Advanced Particle Physics 2

Strong Interactions
and Weak Interactions

L7 – Phenomenology of Weak Interactions

(http://dpnc.unige.ch/~bravar/PPA2/L7)

lecturer

Alessandro Bravar Alessandro.Bravar@unige.ch

tél.: 96210 bureau: EP 206

assistant

Jorge Sabater Jorge.Sabateriglesias@unige.ch

April 04, 2023

A Bit of History of (Electro)Weak Interactions
(not complete)

1896	discovery of radioactivity (Becquerel, Nobel 1903)
1920	β spectrum (~20 years of research and controversy)
1932	Pauli introduces the neutrino
1933	Fermi theory of weak interactions
1953	θ^+ / τ^+ puzzle (parity violation)
1955	$\nu_e \neq \overline{\nu}_e$ (Davis)
1956	Lee-Yang parity violation in weak decays (Nobel 1956)
1956	discovery of neutrino (Cowans and Reines, Nobel 1995)
1957	Wu parity violation experiment (PV in β decay)
1958	Goldhaber measures neutrino helicity
1958	V - A structure of weak interactions (Feynman and Gell-Mann)
1962	$\nu_\mu \neq \nu_e$ (Lederman, Schwartz, and Steinberger, Nobel 1988)
1963	Cabibbo angle ($\rightarrow$ quark mixing)
1964	Higgs mechanism (Englert, Brout, and Higgs, Nobel 2013)
1964	CP violation in K^0 decays (Cronin and Fitch, Nobel 1980)
1964	Solar neutrino problem (Davis†, Nobel 2002)
1967	E-W unification (Glashow, Salam, and Weinberg, Nobel 1979)
1970	Glashow, Iliopulos, Maiani GIM mechanism (charm)
1971	renormalization of Yang-Mills theories (‘t Hooft and Veltman, Nobel 1999)
1973	discovery of Neutral Currents
1973	Kobayashi – Maskawa 3 $\times$ 3 quark mixing matrix (Nobel 2008)

1977 discovery of tau lepton (Perl, Nobel 1995)
1983 discovery of $W^{+/-}$ and Z^0 bosons (Rubbia and van der Meer, Nobel 1984)
1987 observation of neutrinos from supernovae (Koshiba, Nobel 2002)
1988 Z^0 line shape $\rightarrow$ 3 flavor families
1998 neutrino oscillations (Kajita and McDonald, Nobel 2015)
2000 direct observation of tau neutrino (the fermion family is complete)
2001 direct CP violation in the B_0 system
2012 discovery of the Higgs boson
2013 ν_e appearance and θ_{13}
2017 hint of CP violation in ν sector
2019 discovery of CP violation in D^0 system

And many more discoveries to come

direct measurement of neutrino mass
CP violation in the lepton sector
nature of neutrinos (Dirac vs. Majorana)
lepton flavor violation
new particles
…..

The history of weak interactions if tightly connected to the history of neutrinos,
In a sense the history of neutrinos is the history of weak interactions
and the Standard Model is neutrino physics

Properties of Weak Interactions

Strong and Weak Nuclear forces associated with nuclear “properties” ~ ’20

STRONG: binds the nucleons in the nucleus

WEAK: beta decays

new forces $\Rightarrow$ new couplings, new mediators, …

~ ’30 know particles:

p, n, e^-, ν (postulated by Pauli), π (postulated by Yukawa)

anti-particles (Dirac)

no QED and no QFT!

Weak force connected with nuclear decays

β^-: $^{60}Co \rightarrow {}^{60}Ni^{**} + \beta^- + \overline{\nu_e}$ (neutron decay)

$n \rightarrow p\, e^- \overline{\nu}_e$

β^+: $^{22}Na \rightarrow {}^{22}Ne^* + \beta^+ + \nu_e$ (proton “decay”)

$p \rightarrow n\, e^+ n_e\, \nu_e$

EC: $^{37}Ar + e^- \rightarrow {}^{37}Cl + \nu_e$ (electron capture)

$p + e^- \rightarrow n + \nu_e$

[very broad range of lifetimes for β decays: ms $\rightarrow 10^7$ s $\rightarrow 10^{16}$ s]

but also with particle decays

$\mu^+ \rightarrow e^+ \nu_e \overline{\nu_\mu}$ $\tau = 2.2 \times 10^{-6}$ s

$\pi^+ \rightarrow \mu^+ \nu_\mu$ $\tau = 2.6 \times 10^{-8}$ s

Neutrinos participate in all these decays

Weak Interactions intimately associated with neutrinos

(there are also weak decays with no neutrinos, i.e. $\Lambda \to p + \pi^-$)

ν sources

stars, supernovae, "atmosphere", Earth, human body, accelerators, nuclear reactors, Big-Bang (remnants of EW transition ~ meV neutrinos), …

The weak force is the only force with no bound states:

- weakness (very heavy mediators)
- short range ~ 10^{-17} m (proton radius 10^{-15} m)

Weak interaction effects (decays, collisions) observable only when not masked by strong or electromagnetic interactions.

Pure weak probes are neutrinos (carry no color and no electric charge)

Basic symmetries P, C, T are violated by weak interactions

Parity violation maximal

Charge conjugation violation maximal

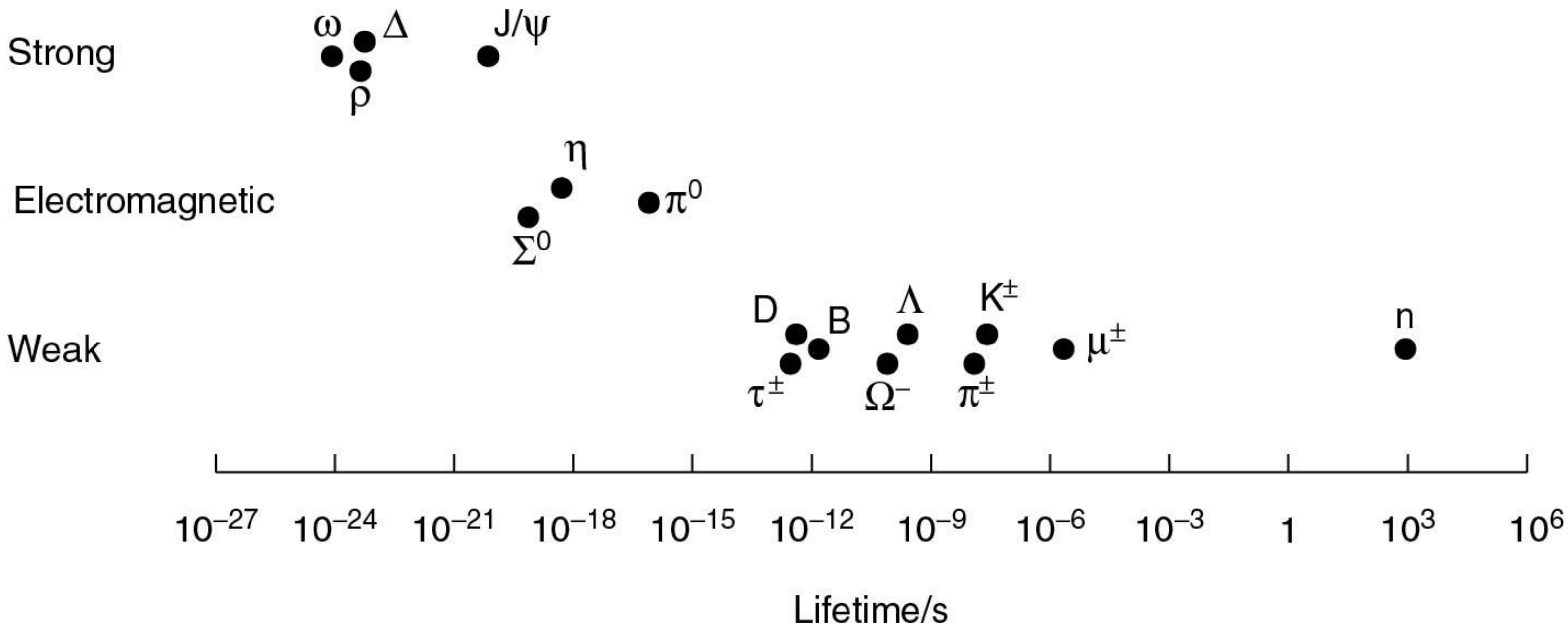
Also CP violation ~ 10^{-3} level (see L10)

transformation properties of **p** and **σ** under P, C, and T

$$\begin{array}{lll} P: & \vec{p} \to -\vec{p}; & \vec{\sigma} \to +\vec{\sigma} \\ C: & \vec{p} \to +\vec{p}; & \vec{\sigma} \to +\vec{\sigma} \\ T: & \vec{p} \to -\vec{p}; & \vec{\sigma} \to -\vec{\sigma} \end{array}$$

Lifetimes

Long lifetimes: huge range of lifetimes >> τ strong and τ EM (excluding the top quark)



the huge range is due to the phase space!
(i.e. energy released in the decay)

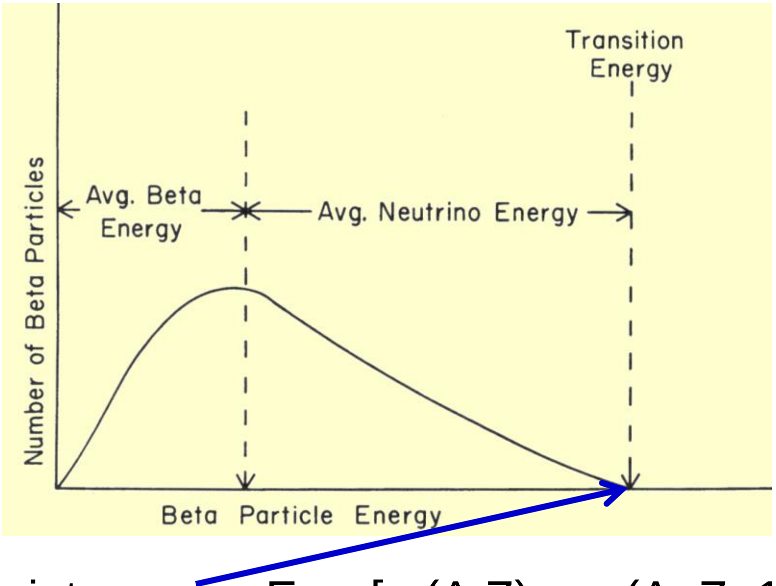
Sargent's law: $\Gamma \sim G_F^2 \times \Delta Q^5$

The Neutrino

β spectrum (~20 years of research and controversy)
the interpretation followed two lines:

- primary electrons with continuous energy (C. D. Ellis)
- secondary processes, which broadens the electron spectrum (L. Meitner)

Introduction of **neutrino** (Pauli 1931) to explain beta spectrum,
the electron energy spectrum is continuous
(at least one more particle participate in the decay).
Energy conservation and total angular momentum conservation: → spin of ν is ½



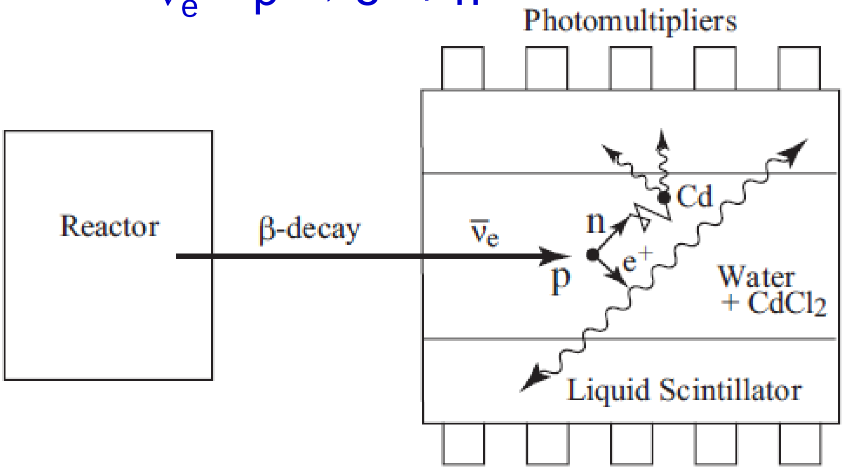
endpoint energy: $E_0 = [m(A,Z) - m(A, Z+1) - m_e - m_\nu]c^2$
(set upper limit on neutrino mass < 1 eV)

The Discovery of the Neutrino

First indications of the existence of neutrinos in EC $^{37}Ar + e^- \rightarrow {}^{37}Cl + \nu_e$
(recoil kinematics of ^{37}Cl consistent with a 2-body process) early 50's

1956: discovery of neutrino @ Savannah nuclear reactor
by Cowans and Reines → Nobel ~40 years later

$\overline{\nu}_e + p \rightarrow e^+ + n$



prompt signal: photons from e^+e^- annihilation
delayed signal (~ 10 ms): γs from neutron capture on Cd: $n + {}^{113}Cd \rightarrow {}^{114}Cd + \#\gamma$

Neutrino vs. Anti-Neutrino

ν source the sun

$\bar{\nu}$ source nuclear reactors

If neutrino and antineutrino were identical particles, both processes

$$\nu_e + n \rightarrow e^- + p \quad \text{and} \quad \bar{\nu}_e + n \rightarrow e^- + p$$

should occur.

Experiment (Davis 1955):

$\bar{\nu}_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ found no evidence of ${}^{37}Ar$ production

while $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ occurs

$$\Rightarrow \bar{\nu}_e \neq \nu_e$$

Solar neutrinos

Same detection principle $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ in a larger scale version used to detect ν_e produced in nuclear reactions inside the sun (solar neutrinos):

experiment observed ~1/3 of expected electron neutrinos (i.e. ${}^{37}Ar$ atoms)

$\rightarrow$ solar neutrino problem, today explained as due to ν oscillations + matter effects (Davis 1964, Nobel Prize 2004)

Neutrino Flavor

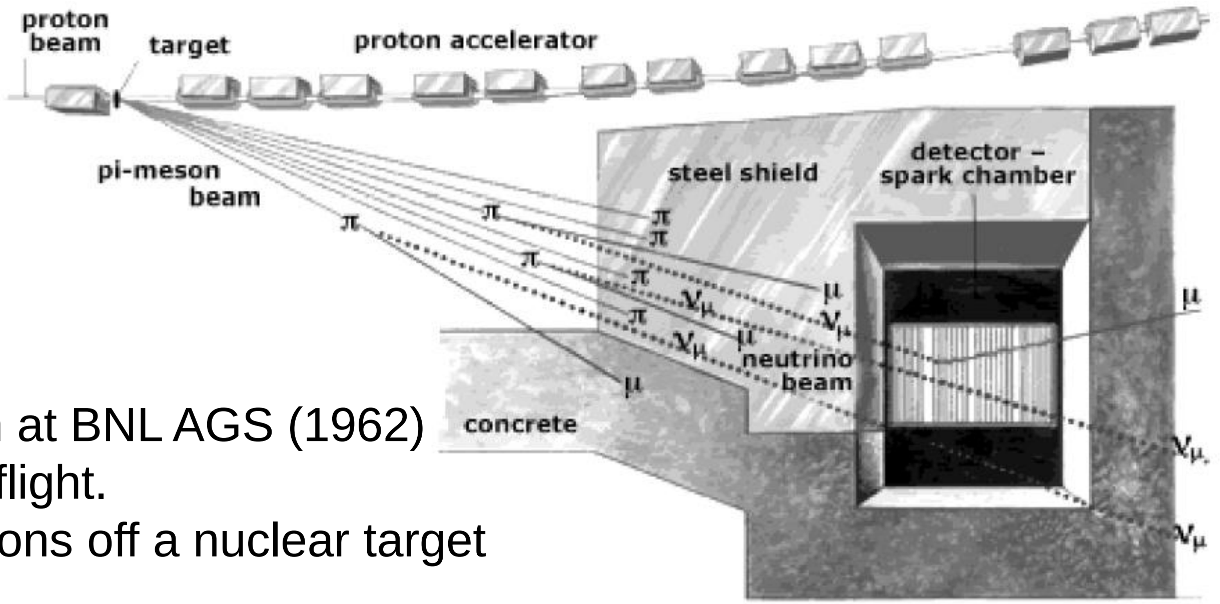
With the advent of accelerator based ν beams, study also scattering of neutrinos:
$\nu_\mu N \to \mu^- X$ (CC) $\nu_\mu e^- \to \mu^- \nu_e$ (CC) $\nu_\mu N \to \nu_\mu X$ (NC) $\nu_\mu e^- \to \nu_\mu e^-$ (NC)

Are the neutrinos produced in β decay and in π decay same or different?
If ν_e and ν_μ were identical, for instance the reactions

$$\nu_\mu + n \to \mu^- + p$$
$$\nu_\mu + n \to e^- + p$$

should result in same rates

$$(\nu_e + n \to e^- + p)$$



First neutrino accelerator beam at BNL AGS (1962) produced ν_μ from π^+ decays in flight.
Only μ^- observed in ν_μ interactions off a nuclear target

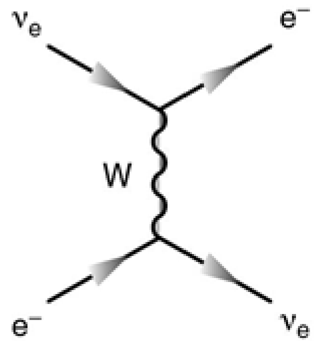
$$\Rightarrow \nu_e \neq \nu_\mu$$

(Lederman, Schwartz, and Steinberg, Nobel Prize 1988)

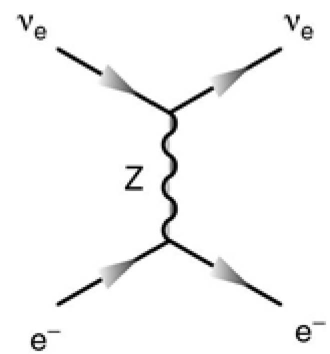
Modern View of Weak Interactions

Three massive spin-1 vector bosons mediate the weak interactions: W^+, Z^0 , and W^- .

charged current



neutral current



In the weak interaction vertex two fermions join a vector boson:

- $W^{\pm}$ – charged current interaction, $\Delta Q = \pm 1$
- Z^0 – neutral current interaction, $\Delta Q = 0$

Z and W bosons can also interact between themselves: i.e. $Z^0 \rightarrow W^+ + W^-$

W^+ and W^- are charged and therefore can also couple to photons

SU(2) non-abelian gauge symmetry (see L11 and L12)

The coupling constants g (W) and g' (Z) are of the same order as the electric charge e:

g ~ g' ~ e

Weakness due to the large mass of W, Z bosons, see propagator: $$\frac{-i\left(g_{\mu\nu} - q^{\mu}q^{\nu}/M^2\right)}{q^2 - M^2}$$

For $q^2 \ll M_W^2$ the propagator shrinks to
(true for all weak decays except the top quark) $$\frac{ig_{\mu\nu}}{M^2} \sim \text{constant}$$

Classification of Weak Processes

Leptonic processes

only leptons are present

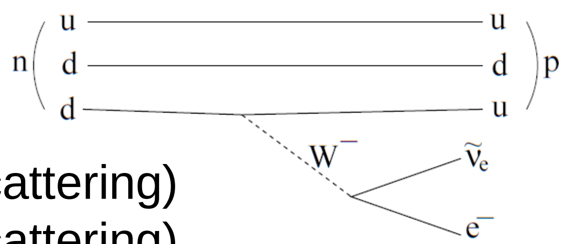
uncontaminated by the strong interaction → high accuracy calculations

$\mu^- \to e^- \nu_\mu \nu_e$ (CC) (decay) $\nu_\mu e^- \to \mu^- \nu_e$ (CC) (scattring) $\nu_\mu e^- \to \nu_\mu e^-$ (NC) (scattring)

Semi-leptonic processes

hadrons and leptons are present

β decay, νN scattering, π decay



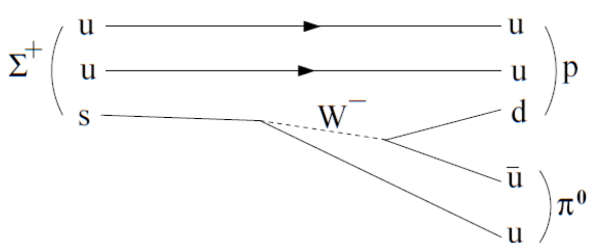
$n \to p\, e^-$ and $\bar{\nu}_e\, \nu_e\, d \to e^-\, u$ (CC) (decay and scattering)

$\Lambda \to p\, e^-$ and $\bar{\nu}_e\, \nu_e\, s \to e^-\, u$ (CC) (decay and scattering)

$\nu\, p \to \nu\, p$ and $\nu_e\, u \to \nu_e\, u$ (NC) (only scattering)

Non-leptonic or hadronic weak processes

only hadrons are involved



$\Lambda \to p + \pi^-$ or $n + \pi^0$ ($s \to u + W^- \to$ $u\, d$) (CC)

$K^+ \to \pi^+ + \pi^0$ (CC)

Lepton number is conserved and baryon number is conserved, always!

In addition, lepton flavor number conserved

Flavor changes:

rule $\Delta S = \Delta Q$, no $\Delta S = \pm 1$ transition with $\Delta Q = 0$ (no FCNC!)

i.e. $s \to d$ decay forbidden, only $s \to u$ decay allowed

Fermi Theory of Weak Interactions

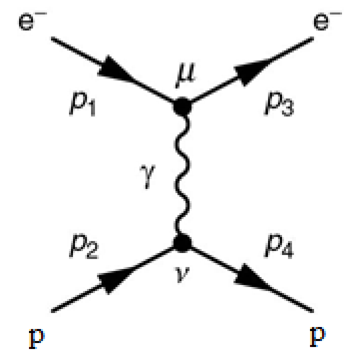
Proposed by Fermi in 1933 and rejected by Nature:
"The paper contains speculations too remote from reality to be of interest to readers"

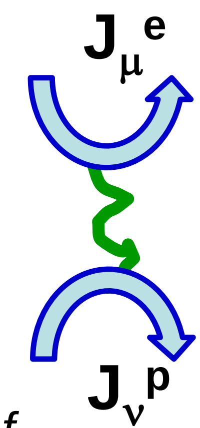
Current – Current Interaction

Consider e p → e p scattering (EM process) described by the invariant amplitude

$$iM_{fi} = \left(\bar{u}_p(-ie)\gamma^{\nu}u_p\right)\frac{ig_{\mu\nu}}{q^2}\left(\bar{u}_e(+ie)\gamma^{\mu}u_e\right)$$

J^{ν}_{p} J^{μ}_{e}





Can define the transition current between the initial and final states *i* and *f*

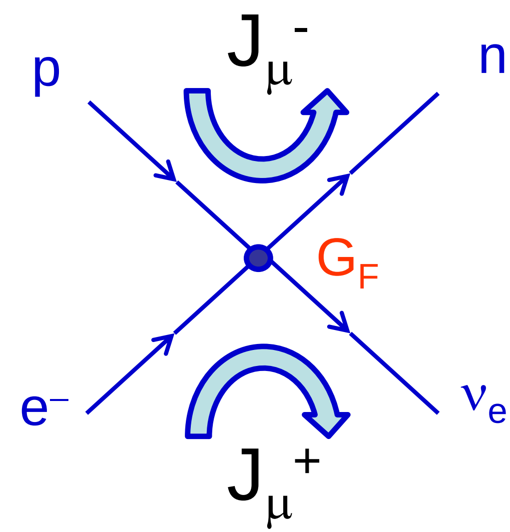
$$ej_{\mu}^{EM} = j_{\mu}^{fi}(0) = -e\bar{u}_f\gamma_{\mu}u_i$$

and the invariant amplitude can be rewritten as

$$M_{fi} = -\frac{e^2}{q^2}\left(J_{\mu}\right)_p\left(J^{\mu}\right)_e$$

In analogy consider the weak process p → n e^+ ν_e
or the crossed process p e^- → n ν_e

$$M_{fi} = G_F\left(\bar{u}_n\gamma_{\mu}u_p\right)\left(\bar{u}_{\nu}\gamma^{\mu}u_e\right)$$



q^2 too small to resolve vertices → no propagator, pointlike 4 fermion interaction

Weak charged currents:

charge lowering	$p \to n$	J_μ^-	$\Delta Q = -1$	(W^+)
charge raising	$e^- \to \nu_e$	J_μ^+	$\Delta Q = +1$	(W^-)

G_F – Fermi weak interaction constant to be determined from experiment, replaces the propagator which is absent

G_F has dimensions $1/[E]^2$

$G_F = 1.16637 \pm 0.00001 \times 10^{-5}$ GeV^{-2} (< 10 ppm)

however Fermi pointlike theory works very well for "ordinary" processes
recall Dirac eq. just discovered, neutrino just postulated, no QFT, …
note that the "distance" is 10^{-17} m, i.e. much shorter than any distance probed so far

consequences of no propagator

low energy limit $$\frac{1}{q^2 - M_W^2} \xrightarrow{q^2 \to 0} \to \frac{1}{M_W^2}$$

"quasi-elastic" $\nu_\mu\, e \to \nu_e\, \mu^-$ scattering (see L9)

$\sigma_{\nu e} = (G_F^2 / \pi) \times s$ rises indefinitely with s (violates unitarity! $\sigma \sim 1/s$)

$s = (\text{c.o.m. energy})^2 \sim 2\, m_e E_\nu$ in the lab. Frame (electron is at rest)

$\sigma_{\nu e} = 1.7 \times 10^{-45}\, E_\nu$ [GeV] m^2 (here m is meters) or $1.7 \times 10^{-17}\, E_\nu$ [GeV] barn dimensional analysis: can we guess the form of the observables (i.e. σ and τ)?

cross section σ (see L9)
s only physical quantity

from dimensional analysis $\sigma \sim G_F^2 \times [?]$

$[\sigma] = L^2 = E^{-2}$

$[G_F^2] = E^{-4}$

need a quantity with dimensions E^2 : $[s] = E^2$

$\Rightarrow \sigma \sim G_F^2 \times s$

μ Lifetime τ (Sargent law, see L8)

$$\Gamma\left(\mu^+ \to e^+ \bar{\nu}_\mu \nu_e\right) = \frac{G_F^2}{192\pi^2} m_\mu^5 (1+\varepsilon) \quad \propto \quad m_\mu^5$$

m_μ only physical quantity

from dimensional analysis $\Gamma \sim G_F^2 \times [?]$

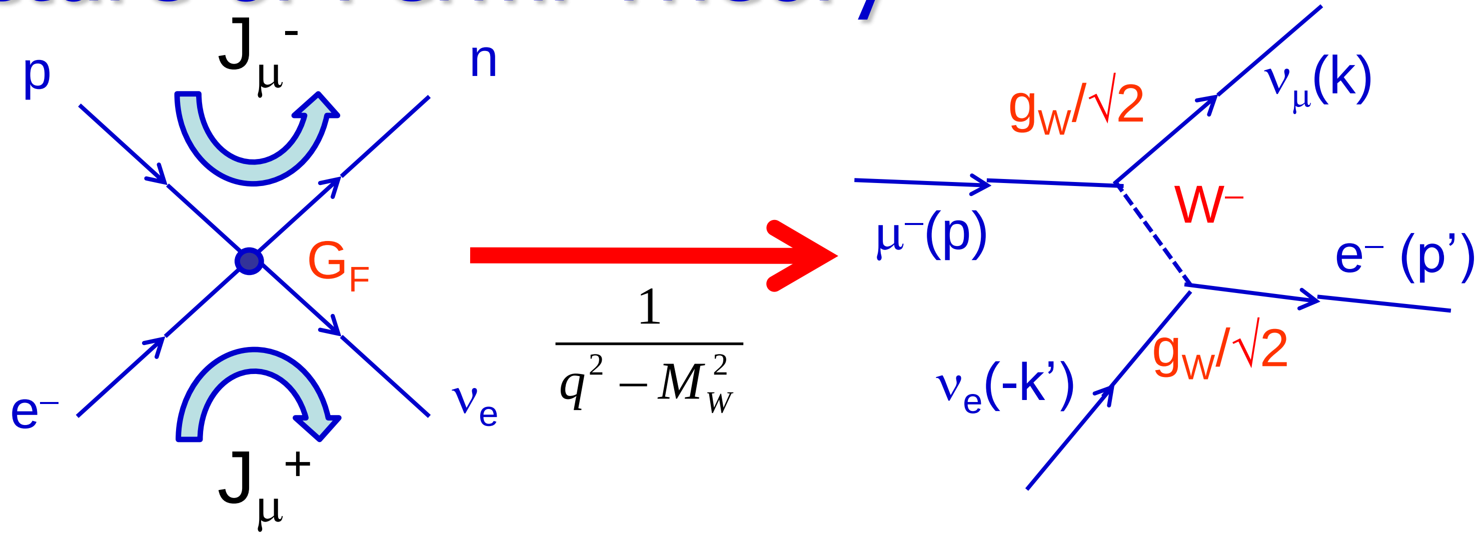
$[\Gamma] = E$

$[G_F^2] = E^{-4}$

need a quantity with dimensions E^5 : $[m_\mu^5] = E^5$

$\Rightarrow \Gamma \sim G_F^2 \times m_\mu^5$

Structure of Fermi Theory



Fermi theory

modern view

most general (Lorentz invariant) form of transition amplitude

$$M_{fi} = \sum_i C^{(i)} \left\langle e^- \bar{\nu}_e \middle| \psi(e)\Gamma^{(i)}\psi(\nu_e) \middle| 0 \right\rangle \left\langle \nu_\mu \middle| \psi(\nu_\mu)\Gamma^{(i)}\psi(\mu^-) \middle| \mu^- \right\rangle$$

$$J^{(+)} = \left\langle \bar{u}(\nu) \middle| \Gamma^{(i)} \middle| u(\mu) \right\rangle$$

$$J^{(-)} = \left\langle \bar{u}(e) \middle| \Gamma^{(i)} \middle| v(\nu) \right\rangle$$

$$M_{fi} = G_F J^{(+)} \cdot J^{(-)}$$

What structure for the interaction, i.e. which $\Gamma^{(i)}$?

Fermi in analogy to QED assumed that the interaction is of vector type, γ^μ.

Already β decays introduced some questions, should explain all transitions.

Arbitrary choice of the interaction form vector × vector (V × V)

to guarantee Lorentz invariance (no P violation).

Dirac bilinears

The most general form for the currents consistent with Lorentz covariance is a linear combination of Dirac bilinear covariants $\Psi\ \Gamma^{(i)}\ \Psi$, where $\Gamma^{(i)}$ is one of the 16 Dirac bilinears (the γ are 4 × 4 matrices, there are 16 linearly independent 4 × 4 matrices):

bilinear	#		P	T	C	spin
$\bar{\psi}1\varphi$	1	scalar	+	+	+	0
$\bar{\psi}\gamma^{\mu}\varphi$	4	vector	–	–	–	1
$\bar{\psi}\sigma^{\mu\nu}\varphi$	6	tensor	+	+	–	2
$\bar{\psi}\gamma^{\mu}\gamma^{5}\varphi$	4	axial vector	+	–	+	1
$\bar{\psi}\gamma^{5}\varphi$	1	pseudoscalar	–	+	+	0

with $\sigma_{\mu\nu} = i/2\ (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ (spin of the exchanged object)

and linear combinations of these bilinears also respect Lorentz invariance, like (V + A) (V plus A) or (V – A) (V minus A)

β Transitions

Fermi transitions: scalar or vector

spin vectors of electron and anti-neutrino are antiparallel ($S = 0$):

$J_f = J_i$, i.e. $0^+ \rightarrow 0^+$ => $\Delta J = 0$ – no change in the nuclear spin

parity is conserved

$^{14}O\ (0^+) \rightarrow {}^{14}N^*\ (0^+) + e^+ + \nu_e$

$^{34}Cl\ (0^+) \rightarrow {}^{34}S\ (0^+) + e^+ + \nu_e$

Gamow – Teller transitions: axial or tensor

spin vectors of electron and anti-neutrino are parallel ($S = 1$):

$\Delta J = 0, \pm 1$, no $0 \rightarrow 0$, i.e. $1^+ \rightarrow 0^+$ – nuclear spin change by 1 unit

parity is conserved

$^{6}He\ (0^+) \rightarrow {}^{6}Li\ (1^+) + e^- + \overline{\nu}_e$

$^{12}B\ (1^+) \rightarrow {}^{12}C\ (0^+) + e^- + \overline{\nu}_e$

mixed Fermi "+" Gamow-Teller transitions

most transitions are mixed

parity is not conserved

$^{21}Na\ (3/2^+) \rightarrow {}^{10}Ne\ (3/2^+) + e^+ + \nu_e \quad \Delta J = 0$

$^{21}Na\ (3/2^+) \rightarrow {}^{10}Ne\ (5/2^+) + e^+ + \nu_e \quad \Delta J = 1$

Parity Violation

1953: θ^+ / τ^+ puzzle; today known as K^+ (see PDG booklet), discovered in cosmic rays

$\theta^+ \rightarrow \pi^+ + \pi^0$ $J^P = 0^+$ (B.R. ~ 20%)

$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ $J^P = 0^-$ (B.R. ~ 6%)

same mass (m = 493.677 MeV) and lifetime ($\tau = 1.238 \times 10^{-8}$ s),
apparently identical particles, but opposite parity
(+1 and -1, s wave final state, parity of π is -1)
$\Rightarrow$ two different particles or non-conservation of parity!

parity of pion consider $\pi^- d \rightarrow n\, n$

d (L=0,2) = 1^+ wave function of (n n) system must be antisymmetric and we must conserve angular momentum

spin of d:

- J = 1 (spin triplet S = 1) + (L = 0) symmetric
- J = 1 (spin triplet S = 1) + (L = 1) antisymmetric
- J = 1 (spin singlet S = 0) + (L = 1) symmetric

spin of (n,n)

$P(\pi)\cdot P(d) = P(nn) \Rightarrow$ parity π^- = -1

$$P\left|\pi^-\right\rangle = -\left|\pi^-\right\rangle$$

Digression on Parity

A wave function does not have necessarily definite parity, however we can always decompose it in two parts with definite parity.

point reflection r → -r (x, y, z) → (-x, -y, -z)
plane (mirror): (x, y, z) → (-x, y, z)
(x, y, z) → (-x, -y, z) rotation! not a parity operation

$$P\psi\left(\vec{r},t\right)=\psi\left(-\vec{r},t\right)=???\psi\left(\vec{r},t\right)$$

$$P^2=1$$

Parity operations distinguish scalars from pseudo-scalars and vectors from axial vectors

$$P(x^0,\vec{x})=(x^0,-\vec{x})$$

$$x'^0=x^0$$

$$x'^k=-x^k$$

$$\psi'(x')=S_P\psi(x)=\gamma^0\psi$$

(S_p – parity operator = γ^0)

$$S\ (I):\qquad \bar{\psi}\psi\rightarrow+\bar{\psi}'\psi'$$

$$V\ (\gamma^\mu):\qquad \bar{\psi}\gamma^0\psi\rightarrow+\bar{\psi}'\gamma^0\psi'$$

$$\bar{\psi}\gamma^k\psi\rightarrow-\bar{\psi}'\gamma^k\psi'$$

$$A\ (\gamma^\mu\gamma^5):\qquad \bar{\psi}\gamma^5\gamma^0\psi\rightarrow-\bar{\psi}'\gamma^5\gamma^0\psi'$$

$$\bar{\psi}\gamma^5\gamma^k\psi\rightarrow+\bar{\psi}'\gamma^5\gamma^k\psi'$$

$$PS\ (\gamma^5):\qquad \bar{\psi}\gamma^5\psi\rightarrow-\bar{\psi}'\gamma^5\psi'$$

The Solution

1956: Lee and Young:

no experimental proof that parity is conserved in weak interactions

They suggested several experiments to test this hypothesis → 6 months later Nobel prize

How to define a parity non invariant quantity?

Consider the β decay

$$A \rightarrow B + e^- + \overline{\nu}_e \quad \text{in the C.M. of A}$$

We have three momenta (four vectors) at disposal: $\mathbf{p}_B$, $\mathbf{p}_e$, $\mathbf{p}_\nu$

scalar products $\mathbf{p}_B \cdot \mathbf{p}_e$ are scalars and conserve parity

mixed products $\mathbf{p}_B \cdot \mathbf{p}_e \times \mathbf{p}_\nu$ are pseudo-scalars and do change sign (parity non invariant)!

but $\mathbf{p}_B \cdot \mathbf{p}_e \times \mathbf{p}_\nu = 0$ because they are coplanar

Need an axial vector, like **J** (spin), and combinations with spin like $\boldsymbol{\sigma} \cdot \mathbf{p}$

$$P(\boldsymbol{\sigma}) = \boldsymbol{\sigma}, \quad P(\mathbf{p}) = -\mathbf{p} \quad \text{and } P(\boldsymbol{\sigma}\cdot\mathbf{p}) = -(\boldsymbol{\sigma}\cdot\mathbf{p})$$

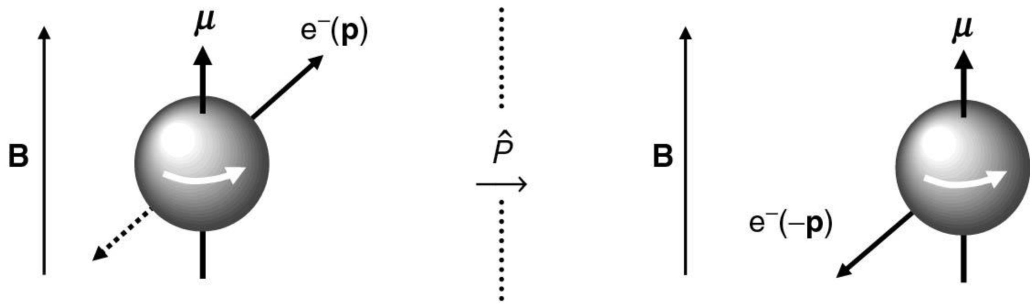
$\langle \mathbf{J} \rangle \cdot \mathbf{p}_e$ = pseudoscalar, it changes sign under parity

If parity is conserved such terms should not exists.

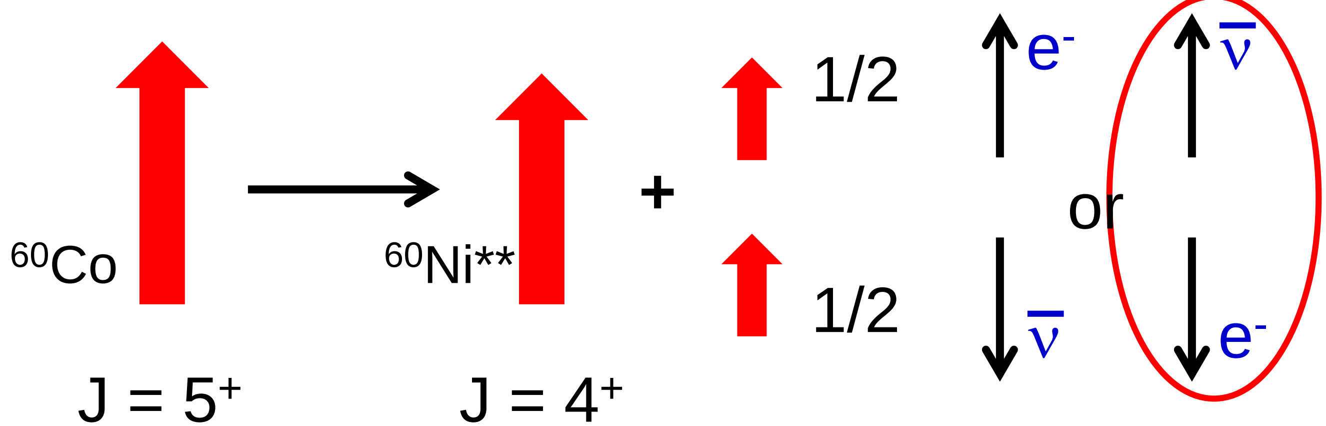
Imagine an experiment to study this …

Parity Violation in β Decay

1957 Wu et al.: DECAY $^{60}Co(5^+) \rightarrow Ni^{**}(4^+) + e^- + \overline{\nu}_e$



Parity conservation ($\theta \rightarrow \pi - \theta$) would require that both "pictures" are present on equal footing.



Study the angular distribution of decay e⁻ with respect to ^{60}Co spin (angle θ)

$$W_e(\vartheta) \propto 1 + P_T \cdot P_e \cdot \alpha \cdot \cos\vartheta$$

where α measures the degree of parity violation ($\alpha = 0 \rightarrow$ no parity violation)
P_T is the polarization of the ^{60}Co nucleus, and P_e the polarization of electron ($P_e = -\beta$).

There are two possibilities according to the figure:

the first case correspond to a decay rate with angular distribution
(the electron is emitted preferentially in the direction of ^{60}Co spin)

$$\frac{d\Gamma}{d\Omega} = \Gamma_0 \left(1 + \frac{\vec{\sigma} \cdot \vec{p}}{E} \right)$$

the second case corresponds to
(the electron is emitted preferentially in the direction opposite to the ^{60}Co spin)

$$\frac{d\Gamma}{d\Omega} = \Gamma_0 \left(1 - \frac{\vec{\sigma} \cdot \vec{p}}{E} \right)$$

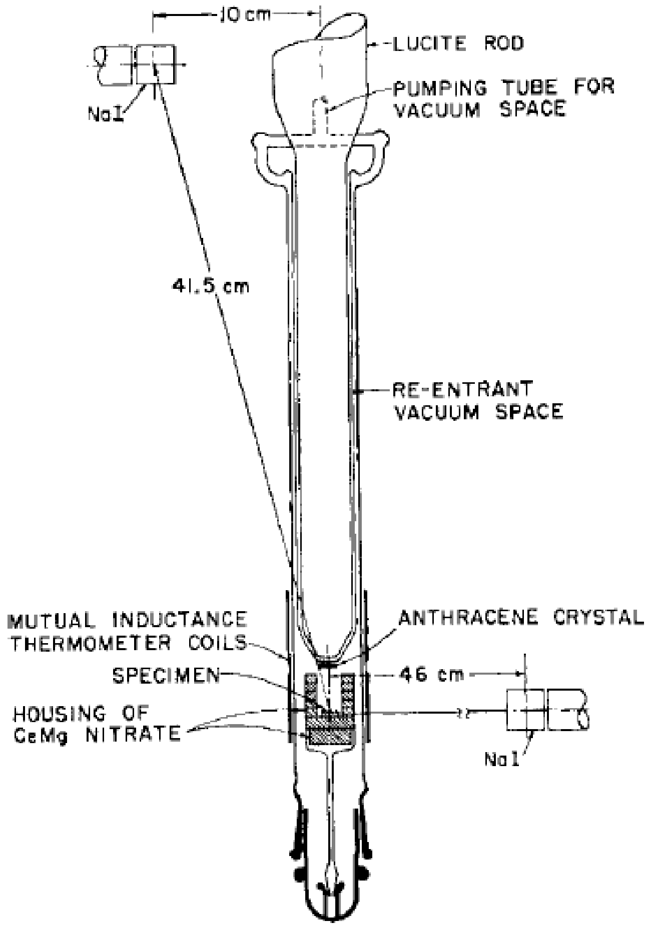
If parity is conserved there should be no angular dependence.
The experiment showed that only the second case is present not even with a small contribution from the first, i.e. $\alpha = -1 \Rightarrow$ maximal violation of parity.

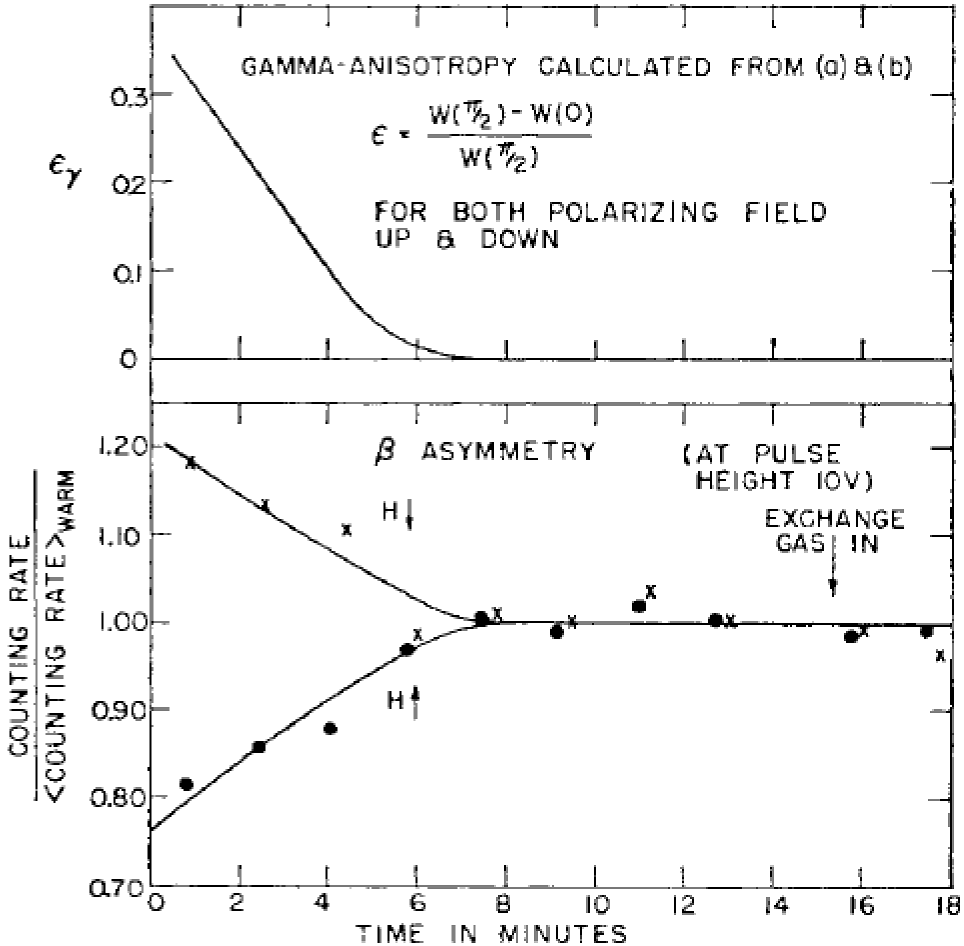
Electrons are left-handed, anti-neutrinos are right-handed (ignoring m_e).
By consequence the electrons are longitudinally polarized ($P_e \propto -\beta_e$), see Lab IV PV exp.

Feynman + Gell-Mann:
Experience showed that the right choice is (V – A) (maximal P violation).
Evidence only ν_L and $\overline{\nu}_R$ are involved in weak interactions:
the absence of the mirror image states $\overline{\nu}_L$ and ν_R is a clear violation of parity invariance (and charge conjugation as well, since C transforms a ν_L state in a $\overline{\nu}_L$ state).

The Wu Experiment (1956)

The experimental challenge is to spin polarize ^{60}Co nuclei because of the small magnetic moment of the nucleus $\propto 1/M_A$.
Polarization of Ni** measured from angular distributions of γ from Ni** decays (quadrupole, not parity violating).





P and C in Pion Decays

We saw that the pion is a Parity eingenstate with eigenvalue P = –1

$$P\left|\pi^-\right\rangle = -\left|\pi^-\right\rangle$$

Only the π^0 is a Charge Conjugation eingenstate (it is its own antiparticle) with C = +1

$$C\left|\pi^+\right\rangle = \left|\pi^-\right\rangle$$

Consider the pion decay

$$P\left[\Gamma(\pi^+ \to \mu^+ + \nu_L)\right] = \Gamma(\pi^+ \to \mu^+ + \nu_R) = 0$$

it is not invariant under parity: there are no right handed neutrinos

$$C\left[\Gamma(\pi^+ \to \mu^+ + \nu_L)\right] = \Gamma(\pi^- \to \mu^- + \bar{\nu}_L) = 0$$

it is not invariant under charge conjugation: there are no left handed anti-neutrinos

Now consider the pion decay under the combined effect of Parity and Charge Conjugatior

$$CP\left[\Gamma(\pi^+ \to \mu^+ + \nu_L)\right] = C\left[\Gamma(\pi^+ \to \mu^+ + \nu_R)\right] = \Gamma(\pi^- \to \mu^- + \bar{\nu}_R)$$

it is invariant under CP

$$= \Gamma(\pi^+ \to \mu^+ + \nu_L)$$

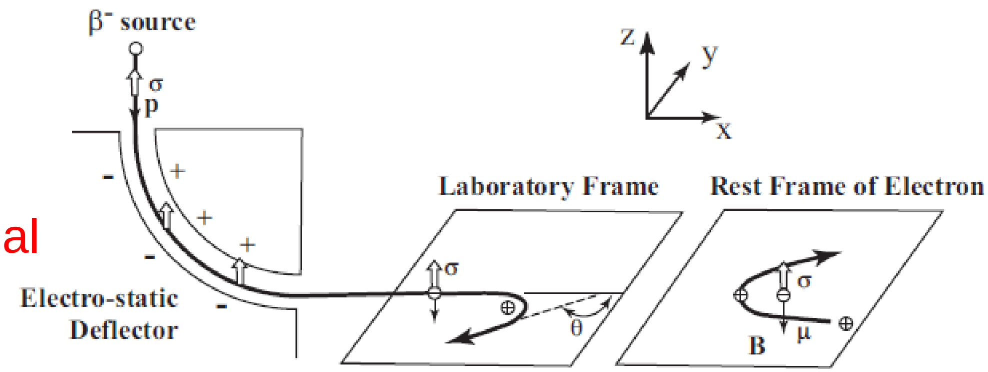
In reality, also CP is violated in weak interactions, but at a much lower level ~ 10^{-3} .

Electron Polarization Measurement

scatter electrons from β decays on an electron target

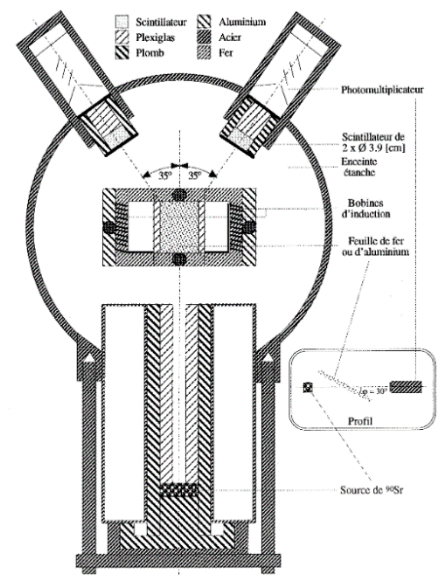
this is a QED process, which conserves parity and therefore
is not sensitive to the longitudinal electron polarization

the observable $\sigma^{\rightarrow} - \sigma^{\leftarrow}$ violates parity



two possible approaches:

a) rotate the electron spin from longitudinal
to transverse in an electric field
(change the direction of electron without
modifying the spin direction)
measure the angular distribution of scattered electrons
(left – right asymmetry which does not violate P)



b) scatter longitudinally polarized electrons
on a longitudinally polarized electron target
(polarized with a magnetic field) and measure

$$\sigma^{\rightarrow\leftarrow} - \sigma^{\leftarrow\leftarrow} / \sigma^{\rightarrow\leftarrow} + \sigma^{\leftarrow\leftarrow} \propto P_e$$

this observable does not violate parity

Neutrino Helicity (Goldhaber 1957)

(some consider this one of the most beautiful particle physics experiment)

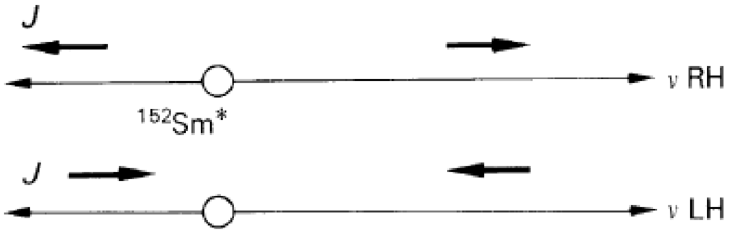
electron capture

$^{152}_{63}Eu^{m}(0) + e^{-} \rightarrow {}^{152}_{62}Sm^{*}(1) + \nu_e$

$\rightarrow {}^{152}_{62}Sm(0) + \gamma$

polarization of $\gamma \equiv$ polarization of $^{152}Sm^{*}(1)$

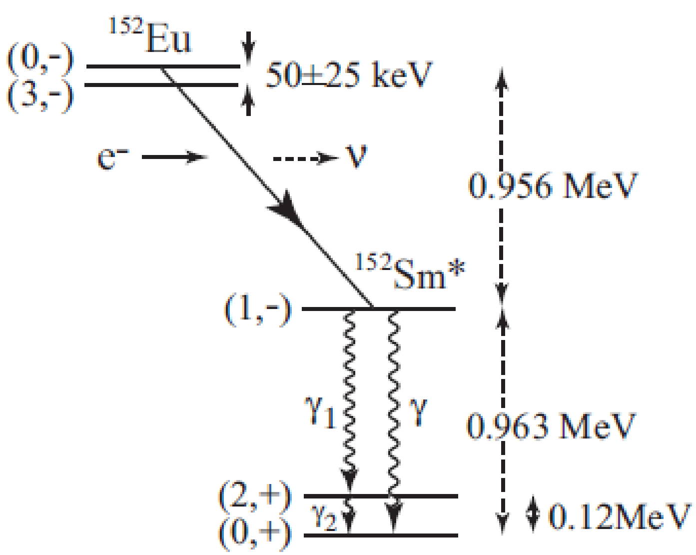
Can we select the γ moving in the opposite direction of the ν_e (E_γ = 960 keV)?
If yes, we can correlate γ polarization to ν_e polarization:
the helicity of γ and ν_e are the same!



By measuring the polarization of the γ we infer the polarization of ν_e .
To analyze the γ, scatter it on $^{152}Sm(0)$.

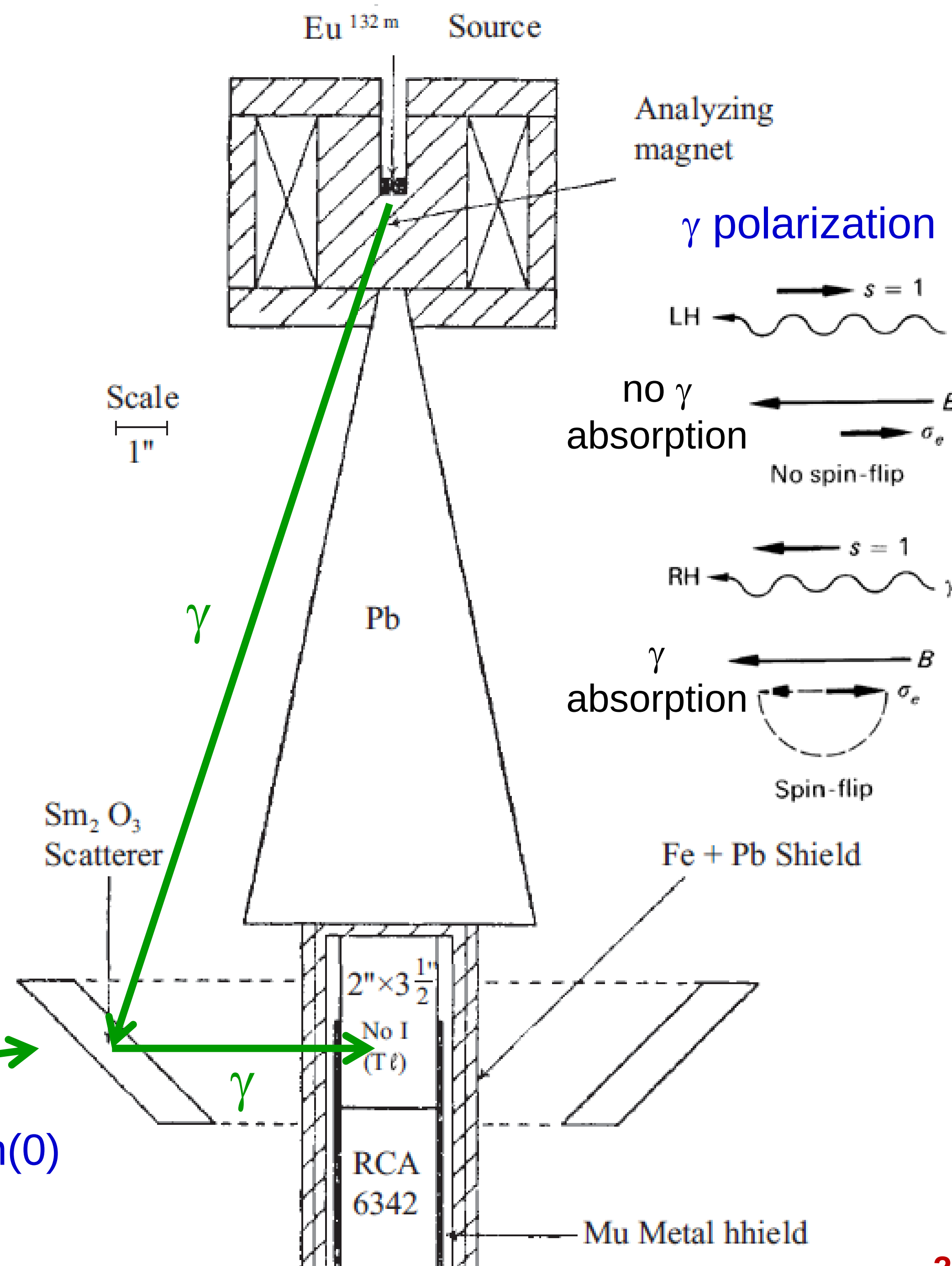
resonant scattering (Mossbauer effect): $\gamma + {}^{152}Sm(0) \rightarrow {}^{152}Sm^{*}(1) \rightarrow \gamma + {}^{152}Sm(0)$
this assures that the γ is moving in the same direction as $^{152}Sm^{*}(1)$,
and opposite to the ν_e,
because the γ can be absorbed by $^{152}Sm(0)$ only if its energy is slightly above the resonance energy (the Sm nucleus must recoil to conserve momentum!).

The Experiment



^{152}Eu (0,-) : Qasi-stable state

^{152}Sm* (1,-) : t = (7±2)x10^{-14} sec

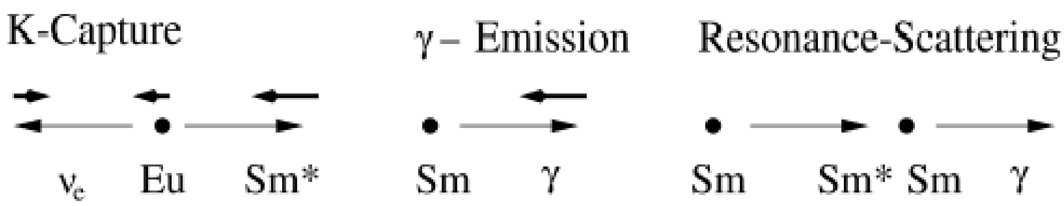


resonant scattering

(Mossbauer effect)

$\gamma + {}^{152}\text{Sm}(0) \rightarrow {}^{152}\text{Sm}^*(1) \rightarrow \gamma + {}^{152}\text{Sm}(0)$

Analysis of the Experiment



$$-\vec{p}_\nu = \vec{p}_{Sm^*} = \vec{p}_\gamma = \vec{p}_{Sm^*} = \vec{p}_\gamma$$

electron capture + resonant scattering

(spin quantization axis along neutrino momentum, S_x 3rd spin component)

S_{e-}	S_{Sm^*}	S_ν	S_γ	h_ν	h_γ	
	+1	-1/2	+1	-1	-1	⇐ experiment!
+1/2	0	+1/2	x			
	-1	x				
	+1	x				
-1/2	0	-1/2	x			
	-1	+1/2	-1	+1	+1	

helicity of γ and ν are the same!

by measuring the polarization of the γ we infer the polarization of ν!

γ transmitted through iron magnet only if γ and e^- spins parallel (no spin flip of electron)

Parity Violating Weak Interaction Vertex

In 1958 Feynman and Gell-Mann proposed the V – A (vector minus axial vector) current structure for the weak interactions.

It is natural to assume that all weak interactions are of the form (V – A) with a universal coupling constant G_F (this assumption will turn out to be correct).

Modify interaction vertex

$$\gamma_\mu \to \frac{1}{2}(\gamma_\mu - \gamma_\mu \gamma_5) = \gamma_\mu \frac{1}{2}(1-\gamma_5)$$

In analogy to EM, introduce in the invariant amplitude M_{fi}

$$\bar{u}_\nu \gamma^\mu u_e \to \bar{u}_\nu \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) u_e = \frac{1}{2}\bar{u}_\nu \gamma^\mu u_e - \frac{1}{2}\bar{u}_\nu \gamma^\mu \gamma^5 u_e$$

Initially it has been assumed $g_V \gamma^\mu + g_A \gamma^\mu \gamma^5$, where g_V is the vector coupling and g_A is the axial vector coupling, since we did not know the relative strengths of V and A couplings.
Experiment has shown that $g_V = -g_A$ for leptons!
while in the electroweak theory $g_V = -g_A$ by construction (e.g. only left handed neutrinos)

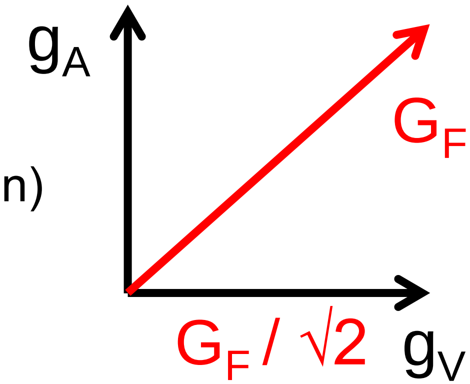
We say that we have an interaction of the type (V – A) × (V – A)

And the transition amplitude becomes

$$M_{fi}\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu\right) = \frac{4G_F}{\sqrt{2}}\left[\bar{u}_\nu \gamma^\sigma \frac{1}{2}\left(1-\gamma^5\right)u_\mu\right]\left[\bar{u}_e \gamma_\sigma \frac{1}{2}\left(1-\gamma^5\right)v_\nu\right]$$

$$M_{fi}\left(p \to n e^+ \nu\right) = \frac{4G_F}{\sqrt{2}}\left[\bar{u}_n \gamma^\sigma \frac{1}{2}\underbrace{\left(g_V + g_A \gamma^5\right)}u_p\right]\left[\bar{u}_\nu \gamma_\sigma \frac{1}{2}\left(1-\gamma^5\right)v_e\right]$$

from nuclear structure
($g_V \sim 1$, $g_A \sim -1.26$)



Note

no propagator

the $1/\sqrt{2}$ is introduced to keep the same definition (normalization) of G_F as before the introduction of the $(1-\gamma^5)$ term (Fermi theory),

4 from the 1/2 in the vertex

And one can use the same Feynman calculus rules as for the EM interactions but with a modified vertex factor:

$$\gamma^\mu \to \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)$$

The amplitude will contain a vector term and an axial-vector term of same size but opposite sign. The structure of the amplitude is therefore (V – A).

Charge Rising and Lowering Weak Currents

Charge rising weak current $\left(\mu \rightarrow \nu_\mu\right)$ $J_\sigma^{(+)} = \left(\bar{u}_\nu \frac{1}{2}\gamma_\sigma(1-\gamma_5)u_\mu\right)$

Charge lowering weak current $\left(\nu_e \rightarrow e\right)$ $J_\sigma^{(-)} = \left(\bar{u}_e \frac{1}{2}\gamma_\sigma(1-\gamma_5)v_\nu\right)$

And the invariant amplitude can be written as $M_{fi} = \frac{4G_F}{\sqrt{2}} J_\sigma^{(+)} \cdot J^{(-)\,\sigma}$

Let's consider the charge raising (1 unit of e) weak current $J^{(+)\sigma} \equiv \bar{u}_\nu \gamma^\sigma \frac{1}{2}\left(1-\gamma^5\right)u_e$

and its hermitian conjugate, which is a charge lowering current

$$\left(J_\sigma^{(+)}\right)^\dagger = \left(\bar{u}_\nu \frac{1}{2}\gamma_\sigma(1-\gamma_5)u_\mu\right)^\dagger = u_e^\dagger \frac{1}{2}\left(1-\gamma^5\right)^\dagger \left(\gamma_\sigma\right)^\dagger \left(\gamma^0\right)^\dagger u_\nu$$
$$= u_e^\dagger \frac{1}{2}\left(1-\gamma^5\right)\gamma^0\gamma_\sigma u_\nu = \bar{u}_e\gamma_\sigma \frac{1}{2}\left(1-\gamma^5\right)u_\nu = \left(J_\sigma^{(-)}\right)$$

Weak interactions amplitudes are then of the form $M_{fi} = \frac{4G_F}{\sqrt{2}} J^\mu J_\mu^\dagger$

Charge conservation requires that M_{fi} is the product of a charge rising and a charge lowering current.

Does the Weak Current Violate Parity?

Let's calculate the "product" of two charge rising currents $J^{\mu} J_{\mu}$

$$\left(g_V j_V^{\mu} + g_A j_A^{\mu}\right)\left(g_V j_{V,\mu} + g_A j_{A,\mu}\right) = g_V^2 j_V^{\mu} j_{V,\mu} + g_A^2 j_A^{\mu} j_{A,\mu} + g_V g_A \left(j_V^{\mu} j_{A,\mu} + j_A^{\mu} j_{V,\mu}\right)$$

The product of two vector currents or two axial vector currents does conserve parity (i.e. does not change sign under a parity operation),
the interference term, i.e. the product of a vector current and axial vector current does not conserve parity (i.e. does changes sign under a parity operation):

j_V does not change sign and the product j_V j_V does not change sign
j_A does change sign, while the product j_A j_A does not change sign,
but the product j_V j_A does change sign

strength of the parity violation

$$\alpha = \frac{2 g_V g_A}{g_V^2 + g_A^2} \qquad \alpha = -1 \Rightarrow g_V = -g_A$$

same α as in Wu experiment.

Polarization, Helicity, Chirality

Let's try to understand the meaning of the new vertex factor $\gamma_\mu \frac{1}{2}(1-\gamma_5)$

polarization

projection of spin (→ third component) on one axis **n** (i.e. **B**, vertical, …) which is physically defined "externally" to the particle

$\boldsymbol{\sigma} \cdot \mathbf{n} \quad \mathbf{n} \parallel \mathbf{z}$ (or $\mathbf{n} \parallel \mathbf{B}$) $\rightarrow \quad \sigma_z$

eigenstates

$$\begin{cases} \frac{1}{2}\sigma_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{1}{2}\sigma_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

helicity

the momentum **p** defines the direction, i.e. polarization along **p**

eigenstates of the helicity operator

$$h = \frac{1}{2}\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|} = \frac{1}{2|\vec{p}|}\begin{pmatrix} \vec{\sigma}\cdot\vec{p} & 0 \\ 0 & \vec{\sigma}\cdot\vec{p} \end{pmatrix} \qquad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

Helicity eigenstates are 2 component spinors that describe a fermion or anti-fermion, not both.

In general not Lorentz invariant, the helicity can change with a Lorentz boost,

but commutes with the Hamiltonian $\left[H_D, \vec{\Sigma}\cdot\vec{p}\right] = 0$

Helicity is a good quantum number only for massless particles (m = 0 → v = c)
(it is Lorentz invariant only for massless particles).

chirality

eigenstates of γ^5 with 2 possible eigenvalues +1 or -1

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad \gamma_5^2 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \qquad P_L = \frac{1}{2}\left(1-\gamma^5\right) = \frac{1}{2}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$$

right chirality projector $P_R = \frac{1}{2}\left(1+\gamma^5\right) \qquad \frac{1}{2}\left(1+\gamma^5\right)\psi = \psi_R$

left chirality projector $P_L = \frac{1}{2}\left(1-\gamma^5\right) \qquad \frac{1}{2}\left(1-\gamma^5\right)\psi = \psi_L$

$$\psi = \psi_L + \psi_R$$

projectors: $P_L + P_R = 1$, $P_L^2 = P_L$, $P_R^2 = P_R$, $P_L P_R = 0$

There are no stationary states because γ^5 does not commute with the Hamiltonian

$$\left[H_D, \gamma_5\right] = \left[\vec{\alpha}\cdot\vec{p} + \beta m, \gamma_5\right] = 2m\beta\gamma_5 \neq 0$$

(mass term!) not good quantum number, only for m → 0 or p → ∞ $[\gamma_5, H_D] = 0$

particles

$$\psi_L = \frac{1}{2}\left(1-\gamma^5\right)\psi$$

anti-particles

$$\bar{\psi}_L = \psi_L^\dagger \gamma^0 = \left[\frac{1}{2}\left(1-\gamma^5\right)\psi\right]^\dagger \gamma^0 = \psi^\dagger \frac{1}{2}\left(1-\gamma^5\right)\gamma^0 = \psi^\dagger \gamma^0 \frac{1}{2}\left(1+\gamma^5\right) = \bar{\psi}\frac{1}{2}\left(1+\gamma^5\right)$$

For instance, the projector $P_L = \frac{1}{2}\left(1-\gamma^5\right)$ selects ν_L or $\bar{\nu}_R$

From the properties of γ matrices $\gamma^\mu\left(1-\gamma^5\right) = \frac{1}{2}\left(1+\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)$

The charge raising current can be rewritten as

$$\bar{u}_\nu \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)u_e = \bar{u}_\nu \frac{1}{2}\left(1+\gamma^5\right)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)u_e = \left(\bar{u}_\nu\right)_L \gamma^\mu \left(u_e\right)_L$$

The charge-raising weak current $\bar{u}_\nu \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)u_e$ therefore couples an

incoming left-handed electron e_L (for v ~ c, h_e = -1) to an outgoing left-handed neutrino ν_L.
Likewise, it couples an incoming right-handed $\bar{\nu}_R$ to an outgoing right-handed $\bar{e}_R$.
i.e. it couples “left” to “left” particles or
“right” to “right” antiparticles!
there is no “left” – “right” coupling!
[angular momentum is conserved at the interaction vertex, it is a vector theory]

Chirality vs. Helicity

Let's start with the Dirac equation $\left(\gamma_\mu p^\mu - m\right)\psi = \left(E\gamma_0 - \vec{p}\cdot\vec{\gamma} - m\right)\psi = 0$

and express the Dirac spinor in terms of the upper and lower components

$$\begin{pmatrix} (E-m)I & -\vec{p}\cdot\vec{\sigma} \\ \vec{p}\cdot\vec{\sigma} & -(E-m)I \end{pmatrix}\begin{pmatrix} \phi \\ \chi \end{pmatrix} = 0 \qquad \Rightarrow \qquad \begin{array}{l} \phi = \dfrac{\vec{p}\cdot\vec{\sigma}}{E-m}\chi \\ \chi = \dfrac{\vec{p}\cdot\vec{\sigma}}{E+m}\phi \end{array}$$

The "left" chiral state is given by

$$\psi_L = \frac{1}{2}\left(1-\gamma^5\right)\psi = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \phi \\ \chi \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \phi-\chi \\ -(\phi-\chi) \end{pmatrix}$$

and we are left with $(\phi-\chi)$combinations only! $\phi - \chi = \phi - \dfrac{\vec{p}\cdot\vec{\sigma}}{E+m}\phi = \left(1 - \dfrac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)\phi$

Now express particle states in terms of helicity eigenstates $\phi = \phi^+ + \phi^-$

$$\frac{1}{2}(\phi-\chi) = \frac{1}{2}\left(1-\frac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)\phi = \frac{1}{2}\left(1-\frac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)\left(\phi^+ + \phi^-\right)$$

left handed
particle state

$$\frac{1}{2}(\phi-\chi) = \frac{1}{2}\left(1-\frac{p_z}{E+m}\right)\phi^+ + \frac{1}{2}\left(1+\frac{p_z}{E+m}\right)\phi^-$$

The upper component of the "left" spinor is not a helicity eigenstate!

Only for m = 0 and p_z = E $\frac{1}{2}(\phi - \chi) = \phi^-$ the chirality and helicity eigenstates coincide.

However, even for E >> m, the "wrong" helicity can be still important

$$\frac{1}{2}(\phi - \chi) = \frac{1}{2}\frac{m}{E}\phi^+ + \phi^- \rightarrow \text{ negative helicity}$$

left chirality bi-spinor ~> negative helicity

antiparticle state

$$\frac{1}{2}(\chi - \phi) = \frac{1}{2}\left(1 - \frac{\vec{p}\cdot\vec{\sigma}}{E-m}\right)\chi = \frac{1}{2}\left(1 - \frac{p_z}{E-m}\right)\chi^+ + \frac{1}{2}\left(1 + \frac{p_z}{E-m}\right)\chi^-$$

(for E >> m, E = -p_z, negative energy solution)

$$\frac{1}{2}(\chi - \phi) = \chi^+ + \frac{1}{2}\frac{m}{E}\chi^- \rightarrow \text{ positive helicity}$$

right chirality bi-spinor ~> positive helicity

apply to EM case

assume massless electron: helicity = chirality and consider both helicities positive

$$f_R \equiv \frac{1}{2}\left(1+\gamma^5\right) f \qquad \bar{f}_L \equiv \bar{f}\,\frac{1}{2}\left(1+\gamma^5\right)$$

$$\bar{f}_L \gamma_\mu f_R = \bar{f}\,\frac{1+\gamma^5}{2}\gamma_\mu \frac{1+\gamma^5}{2} f = \bar{f}\gamma_\mu \frac{1-\gamma^5}{2}\frac{1+\gamma^5}{2} f = 0$$

(no helicity flip for massless particles)

e^+e^- annihilates in opposite helicity states!

Electron Polarization in β Decays

According to the V – A theory, electrons emitted in weak decays are left handed, i.e. they are eigenstates of the chirality projector $P_L = ½(1 - \gamma_5)$:

$$e_L(p) = P_L e(p) = \frac{1}{2}(1-\gamma_5)e(p)$$

To calculate the electron polarization, decompose the chirality eigenstate into helicity eigenstates:

$$e_L(p) = \frac{1}{2}\left(1-\frac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)e = \frac{1}{2}\left(1-\frac{p_z}{E+m}\right)e_{+\frac{1}{2}} + \frac{1}{2}\left(1+\frac{p_z}{E+m}\right)e_{-\frac{1}{2}} \xrightarrow[m\to 0]{p\to\infty} \frac{1}{2}\frac{m}{E}e_{+\frac{1}{2}} + e_{-\frac{1}{2}}$$

The "polarization" $\langle h\rangle = \frac{\Pi^+ - \Pi^-}{\Pi^+ + \Pi^-}$ measures the alignment of the electron spin w.r.t. its momentum.

Π^+ probability to be in + helicity state $\quad \Pi^+ = \left|+\frac{1}{2}\left(1-\frac{p}{E+m}\right)\right|^2$

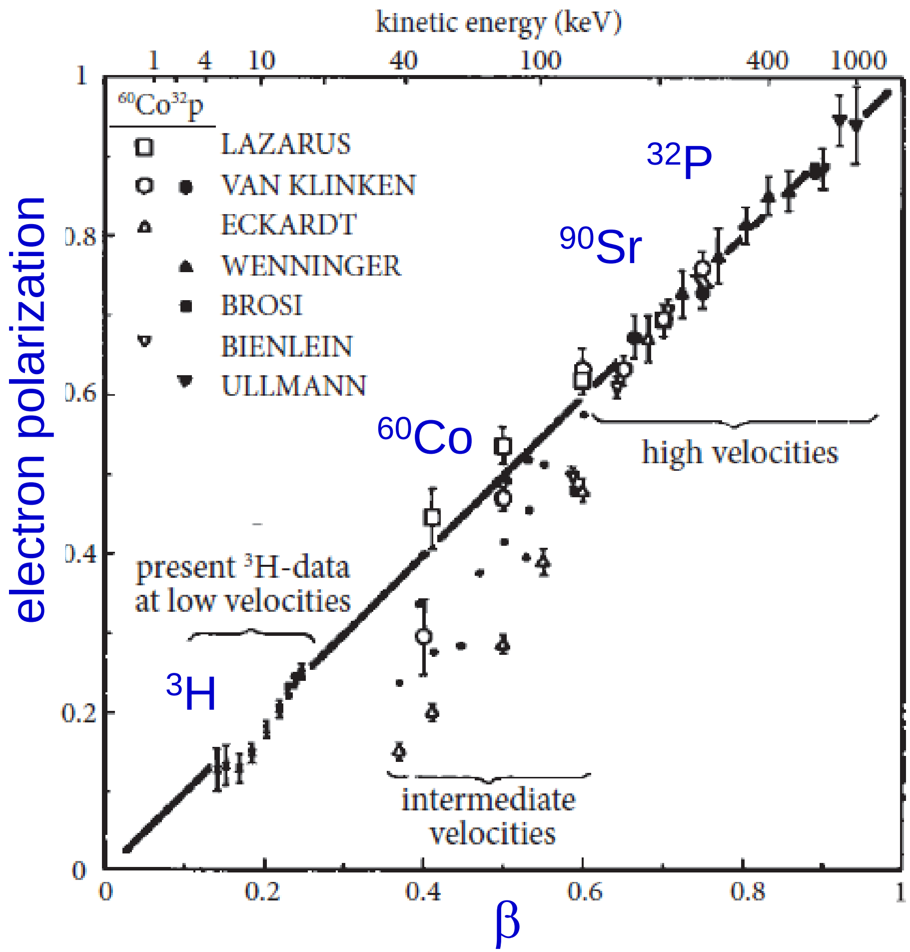
Π^- probability to be in a – helicity state $\quad \Pi^- = \left|+\frac{1}{2}\left(1+\frac{p}{E+m}\right)\right|^2$

It follows that

$$\langle h\rangle = \frac{\Pi^+ - \Pi^-}{\Pi^+ + \Pi^-} = \frac{(E+m-p)^2-(E+m+p)^2}{(E+m-p)^2+(E+m+p)^2} = -\frac{p}{E} = -\beta$$

where β is the speed of the electron.

Electron Polarization Measurement Results



$$P_e = -\beta$$

Interpretation of G_F

Probability amplitude $\propto G_F$, G_F fundamental constant

Extending the analogy with the EM interactions (O. Klein 1938) suggested the existence of intermediate vector bosons $W^{+/-}$ as mediators of the weak force.

$$M_{fi} \propto G_F \qquad\qquad M_{fi} \propto \frac{g_W g_W}{q^2 - M_W^2}$$

For instance the μ decay amplitude becomes

$$M_{fi}\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu\right) = \left[\frac{g_W}{\sqrt{2}} \bar{u}_\nu \gamma^\sigma \frac{1}{2}\left(1-\gamma^5\right) u_\mu\right] \frac{i\left(-g^{\sigma\tau} + q^\sigma q^\tau / M_W^2\right)}{q^2 - M_W^2} \left[\frac{g_W}{\sqrt{2}} \bar{u}_e \gamma_\tau \frac{1}{2}\left(1-\gamma^5\right) v_\nu\right]$$

where $g_W/\sqrt{2}$ is the dimensionless weak coupling constant.

The vertex factor with the coupling constant g_W is

$$-i\frac{g_W}{\sqrt{2}} \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)$$

The resulting current density is partially a vector because of γ^μ term and a pseudo-vector (axial vector) because of $\gamma^\mu \gamma^5$ term. Both enter with equal weight.

Weak interactions don't deserve their name: their apparent weakness at low energies come from the fact that they are mediated by very heavy particles (W and Z), which weakens the amplitude at low momentum transfer

$$\frac{i\left(-g_{\mu\nu} + q^{\mu}q^{\nu} / M^2\right)}{q^2 - M^2}$$

(problem with propagator for M → 0: 3 polarization states → 2 states!)

For $q^2 \ll M_W^2$ the propagator shrinks to $1/M_W^2$ = constant (true for all decays except the top quark, and true for neutrino scattering for E_ν < 1000 GeV).
By comparison with previous expressions

$$\frac{4G_F}{\sqrt{2}} = \frac{1}{2} g_W^2 \frac{1}{M_W^2} \quad \text{or} \quad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} \quad \text{and} \quad g_W = \sqrt{\frac{8G_F}{\sqrt{2}}} M_W$$

Low energy processes uncontaminated by strong processes (and EM radiative corr.) can be calculated with high accuracy (probabilities, cross sections, decay rates, …)
G_F can be determined from the observed rates in β decays, or μ lifetime, …

$G_F = 1.16637 \pm 0.00001 \times 10^{-5}$ GeV^{-2} (< 10 ppm)

$\Rightarrow g_W \sim 0.66$, while e ~ 0.30
and $\alpha_W = g_W^2 / 4\pi = 1/30$ while $\alpha_{EM} = e^2 / 4\pi = 1/137$

The fact that $g_W \sim e$ will allow us to unify the weak interaction with the EM in the electroweak theory.

Weak vs. EM

The comparison with the electromagnetic interactions shows that the structure of the amplitude is quite similar to the electromagnetic one. The neutral component (Z^0, see L10) can even interfere with EM interactions, when charged particles are involved.
There are however major differences between the two:
- weak charge has two components (e and ν_e), weak isospin (see L11)
we classify particles according to their weak isospin T and 3rd component T_3

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad T = \frac{1}{2} \qquad T_3 = \begin{matrix} +1/2 \\ -1/2 \end{matrix} \qquad\qquad e_R \qquad T = 0 \qquad T_3 = 0$$

Right handed neutrinos do not appear in this representation, while those of charged fermions are present. This is due to the fact that neutrinos are neutral and colorless and feel only the weak force. Since the weak force does not couple to right handed fields, neutrinos cannot be produced by any known force. As long as their mass is zero, they do not exist!
- the weak force is mediated by vector bosons, similar to photons and gluons, but very massive. The weak bosons carry the weak isospin. Charged bosons can interact with photons.
- weak interactions do not conserve parity (P) and charge conjugation (C)
They can produce longitudinal polarization effects even if the initial state is unpolarized.

Lepton universality

Charged current weak interactions is universal and is equal for all fermions (when corrected for the masses of fermions) (see L8).

For Next Week

Study the material and prepare / ask questions
Study ch. 12 (sec. 1, 2) in Halzen & Martin and / or ch. 11 (sec. 1 to 7) in Thomson

Do the homeworks

Next week we will study (calculate) some weak interactions
have a first look at the lecture notes, you can already have questions
read ch. 12 (sec.3, 5, 6) in Halzen & Martin
and / or ch. 11 (sec. 6) and ch. 12 (sec. 1) in Thomson