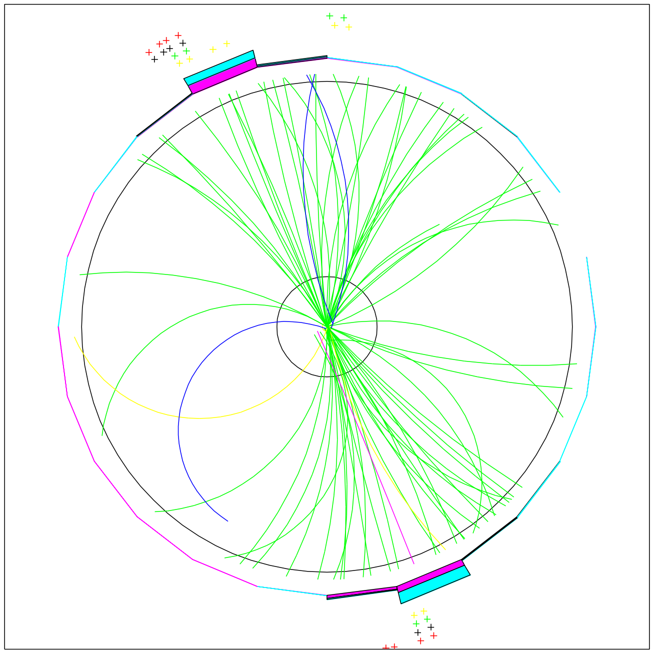


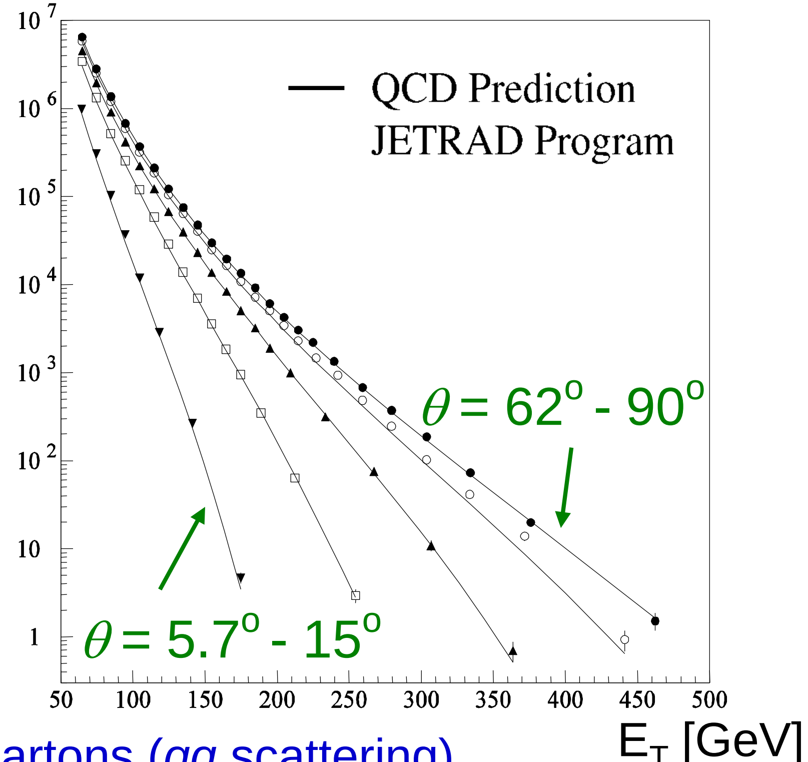
Measure cross-section in terms of

transverse energy $E_T = E_{\text{jet}} \sin \vartheta$

rapidity $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$

pseudorapidity $\eta = \ln\left[\cot\left(\vartheta / 2\right)\right]$

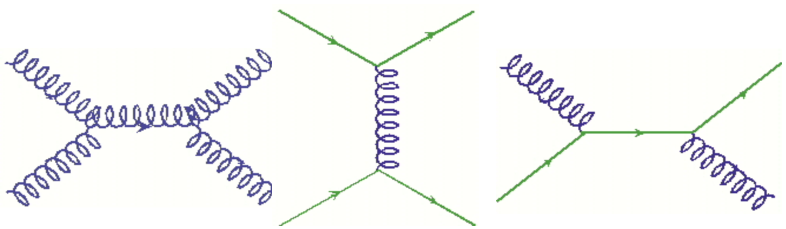




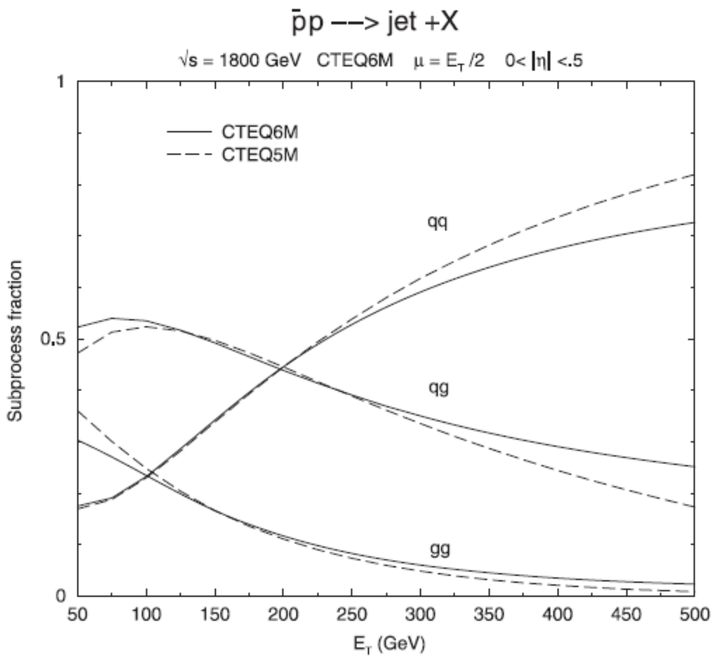
at low E_T cross-section is dominated by low x partons (gg scattering)

at high E_T cross-section is dominated by high x partons ($q\bar{q}$ scattering)

Inclusive Jet Production



gg → gg   qq → qq   qg → qg

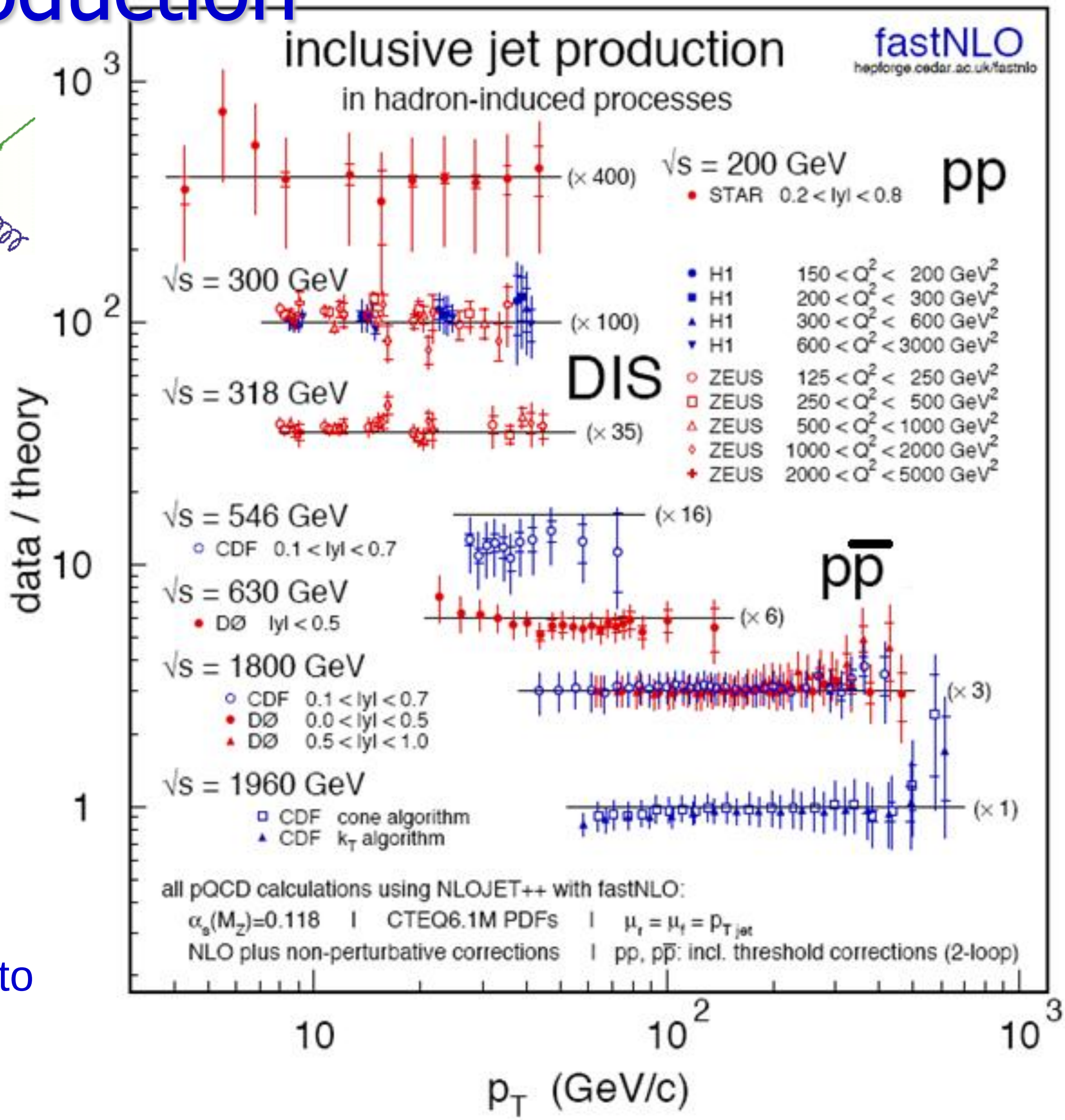


sub-process fraction contributing to

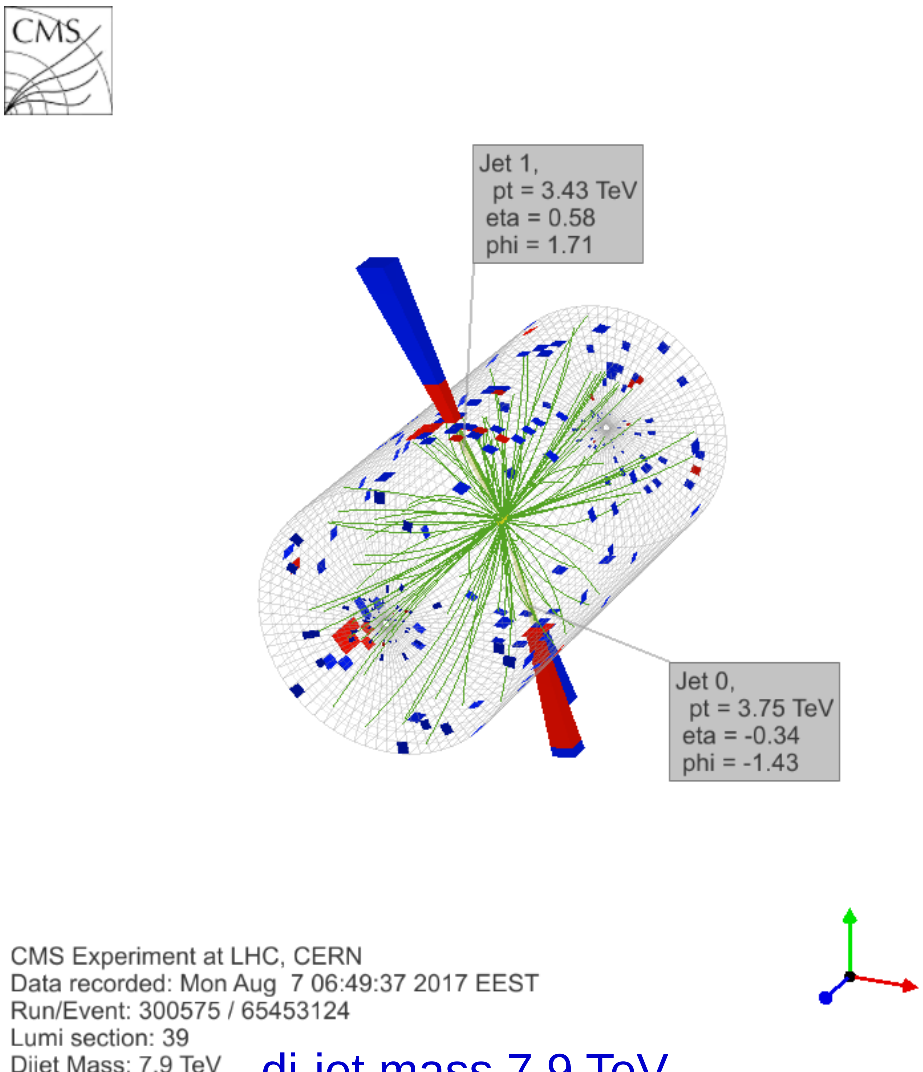
$\overline{p} + p \to \text{jet} + X$

(qq, qg, gg scattering)

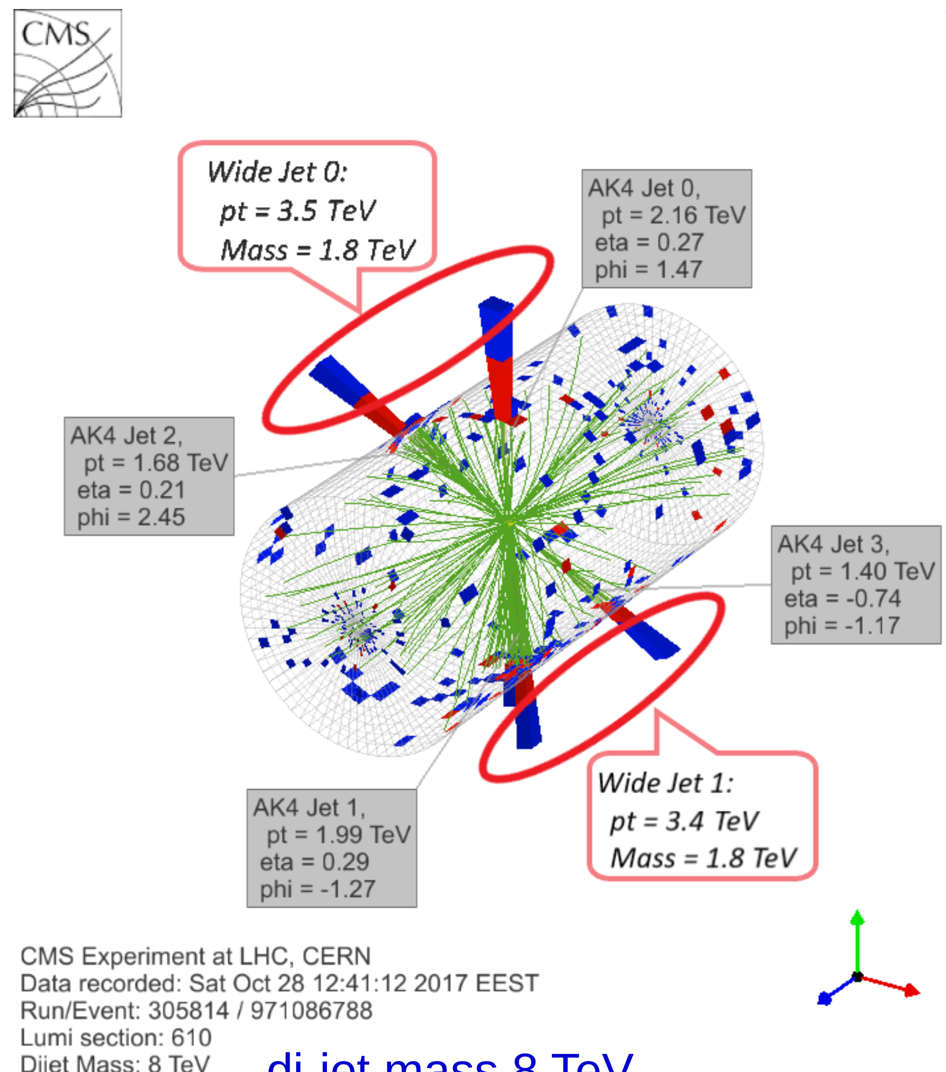
at Tevatron ($\sqrt{s}$ ~ 2 TeV)



Di-Jet Events at CMS



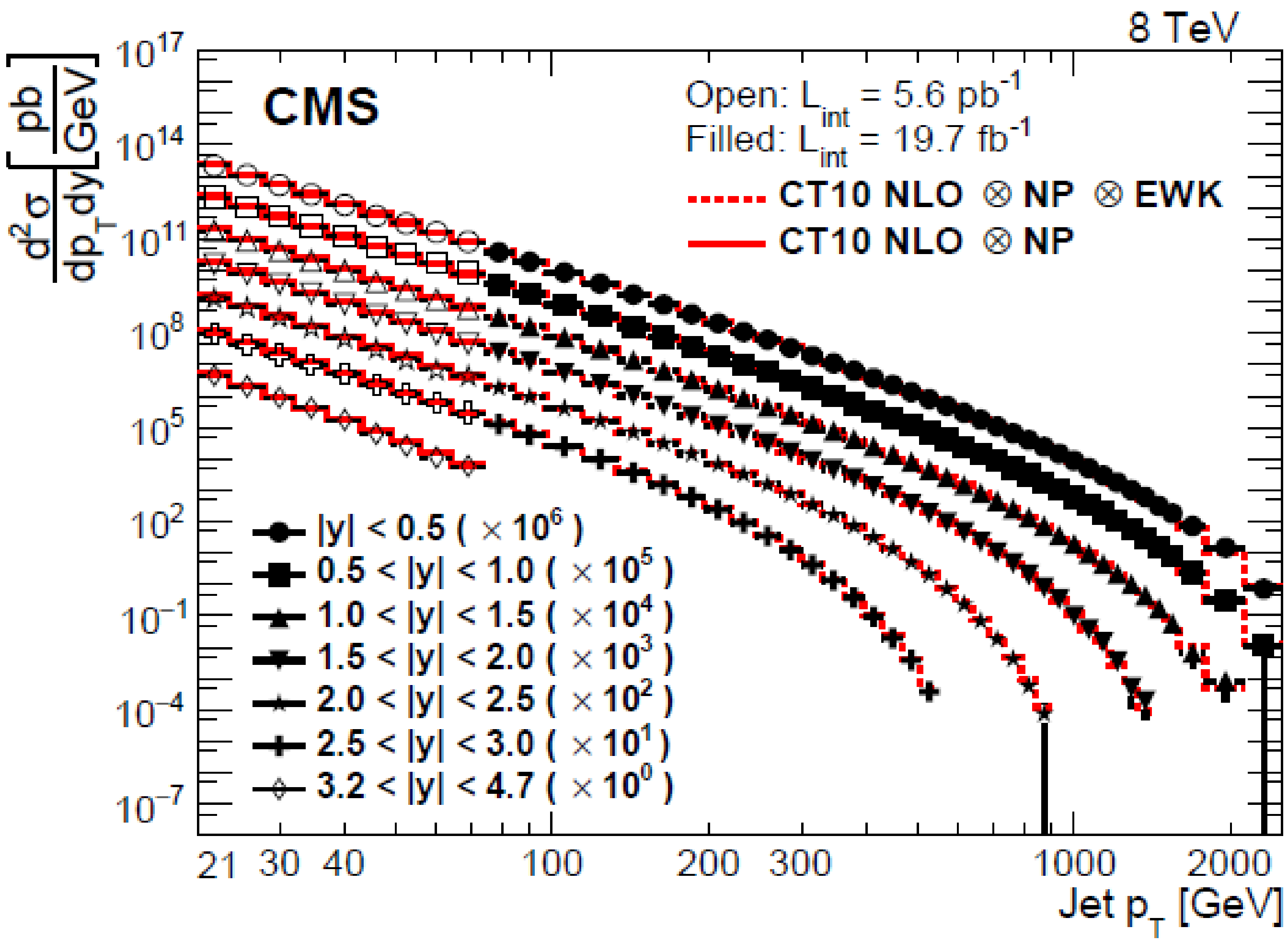
di-jet mass 7.9 TeV



di-jet mass 8 TeV

$$M_{jj}^2 = \hat{s} = x_1 x_2 s \quad \rightarrow \quad x_{1,2} = ?$$

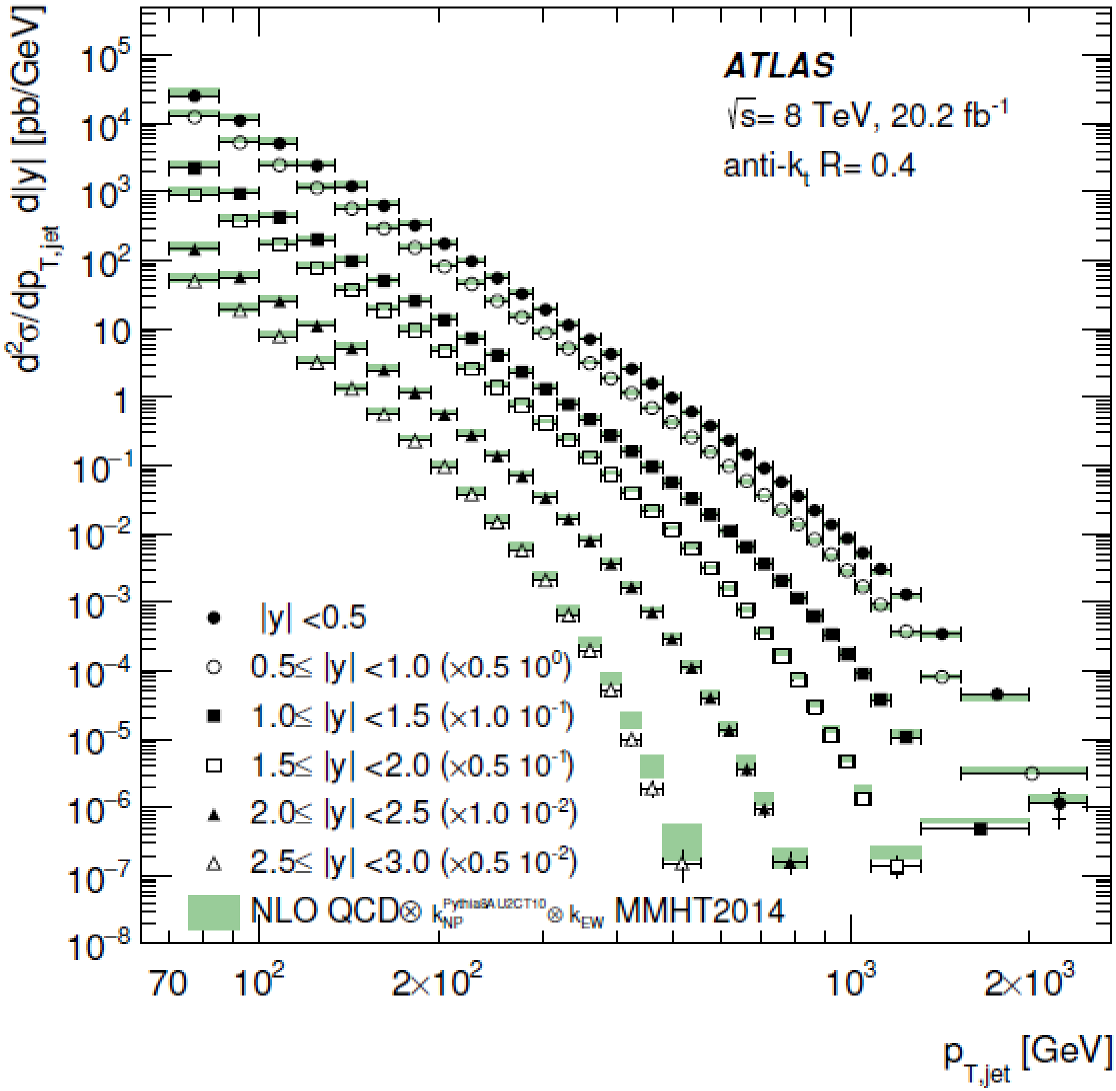
Inclusive Jet ×-section dp_T dy (CMS)

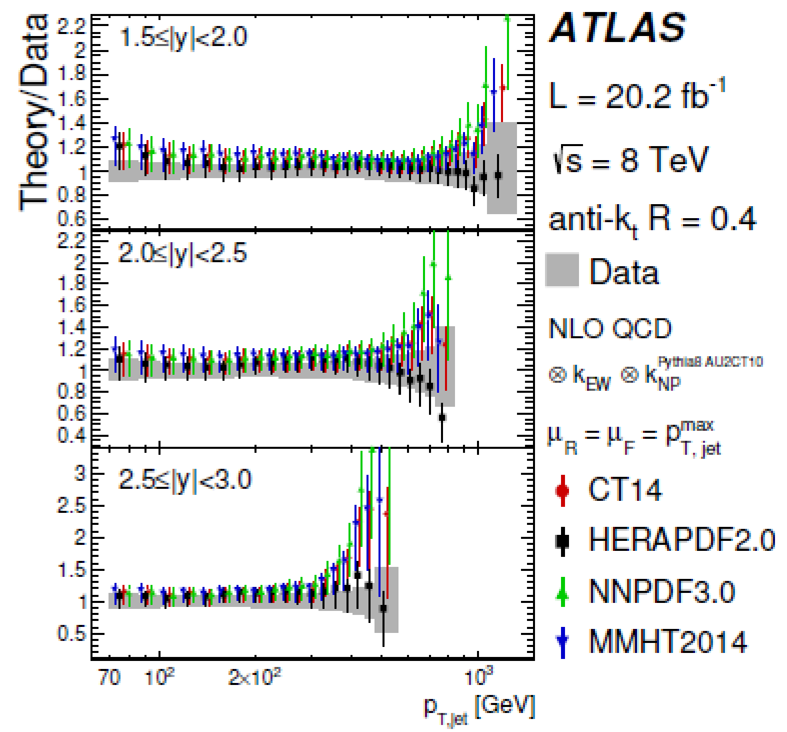


p_T – transverse momentum

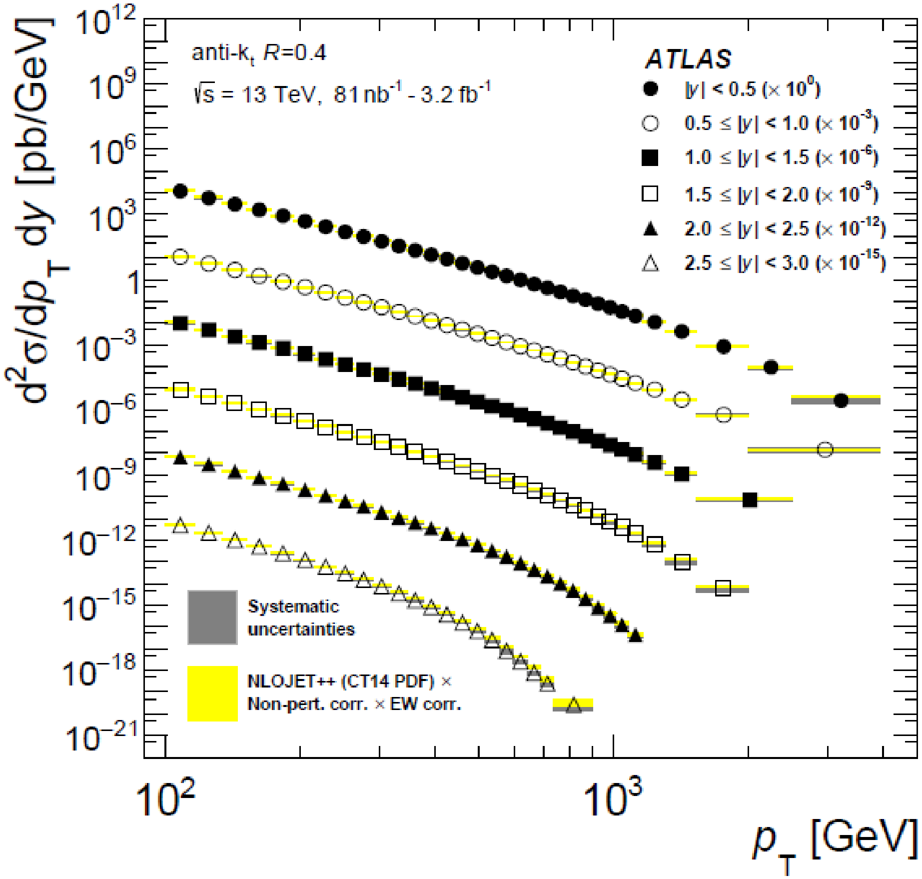
y – rapidity: $$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \tanh^{-1}(p_z/E) \approx \ln\left[\cot(\vartheta/2)\right] = \eta$$

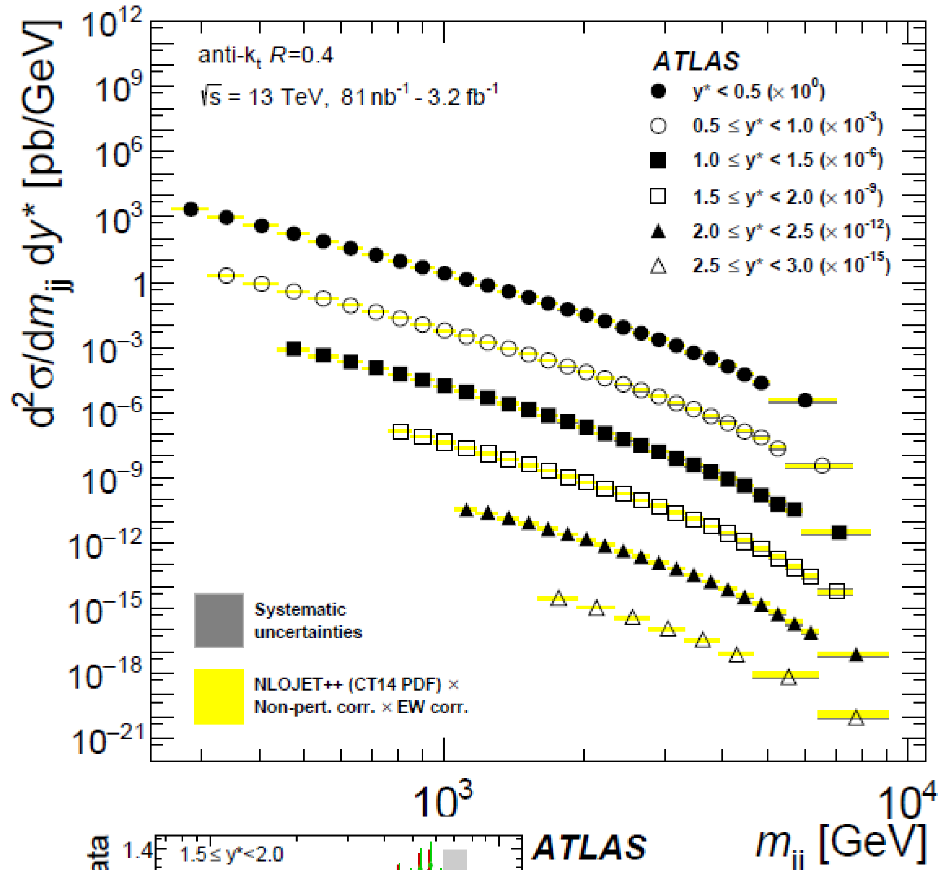
Inclusive Jet ×-section dp_T dy (ATLAS)

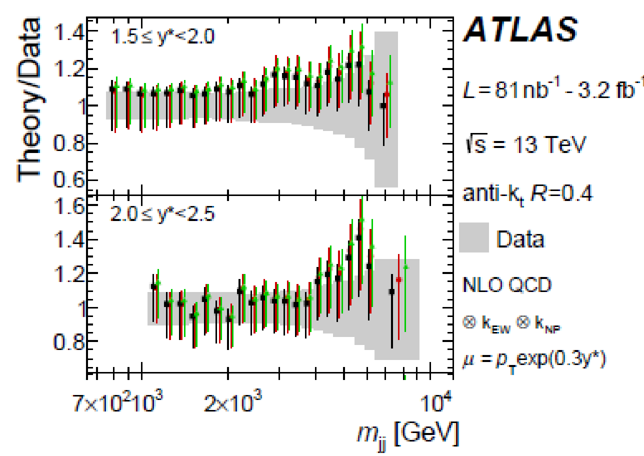




Jets in Atlas





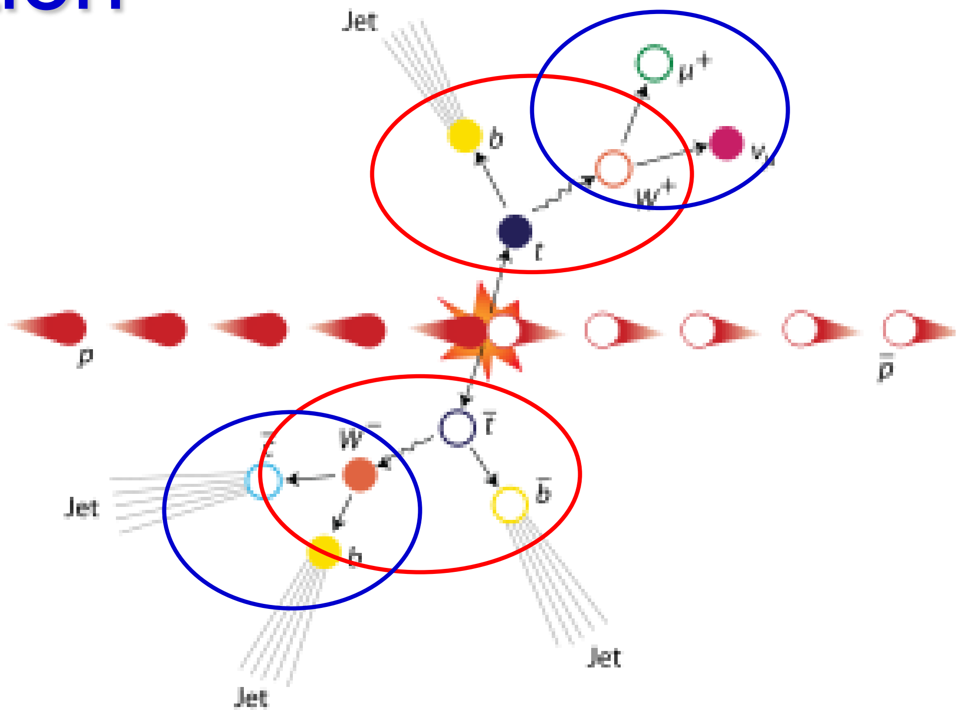


search for quark sub-structure
enhancement at large p_T?

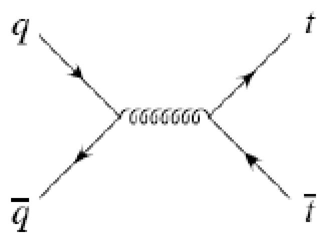
search for new particles / resonances
$X \rightarrow jet_1 + jet_2$
structure (peaks) in M_{jj} spectrum

Top Pair Production

top – $\overline{top}$ production
in proton – anti-proton
collisions at the Tevatron

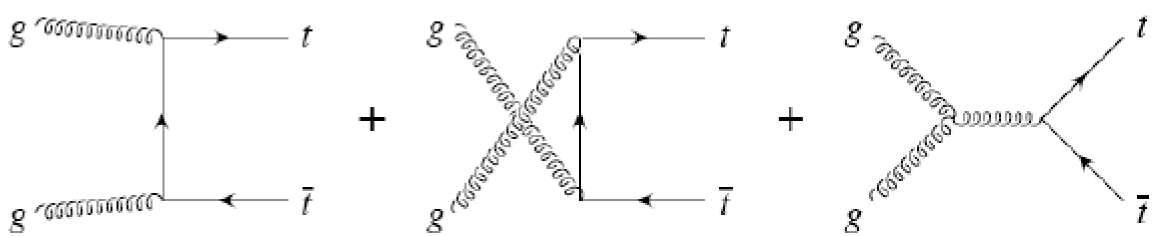


$q\bar{q}$ annihilation

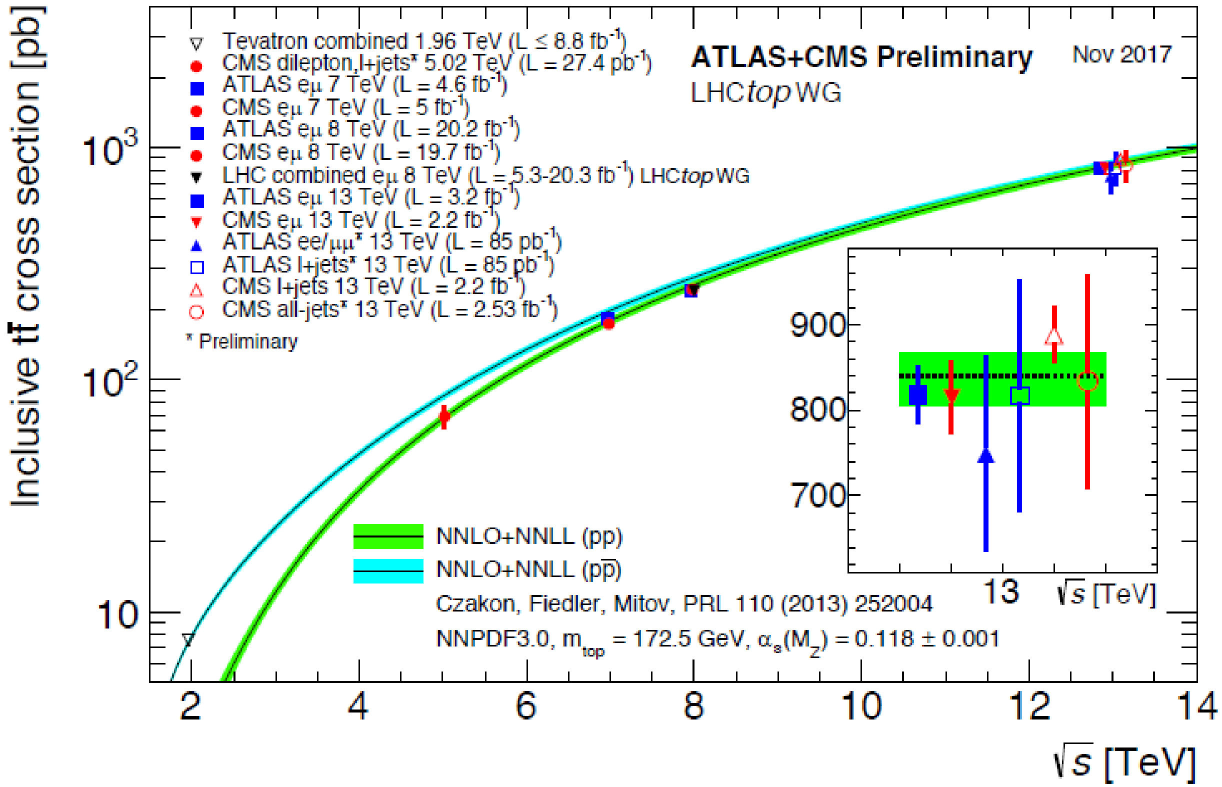


$$\frac{4}{9}\frac{t^2+u^2}{s^2}$$

gg fusion



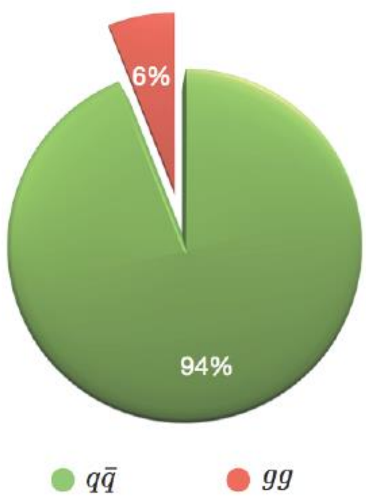
$$\frac{1}{6}\frac{u^2+t^2}{ut}-\frac{3}{8}\frac{u^2+t^2}{s^2}$$



at Tevatron

$p\bar{p}$ collisions

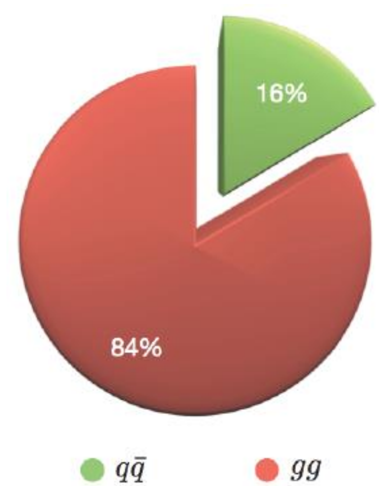
E_{CM} ~ 2 TeV
σ ~ 10 pb



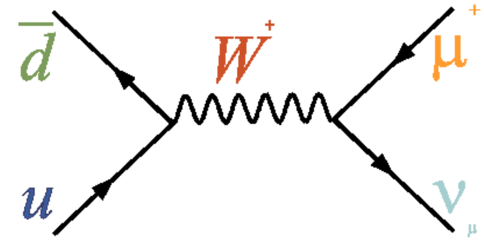
at LHC

pp collisions

E_{CM} ~ 14 TeV
σ ~ 400 pb



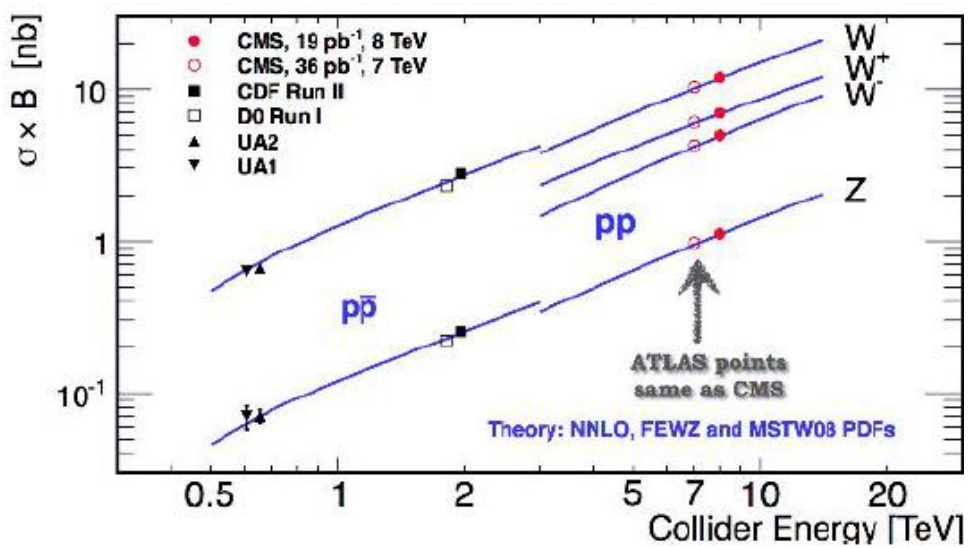
W & Z Production



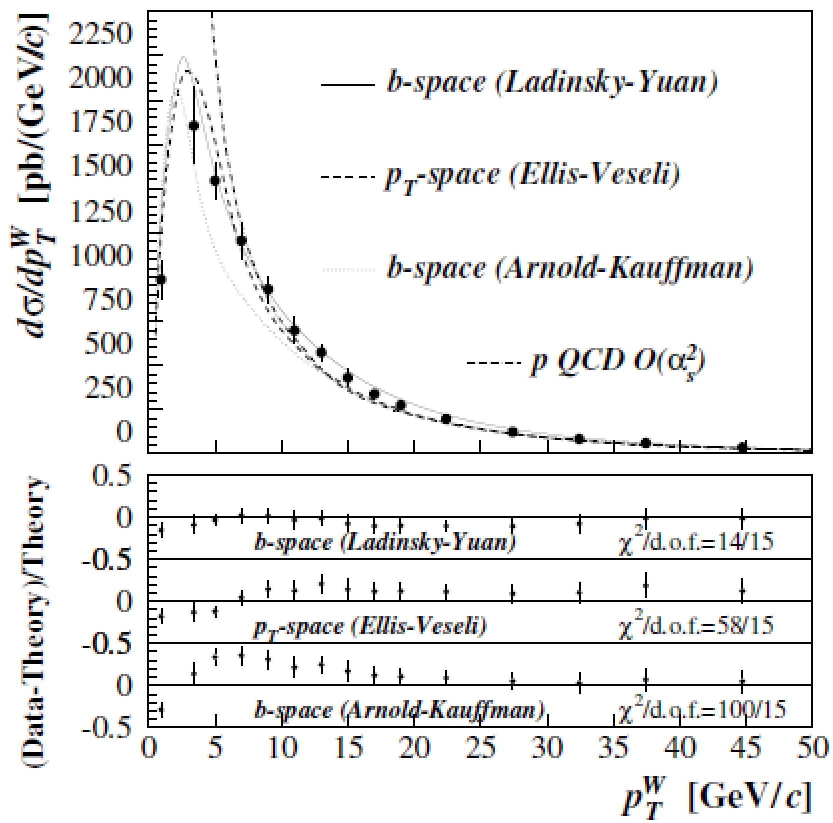
$$\sigma(pp \to W^+ + \text{jet}) \sim \int_0^1 dx_1 \int_0^1 dx_2\ [u(x_1)\bar{d}(x_2) + 1 \leftrightarrow 2]\ \sigma(u\bar{d} \to W^+)\delta(\hat{s} - M_W^2)$$

$$\delta(\hat{s} - M_W^2) = \delta(x_1 x_2 s - M_W^2) = \frac{1}{s}\delta(x_1 x_2 - M_W^2 / s)$$

W & Z cross sections

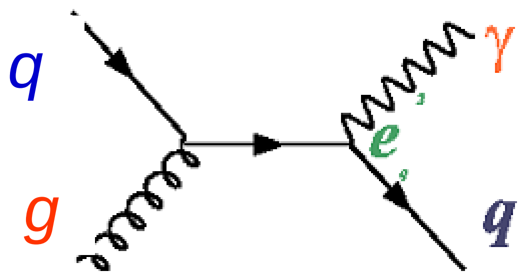


W p_T distribution ($\sqrt{s}$ ~ 1.8 GeV)

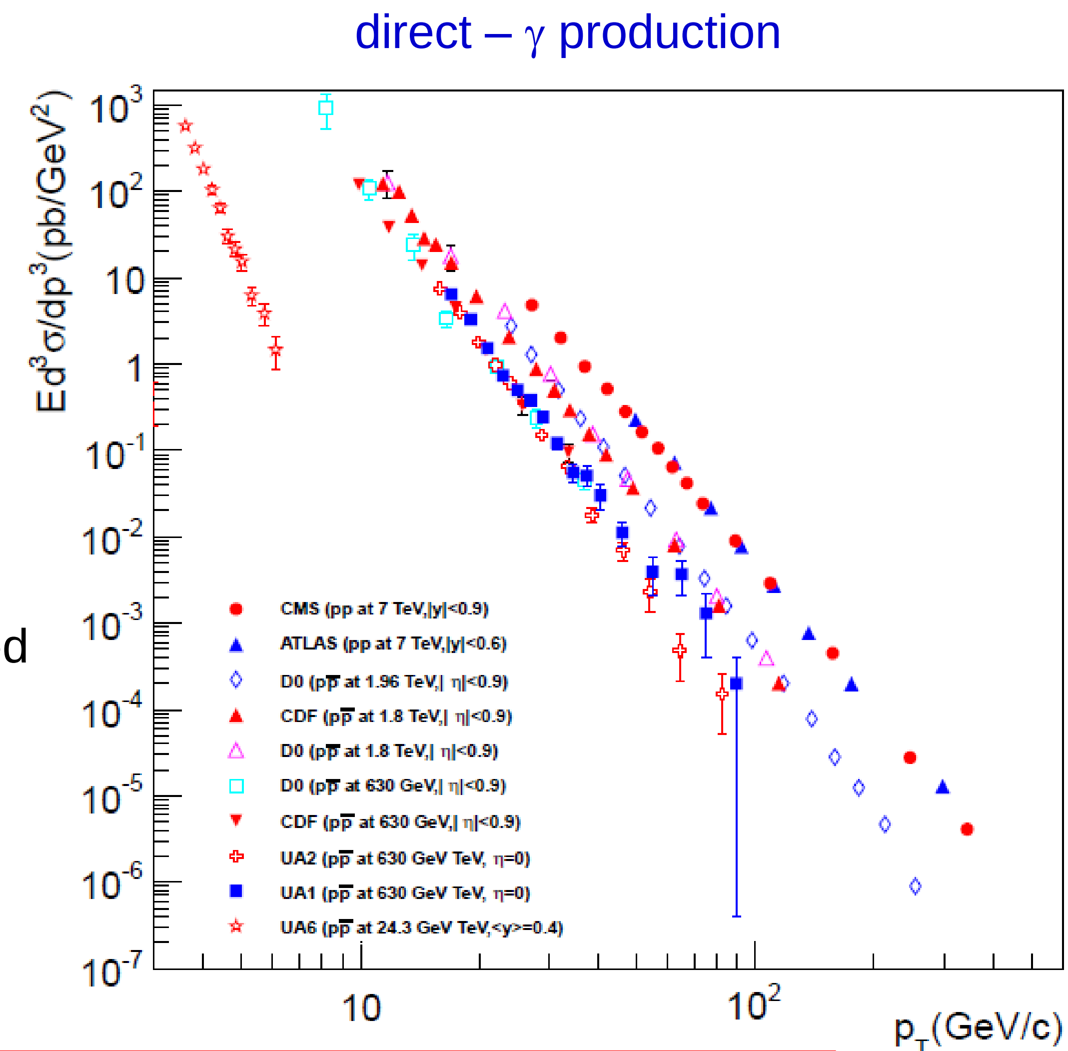


Direct – γ Production

This QCD – Compton diagram gives direct access to the gluons inside the hadrons



By detecting in coincidence an isolated high-p_T photon and the away side jet originated by the fragmenting quark, one can fully reconstruct the sub–process kinematics (recall di-jet kinematics)

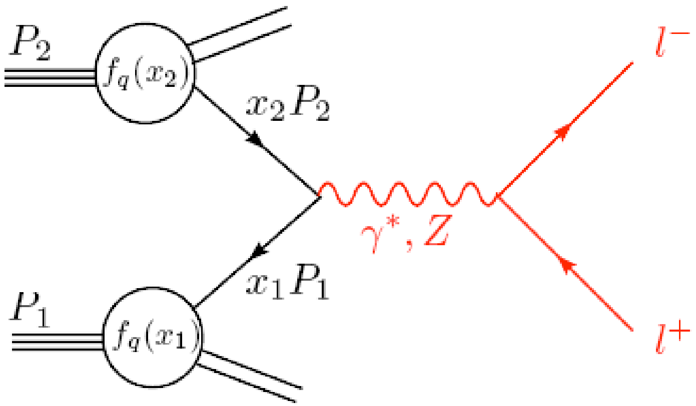


$$\sigma(pp \to \gamma + \text{jet}) \sim \int_0^1 dx_1 \int_0^1 dx_2 \; [g(x_1)q(x_2) + g(x_2)q(x_1)] \; \sigma(qg \to q\gamma)$$

The Drell–Yan Process

The Drell – Yan process is the reaction

$$h_A + h_B \rightarrow \gamma^* + X$$
$$\rightarrow l^+l^-$$



The mass of the lepton pair l^+l^- (e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$)
$M_{ll} \neq 0$ since the photon is virtual ($Q^2 > 0$!)

One assumes that the leptons are produced directly by the electroweak interaction
$q\bar{q} \rightarrow l^+l^-$.

$$\sigma_{AB\rightarrow l^+l^-} = \int dx_a dx_b\, f_{a/A}(x_a)\, f_{b/B}(x_b) \times \hat{\sigma}_{ab\rightarrow l^+l^-}$$

To annihilate, the quark and anti-quark must have same flavor and color.

Very clean, golden process in QCD, because

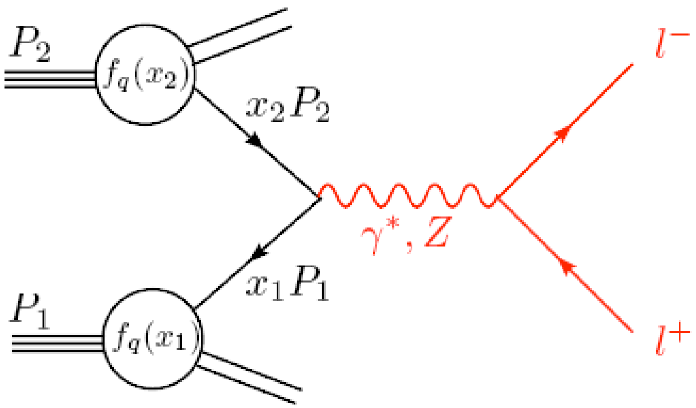
1. dominated by quarks in initial state
2. no quarks or gluons in the final state (small QCD corrections)
3. experimentally leptons are easier to measure
4. very important and precise SM test at LHC

The process is similar to the annihilation $e^+e^- \rightarrow q\bar{q}$ and to the DIS eq $\rightarrow$ eq.
The quarks inside the hadrons are described by the hadron structure functions $f(x,Q^2)$.

The Drell-Yan Cross Section

Start with the partonic sub–process (QED)

$$\hat{\sigma}(q\bar{q} \to l^+l^-) = \frac{1}{N_c}\frac{4\pi\alpha^2}{3\hat{s}}e_q^2$$



To embed the partonic cross section in the hadronic process, we rewrite it as a differential cross section $d\sigma / dQ^2$ for the production of a lepton pair with invariant mass $M_{ll}^2 = Q^2 = \hat{s}$

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{1}{N_c}e_q^2\frac{4\pi\alpha^2}{3Q^4}\delta\left(Q^2 - \hat{s}\right)$$

The hadronic cross section can be obtained by adding incoherently the contributions of different quarks and convoluting the partonic $d\sigma / dQ^2$ with the structure functions $f_q(x)$

$$\frac{d\sigma\left(pp \to l^+l^-X\right)}{dQ^2} = \frac{1}{3}\cdot\frac{1}{3}\cdot 3\sum_q\int_0^1 dx_1\int_0^1 dx_2\, f_q\left(x_1, Q^2\right) f_{\bar{q}}\left(x_2, Q^2\right) e_q^2\frac{4\pi\alpha^2}{3Q^4}\delta\left(Q^2 - \hat{s}\right)$$

The factors 1/3 come from averaging over initial state colors and the factor 3 from summing over $q\bar{q}$ same color combinations. If the quarks carried no color, the cross section would be 3 times larger (another evidence for color !).

The kinematics is similar to the di-jet production in hadron – hadron collisions.
We replace the outgoing jet_1 and jet_2 with the lepton pair l^+ and l^- and can determine the c.o.m. kinematics:

$$x_F = x_1 - x_2 = p_{//} / p_{cm}$$

$$M_{12} \equiv \sqrt{x_1 x_2 s} = \sqrt{\hat{s}}$$

$$\tau \equiv M_{12}^2 / s = \hat{s} / s = x_1 \cdot x_2$$

Next, we can replace $\hat{s} = x_1 x_2 s$

$$\frac{d\sigma\left(pp \to l^+ l^- X\right)}{dQ^2} = \frac{1}{N_c}\frac{4\pi\alpha^2}{3Q^4}\sum_q e_q^2 \int_0^1 dx_1 \int_0^1 dx_2\, f_q\left(x_1, M^2\right) f_{\bar{q}}\left(x_2, M^2\right) \delta\left(1 - x_1 x_2 \frac{s}{Q^2}\right)$$

and in terms of x_1 and x_2 (integrate over Q^2, drop the integral over x_1 and x_2)

$$\frac{d^2\sigma\left(pp \to l^+ l^- X\right)}{dx_1 dx_2} = \frac{1}{N_c}\frac{4\pi\alpha^2}{3 x_1 x_2 s}\sum_q e_q^2 \left[f_q^A\left(x_1\right) f_{\bar{q}}^B\left(x_2\right) + f_{\bar{q}}^A\left(x_1\right) f_q^B\left(x_2\right) \right]$$

or in terms of x_F and M_{12}^2 (= Q^2)

$$\frac{d^2\sigma\left(pp \to l^+ l^- X\right)}{dM_{12}^2 dx_F} = \frac{1}{N_c}\frac{4\pi\alpha^2}{3 M_{12}^4}\frac{x_1 x_2}{x_1 + x_2}\sum_q e_q^2 \left[f_q^A\left(x_1\right) f_{\bar{q}}^B\left(x_2\right) + f_{\bar{q}}^A\left(x_1\right) f_q^B\left(x_2\right) \right]$$

The Drell-Yan process is extremely important, because it allows us to access the anti-quark distributions directly.

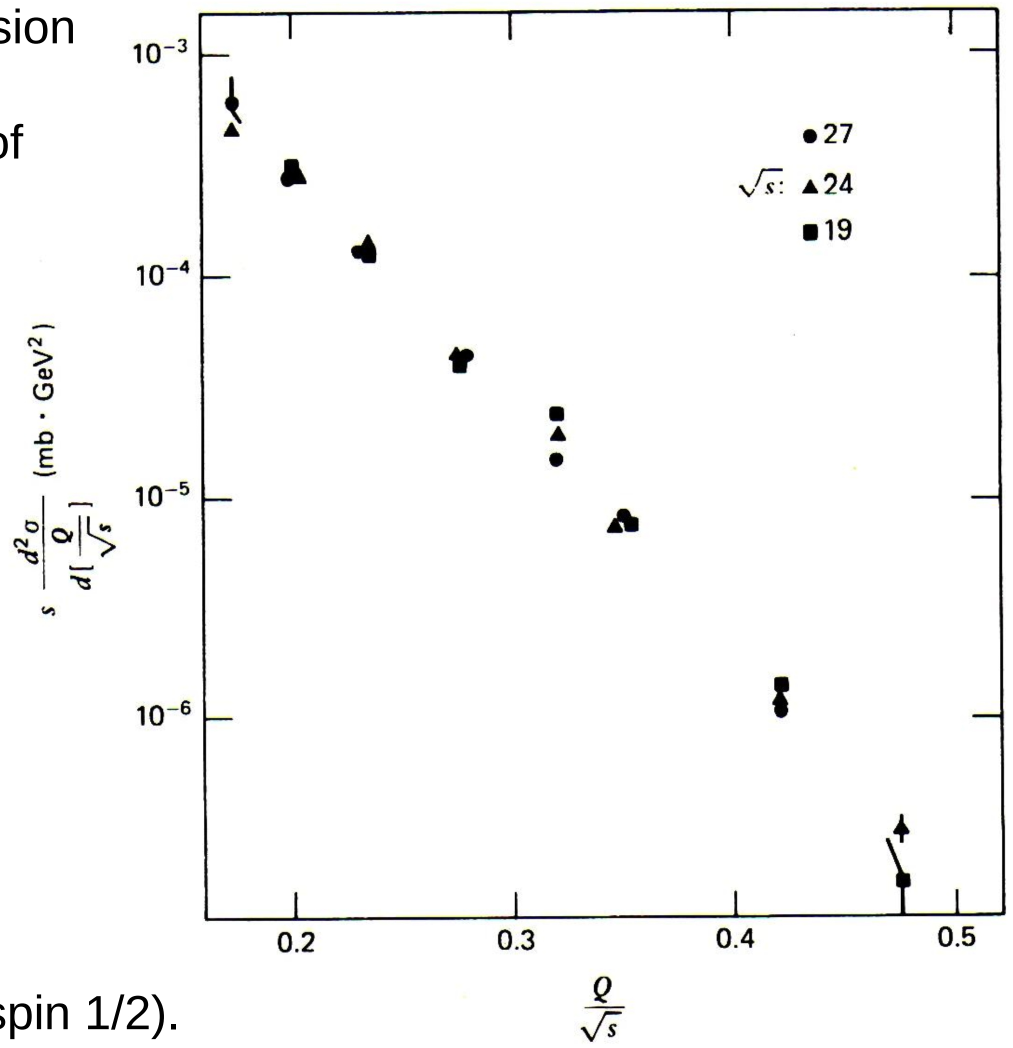
In lowest QCD order with no gluon emission we expect a scaling result:
although the cross section is a function of s and the lepton pair mass Q^2,
the quantity

$$Q^4 \, d\sigma/dQ^2$$

is a function of the ratio s/Q^2 only.
(NB the LO diagram does not depend on any scale).

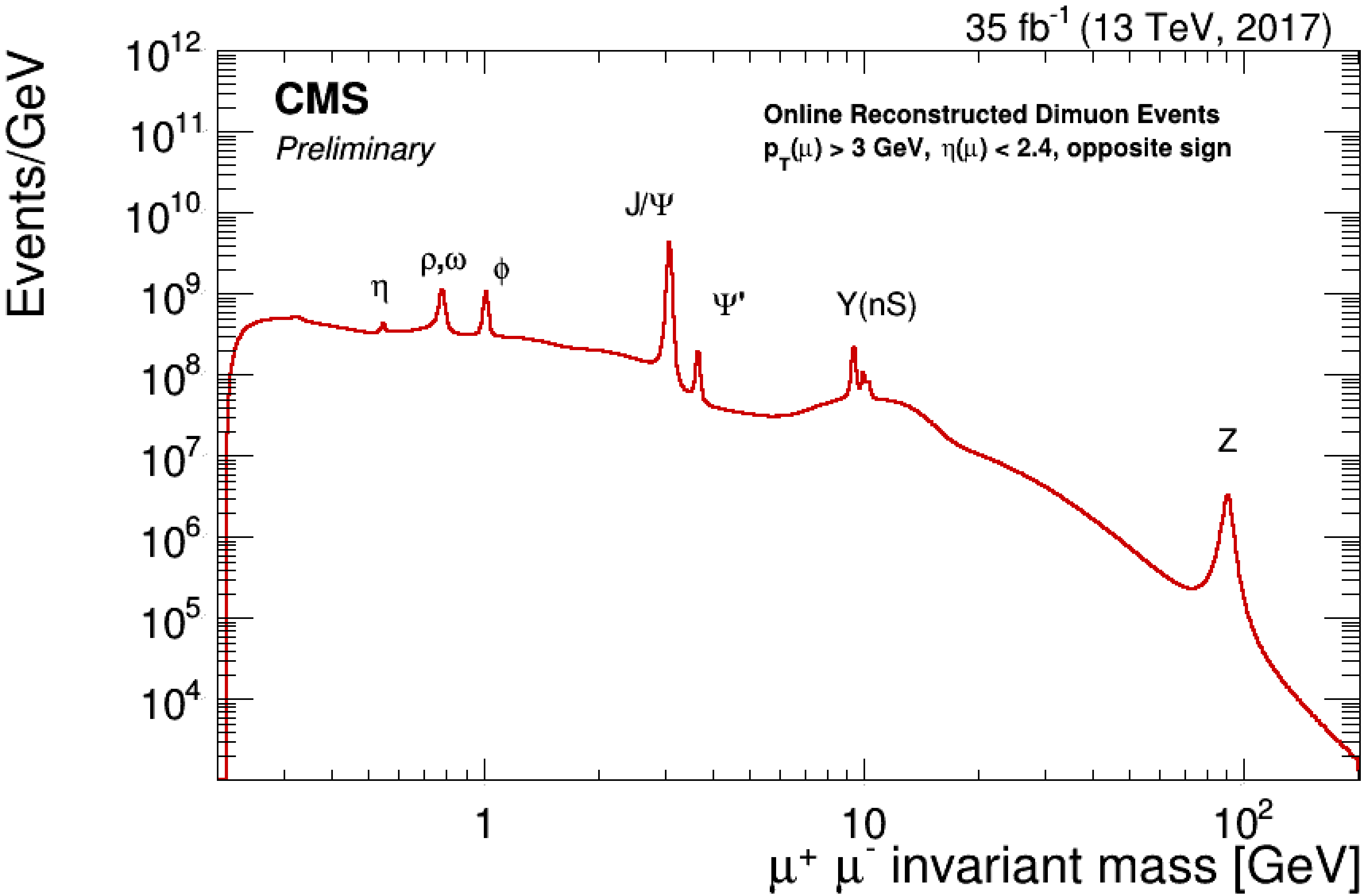
Several experiments verified this scaling invariance with a precision of 10 – 20%.

The angular distribution of the leptons in the lepton pair c.o.m. follows well the $(1 + \cos^2\theta)$ distribution
(yet another proof that the quarks have spin 1/2).



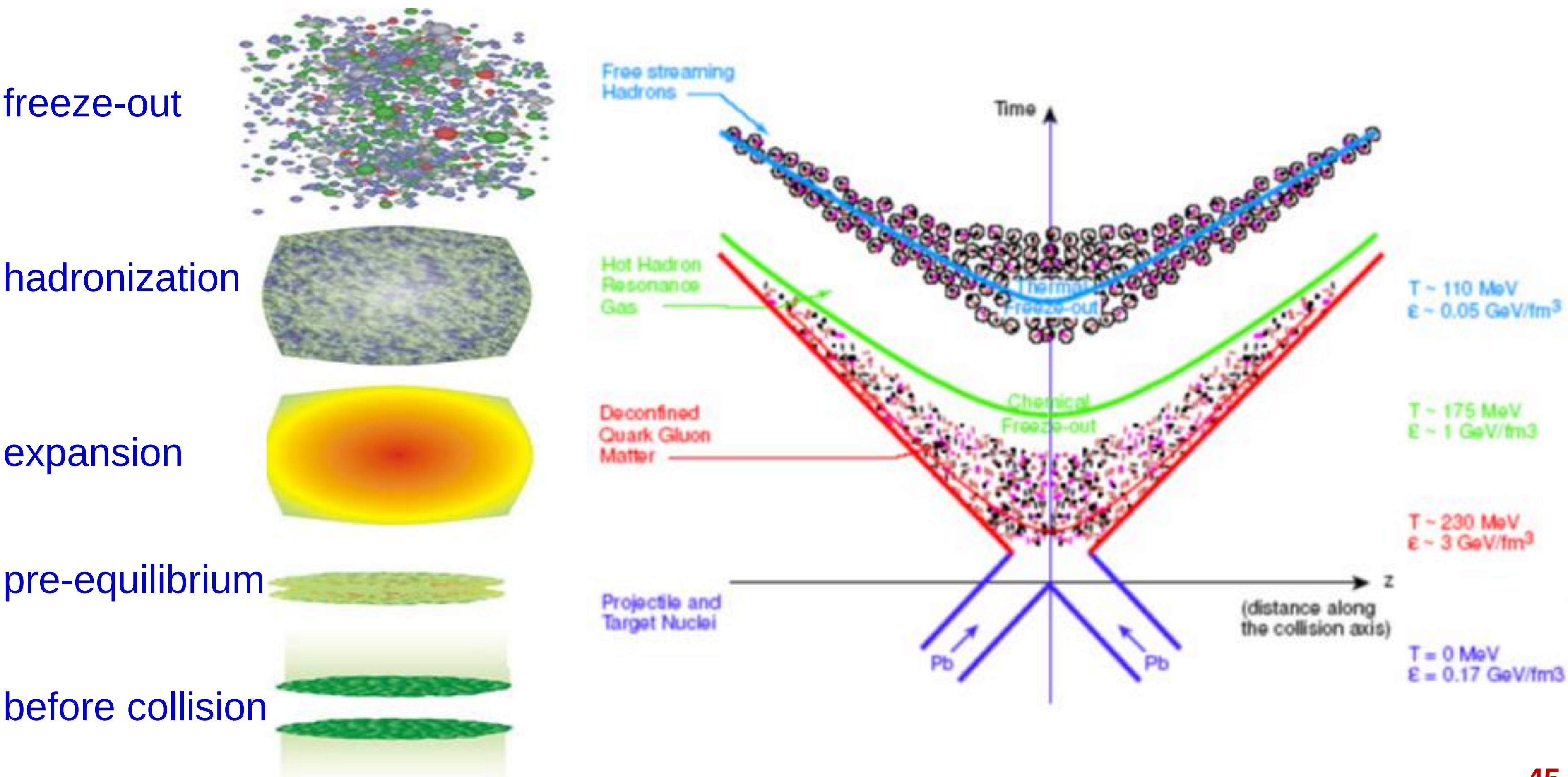
At NLO we have to consider also gluon radiation from the interacting quarks.
This will introduce logarithmic scaling violations similar to those observed in DIS.

$p + p \rightarrow \mu^+ + \mu^- + X$

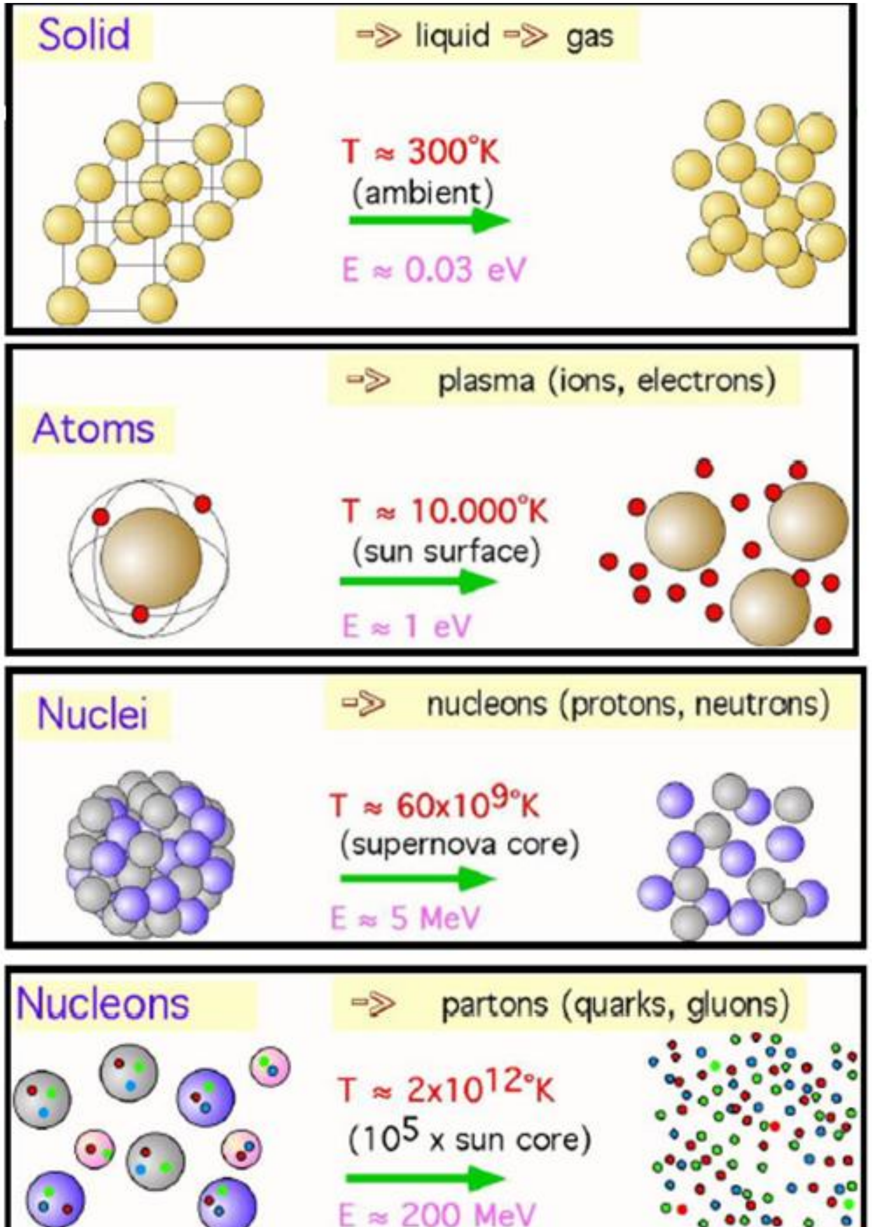


Schematic of a Relativistic Heavy Ion Collision

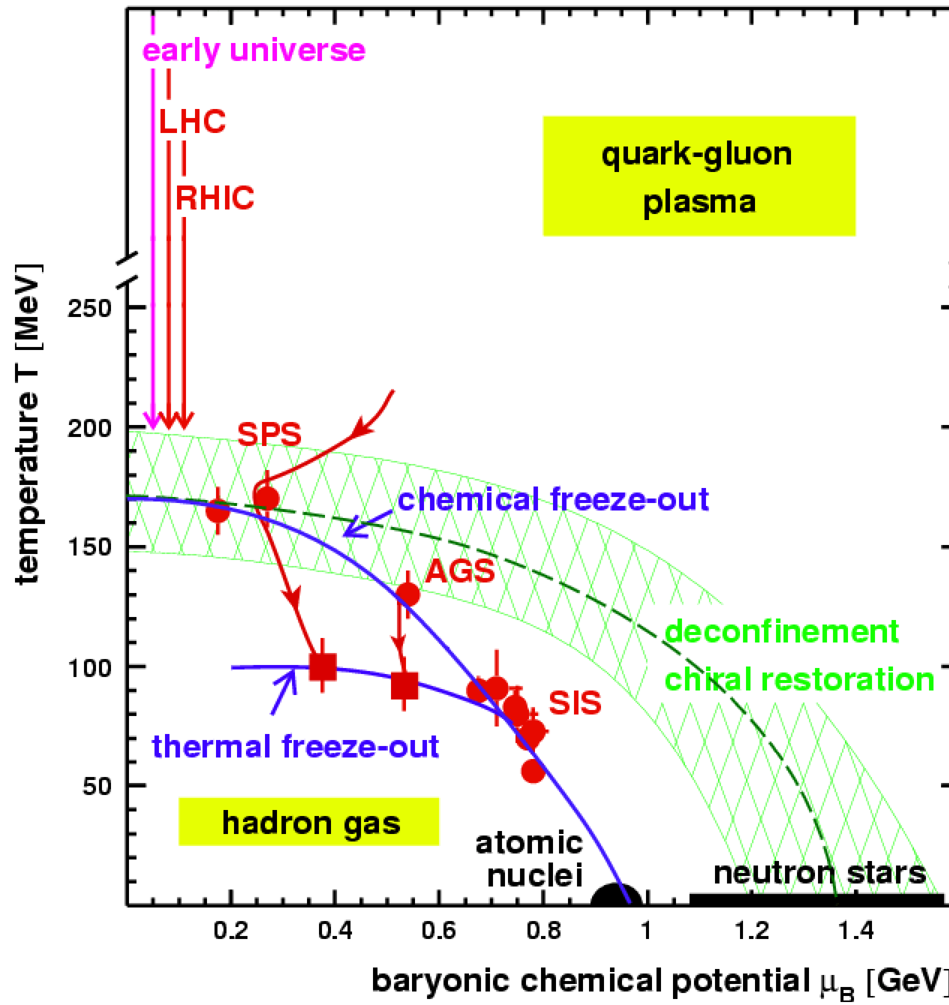
Collide relativistic heavy ions (i.e Pb – Pb) to heat and compress nuclear matter.
When the temperature and density are above a critical value (ε>3 GeV/fm^3, T>200 MeV), normal nuclear matter results in deconfined quark-gluon matter (plasma).
The constituents of normal nuclear matter are hadrons while the constituents of quark-gluon matter are quarks and gluons in a deconfined state.



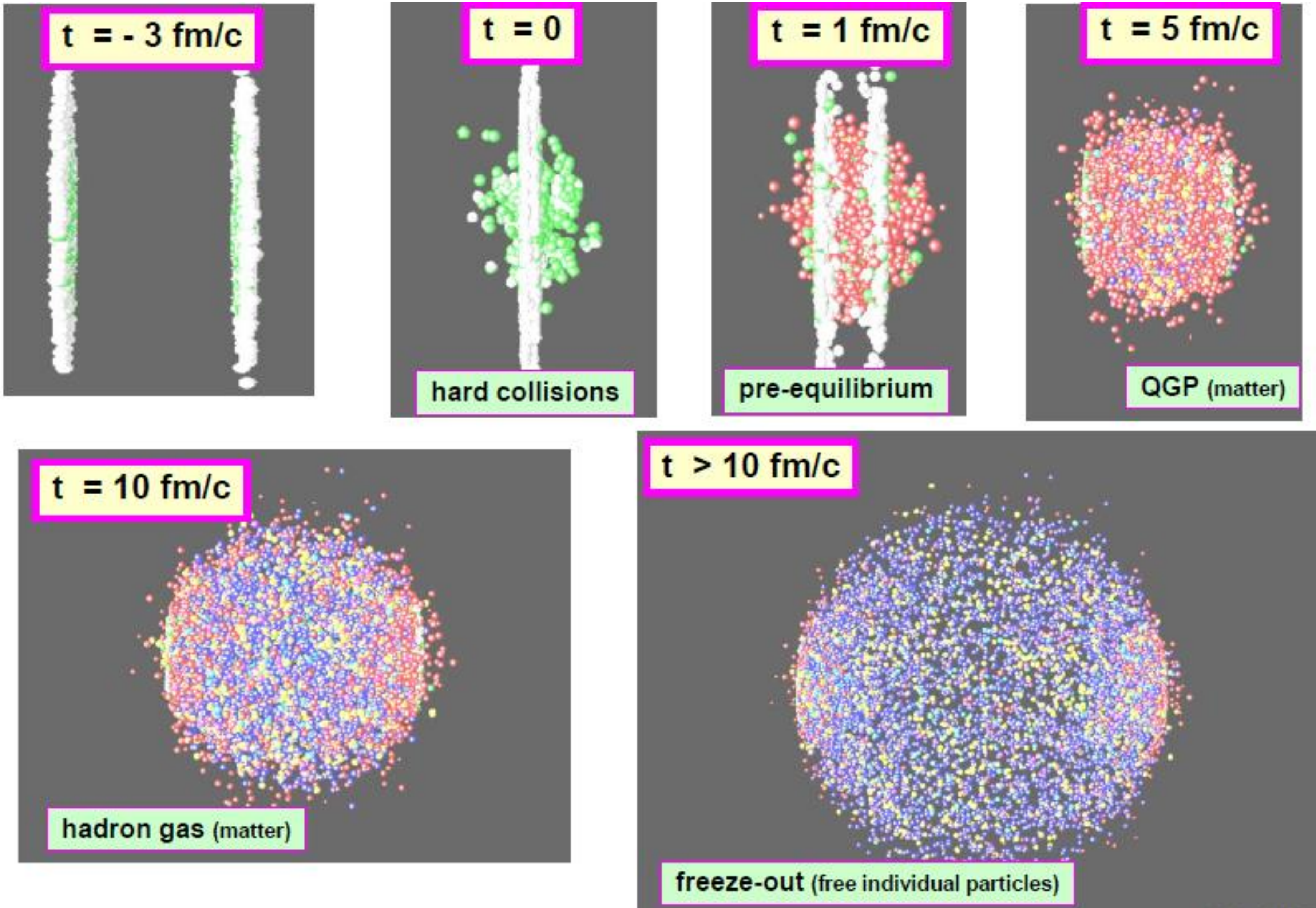
Melting Matter



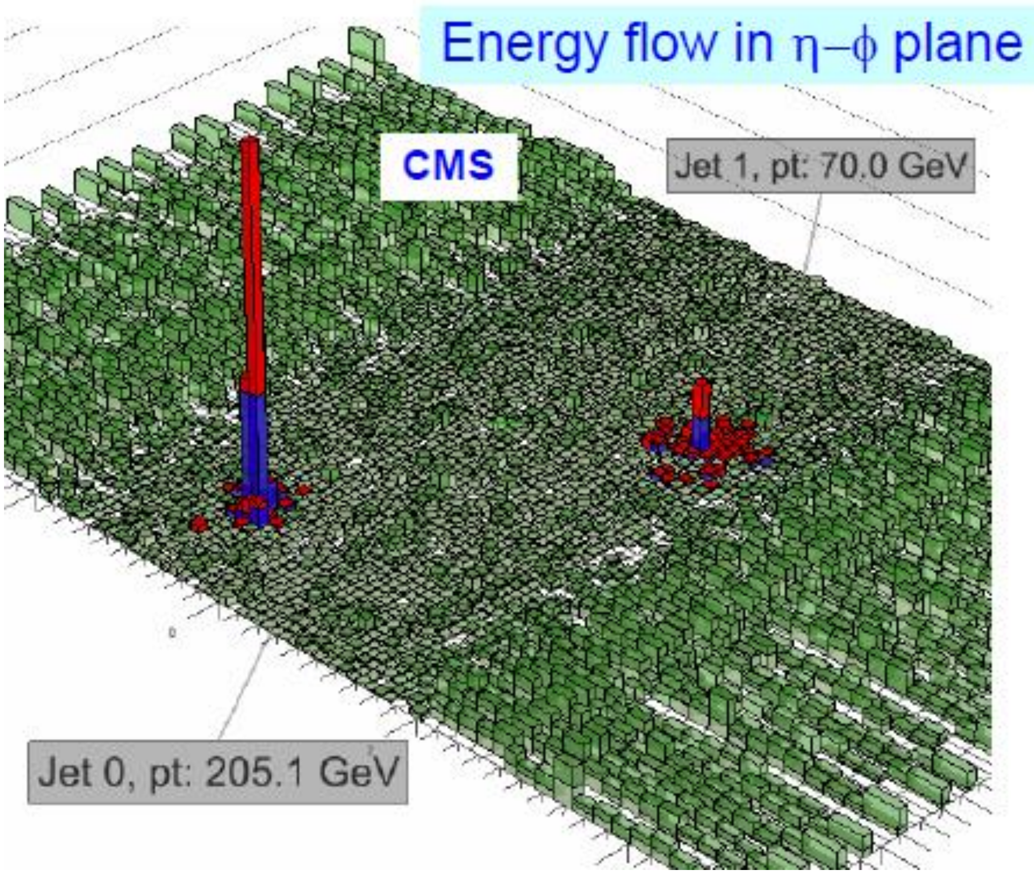
phase diagram of hadron matter

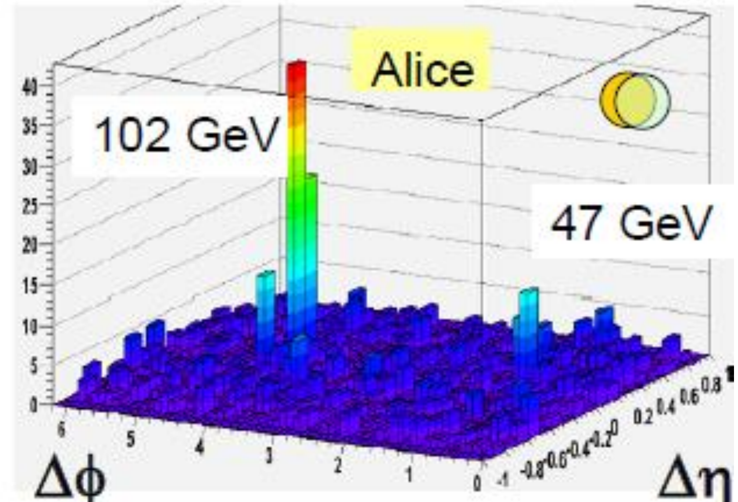


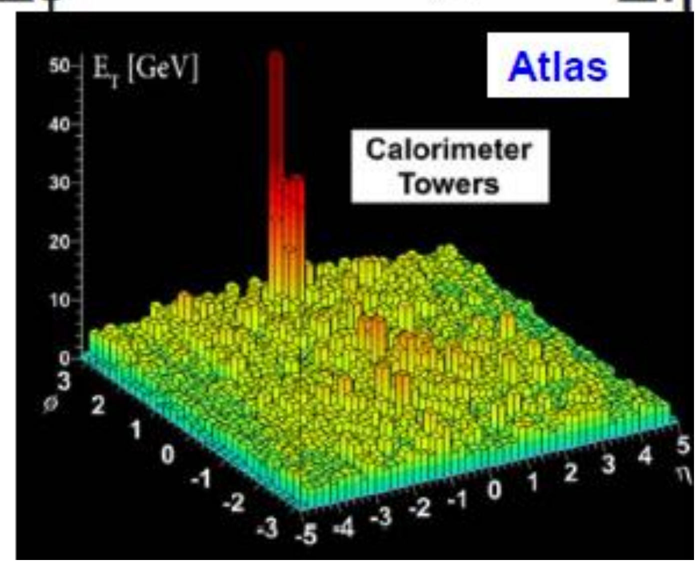
Heating Matter in Heavy Ion Collisions



Jet Quenching at LHC







For Next Week

Study the material and prepare / ask questions
Study ch. 11 (sec. 8, 9) in Halzen & Martin and / or ch. 10 (sec. 9) in Thomson

Do the homeworks

Next week we will start the weak interactions
  have a first look at the lecture notes, you can already have questions
  read ch. 12 (sec. 1, 2) in Halzen & Martin and / or ch. 11 (sec. 1 to 7) in Thomson