Advanced Particle Physics 2

Strong Interactions
and Weak Interactions

L8 –Weak Decays

(http://dpnc.unige.ch/~bravar/PPA2/L8)

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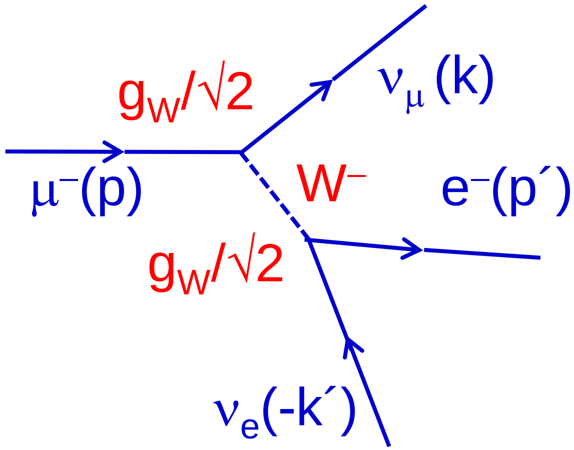
April 20, 2023

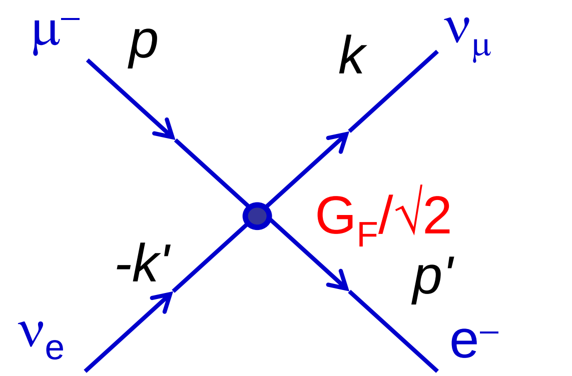
"Prototype" Weak Interaction

The prototype of a weak process mediated by a $W^{\pm}$ exchange is the muon decay

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$



effective 4 fermion interaction



$$\frac{1}{q^2 - M_W^2} \xrightarrow[q^2 \to 0]{\text{propagator}} \frac{1}{M_W^2}$$

$$\boxed{M_{fi} = \frac{4G_F}{\sqrt{2}} J^\mu J_\mu^\dagger}$$ with $$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

Charge rising weak charged current $$J^{(+)\mu} = J^\mu = \bar{u}(k)_{\nu_\mu} \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) u(p)_{\mu^-}$$

Charge lowering weak charged current $$J_\mu^{(-)} = J_\mu^\dagger = \bar{u}(p')_e \gamma_\mu \frac{1}{2}\left(1-\gamma^5\right) v(k')_{\bar{\nu}_e}$$

Estimate G_F from μ lifetime and compare for instance β decay to μ decay

$\to$ is G_F universal?

Fermi Golden Rule

interaction rate per target particle transition amplitude

$$\boxed{W_{fi} = 2\pi\overline{\left|T_{fi}\right|^2}\rho(E) \qquad T_{fi} = -i\left(2\pi\right)^4 \delta^4\left(p_p - p_n - p_e - p_\nu\right)M_{fi}}$$

Derived by Fermi to calculate decay rates

$$\boxed{\mathrm{d}\Gamma = \frac{1}{2E_A}\overline{\left|T_{fi}\right|^2}\mathrm{d}Q = \frac{1}{2E_A}\overline{\left|M_{fi}\right|^2}\frac{\mathrm{d}^3 p_1}{\left(2\pi\right)^3 2E_1}\cdots\frac{\mathrm{d}^3 p_n}{\left(2\pi\right)^3 2E_n}\left(2\pi\right)^4\delta^4\left(p_A - p_1 - \cdots - p_N\right)}$$

$2E_A$ (= m_A) ≡ number of decaying nuclei per unit volume (normalization of the wave fun.)

Decay A → 1 + 2 $\qquad \Gamma(A\to 1+2) = \int \mathrm{d}\Gamma = \cdots = \frac{p_f}{32\pi^2}\int\overline{\left|T_{fi}\right|^2}\mathrm{d}\Omega$

If several decay channels contribute, add all decay rates to obtain the total decay rate (different decay modes are orthogonal → add the amplitudes modulo square!)

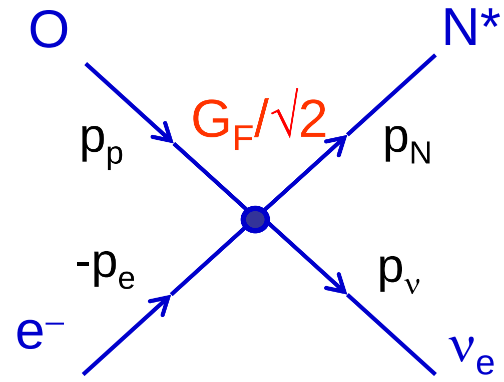
Lifetime given by $\boxed{\frac{1}{\tau} = \Gamma = \sum_i \Gamma_i}$ $\qquad \Gamma_i$ = partial width

τ is the same for all decay modes, the particle does not know a priori in which channel it will decay

Experimentally measure lifetime and branching ratios

$$\boxed{\tau = \frac{1}{\Gamma} \qquad \mathrm{BR}(A\to i) = \frac{\Gamma_i}{\Gamma} \qquad \Gamma_i = \frac{\mathrm{BR}(A\to i)}{\tau}}$$

β Decay



As an example of β decay, let's study the decay

$^{14}O \rightarrow {}^{14}N^* + e^+ + \nu$ ($0^+ \rightarrow 0^+$ Fermi transition)

(at nucleon level: $p \rightarrow n + e^+ + \nu_e$)

(at quark level: $u \rightarrow d + e^+ + \nu_e$)

(same formalism for other β decays)

Transition amplitude in configuration space

$$T_{fi} = -i \frac{4G_F}{\sqrt{2}} \int \mathrm{d}^4x \; J_\mu^{(N)\dagger}(x) \cdot J^{(l)\mu}(x)$$
$$= -i \frac{4G_F}{\sqrt{2}} \int \mathrm{d}^4x \left[\bar{\psi}_n(x) \gamma_\mu \frac{1}{2}\left(1-\gamma^5\right) \psi_p(x) \right] \left[\bar{\psi}_\nu(x) \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) \psi_e(x) \right]$$

approximations: other nucleons are spectators (i.e. do not participate in the transition)
"point" interaction, ignore longer range strong interactions
$0^+ \rightarrow 0^+$ no change in nuclear wave function (Fermi transition), only V
$E \sim 2$ MeV $\rightarrow$ $\lambda_e \sim 10^{-11}$ cm big compared to $R_A \sim 3$ fm
non-relativistic for nucleus / nucleons ($\mathbf{p} \rightarrow 0$)
$\rightarrow$ only γ^0 contributes: $\bar{\psi}_n \gamma^\mu \psi_p \rightarrow \bar{\psi}_n \gamma^0 \psi_p = \psi_n^\dagger \gamma^0 \gamma^0 \psi_p = \psi_n^\dagger \psi_p$

$$T_{fi} \approx -i \frac{G_F}{\sqrt{2}} \left[\bar{u}_\nu\left(p_\nu\right) \gamma^0 \left(1-\gamma^5\right) v_e\left(p_e\right) \right] \int \mathrm{d}^4x \; \psi_n^\dagger(x) \psi_p(x) \cdot e^{-i\left(p_\nu + p_e\right)\cdot x}$$

recall: $\psi\left(x\right) = u(p) e^{-ip\cdot x}$ integral in $d^4x \equiv$ Fourier transform $\rightarrow$ momentum space

A first integration over dt $\int \mathrm{d}t\; e^{-iEt} = 2\pi\delta(E_i - E_f)$ gives

$$T_{fi} = -i\frac{G_F}{\sqrt{2}}\left[\bar{u}_\nu(p_\nu)\gamma^0(1-\gamma^5)v_e(p_e)\right](2\pi)\delta(E_0 - E_e - E_\nu)\int \mathrm{d}^3x\; \psi_n^\dagger(x)\psi_p(x)\cdot \overbrace{e^{-i(\vec{p}_\nu+\vec{p}_e)\cdot\vec{x}}}^{e^{i(\vec{p}_\nu+\vec{p}_e)\cdot\vec{x}} \approx 1}$$

The integration over d^3x gives

energy released in the decay $E_0 = \sqrt{m_n^2 - m_p^2 - m_e^2}$

$$= -i\frac{G_F}{\sqrt{2}}\left[\bar{u}_\nu(p_\nu)\gamma^0(1-\gamma^5)v_e(p_e)\right](2m_N)(2\pi)\delta(E_0 - E_e - E_\nu)$$

$2m_N \equiv$ normalization of nucleon wave functions $\int \mathrm{d}^3x\; \psi_n^\dagger\psi_p = 2m_N(/V)$

^{14}O has 8 protons: do all these protons contribute to the decay on equal footing?

^{14}O is part of the isospin triplet (^{14}O, ^{14}N*, ^{14}C),
interpreted as (|pp>, 1/√2(|pn> + |np>) , |nn>) around an isosinglet ^{12}C core.
The decay $^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N}^* + e^+ + \nu$ is described as a transition inside the isospin multiplet |pp> → 1/√2(|pn> + |np>) + e^+ + ν
only the two “external” protons (2 out of 8) participate in the transition
→ sum of amplitudes.

transition amplitude $$\boxed{T_{fi} \approx \frac{G_F}{\sqrt{2}}\left[\bar{u}(p_\nu)\gamma^0(1-\gamma^5)v(p_e)\right](2m_N)\left(2\frac{1}{\sqrt{2}}\right)}$$

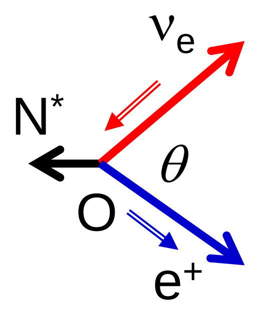
2 from the sum of amplitudes (don’t know which p), 1/√2 from isospin

Invariant Amplitude

$$\boxed{M_{fi} \approx \frac{G_F}{\sqrt{2}}\left[\bar{u}(p_\nu)\gamma^0\left(1-\gamma^5\right)v(p_e)\right](2m_N)\left(2\frac{1}{\sqrt{2}}\right)(2m_N)}$$

Add over final state spins (only lepton spins, ^{14}O and $^{14}N^*$ are spinless)

$$\overline{\left|M_{fi}\right|^2} = \frac{G_F^2}{\cancel{2}}\sum_{\text{spins}}\left|\bar{u}(p_\nu)\gamma^0\left(1-\gamma^5\right)v(p_e)\right|^2 (2m_N)^2\left(2\cancel{\frac{1}{\sqrt{2}}}\right)^2$$



The sum over spins (exercise)

$$\sum_{\text{spins}}\left|\bar{u}(p_\nu)\gamma^0\left(1-\gamma^5\right)v(p_e)\right|^2$$

$$= \sum_{s,t}\left(\bar{u}^{(s)}(p_\nu)\gamma^0\left(1-\gamma^5\right)v^{(t)}(p_e)\right)\left(\bar{v}^{(t)}(p_e)\left(1+\gamma^5\right)\gamma^0 u^{(s)}(p_\nu)\right)$$

$$= Tr\left(\not{p}_\nu\gamma^0\left(1-\gamma^5\right)\not{p}_e\left(1+\gamma^5\right)\gamma^0\right)$$

gives $$= 8\left(E_e E_\nu + \vec{p}_e\cdot\vec{p}_\nu\right) = \boxed{8E_e E_\nu\left(1+\overset{\sim 1}{\cancel{\beta_e}}\cos\vartheta\right)}$$

Maximal when emitted in same direction (opposite to recoiling nucleus)

Transition Rate Γ

Putting everything together

$$\boxed{\mathrm{d}\Gamma = \frac{1}{2E_p}\overline{\left|M_{fi}\right|^2}\frac{1}{2E_n}\frac{\mathrm{d}^3 p_e}{(2\pi)^3 2E_e}\frac{\mathrm{d}^3 p_\nu}{(2\pi)^3 2E_\nu}2\pi\delta\left(E_0 - E_e - E_\nu\right)}$$

[integrating over d^4x is equivalent to integrating over d^4p_N

$$\frac{\mathrm{d}^3 p_n}{(2\pi)^3 2E_n}(2\pi)^4\delta^4\left(p_A - p_1 - \cdots - p_N\right) \to \frac{1}{2E_n}2\pi\delta\left(E_0 - E_e - E_\nu\right)$$

]

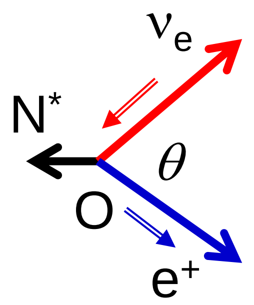
gives

$$\boxed{\mathrm{d}\Gamma = \frac{1}{\cancel{2E_p}}G_F^2 8E_e E_\nu\left(1+\cos\vartheta\right)(2\cancel{m_N})^2\frac{1}{\cancel{2E_n}}\frac{\mathrm{d}^3 p_e}{(2\pi)^3 2E_e}\frac{\mathrm{d}^3 p_\nu}{(2\pi)^3 2E_\nu}2\pi\delta\left(E_0 - E_e - E_\nu\right)}$$

express $\mathrm{d}^3 p_e\ \mathrm{d}^3 p_\nu$ in spherical coordinates $p_e^2\mathrm{d}p_e\mathrm{d}\left(\cos\vartheta_e\right)\mathrm{d}\varphi_e\ E_\nu^2\mathrm{d}E_\nu\mathrm{d}\left(\cos\vartheta_\nu\right)\mathrm{d}\varphi_\nu$

2 integrations in dϕ and 1 in d(cosθ_ν) give $2\pi p_e^2\mathrm{d}p_e\mathrm{d}\left(\sin\vartheta_e\right)2\pi 2E_\nu^2\mathrm{d}E_\nu$

$$\boxed{\mathrm{d}\Gamma = \frac{4G_F^2}{(2\pi)^3}\left(1+\cos\vartheta\right)\left(\mathrm{d}\left(\cos\vartheta\right)p_e^2\mathrm{d}p_e\right)\left(E_\nu^2\mathrm{d}E_\nu\right)\delta\left(E_0 - E_e - E_\nu\right)}$$



with $E_0 = \sqrt{m_n^2 - m_p^2 - m_e^2}$ the energy released in the decay

β Energy Spectrum

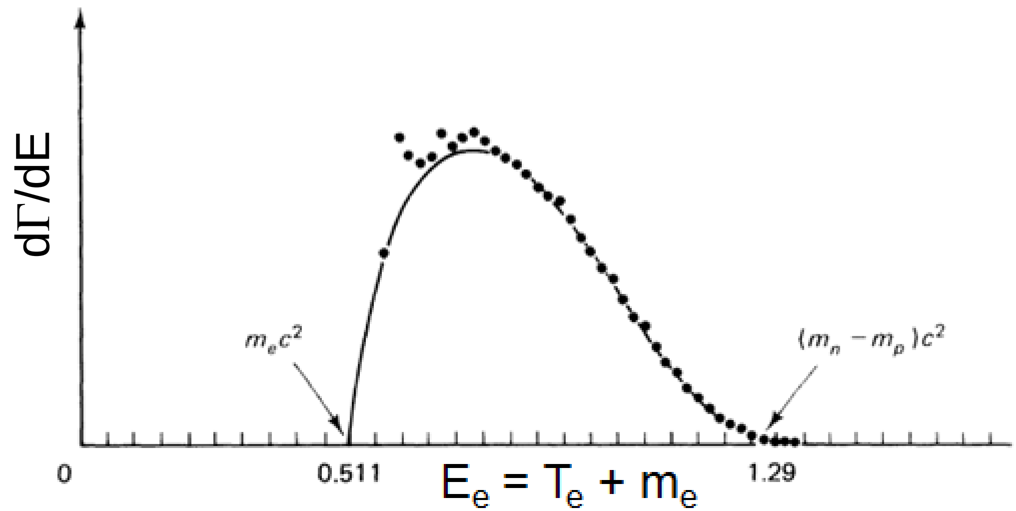
Integrating over E_ν and $\cos\theta$ gives the energy spectrum of the decay electron ($p_e \sim E_e$)

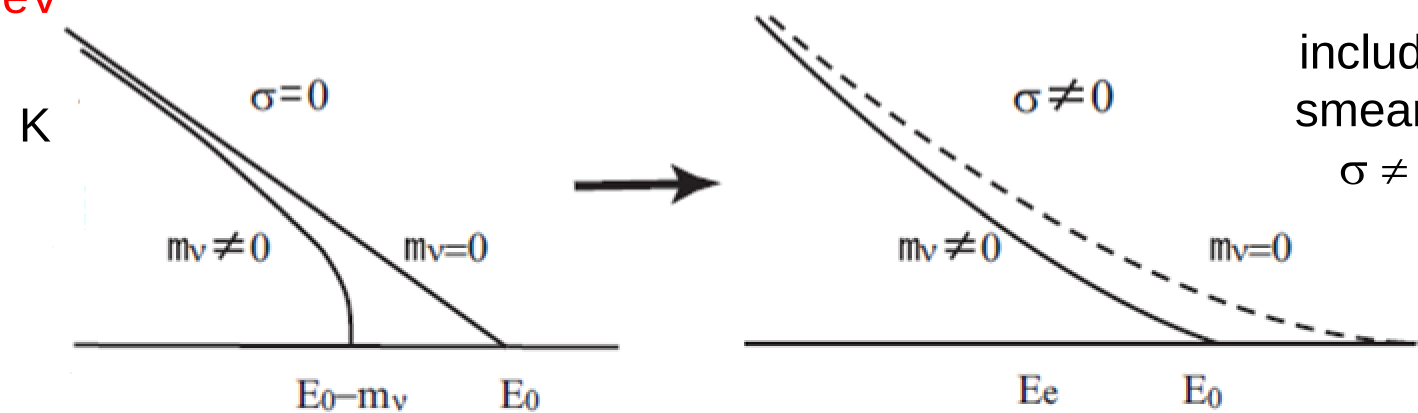
$$\boxed{\frac{d\Gamma}{dE_e} = \frac{4G_F^2}{(2\pi)^3} E_e^2 (E_0 - E_e)^2 \int_0^\pi d\cos\vartheta (1+\cos\vartheta) = \frac{G_F^2}{\pi^3} E_e^2 (E_0 - E_e)^2}$$

To study the spectrum end-point,
can rewrite $d\Gamma/dE$ in the following way

$$\boxed{K = \sqrt{\frac{d\Gamma}{dE_e}} \Big/ E_e = \frac{G_F}{\pi^{3/2}} (E_0 - E_e)}$$

Kurie plot (spectrum): deviations → m_ν
exp.: $m_\nu < 0.8$ eV

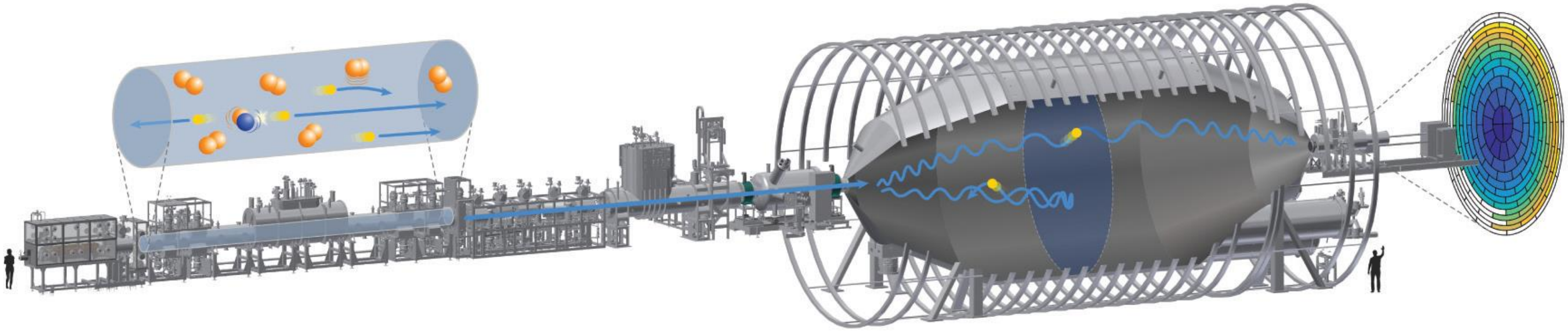




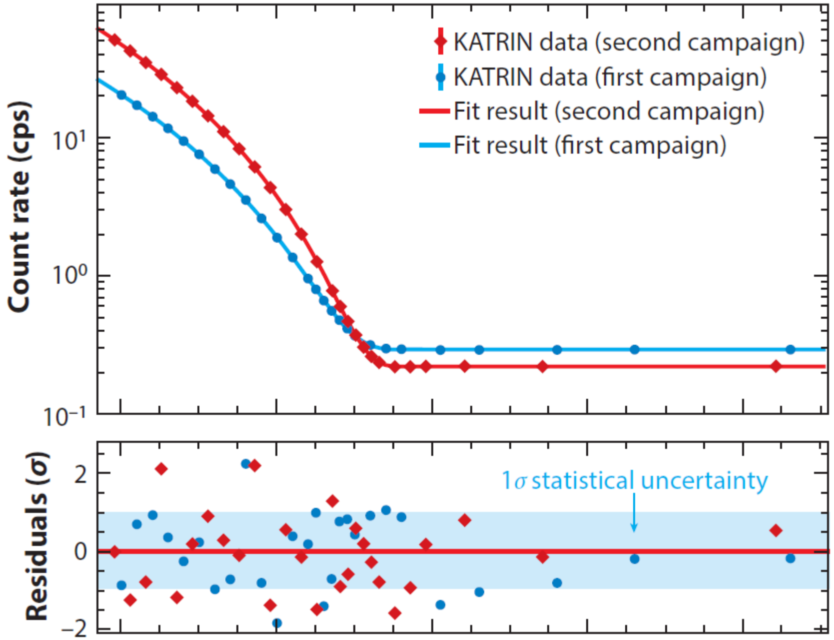
including
smearing
$\sigma \neq 0$

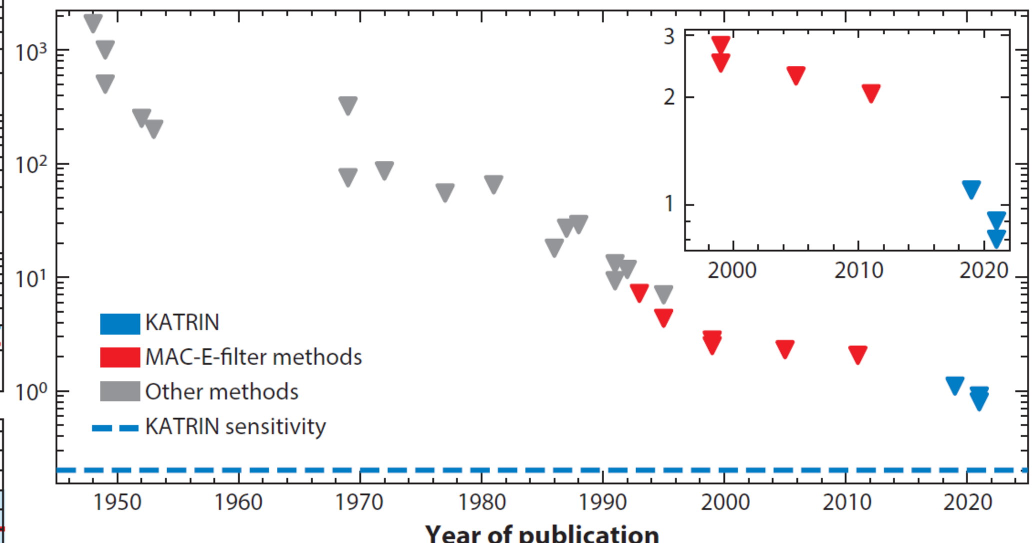
Neutrino Mass Measurement

KATRIN experiment ($^3H \rightarrow {}^3He + e^- + \overline{\nu}_e$)



endpoint spectrum





m_ν < 0.8 eV (90% CL) (2022)

β Lifetime

So far ignored nuclear / Coulomb effects

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} = \frac{G_F^2}{\pi^3} E_e^2 \left(E_0 - E_e\right)^2 \underbrace{F(Z, E_0)}_{\sim 1}$$

By integrating over E_e we can relate the transition rate ($\Gamma = 1/\tau$) to G_F

$$\frac{1}{\tau} = \Gamma = \frac{G_F^2}{\pi^3} \int_0^{E_0} E_e^2 \left(E_0 - E_e\right)^2 \mathrm{d}E_e = \frac{G_F^2}{30\pi^3} E_0^5$$

$$\boxed{\Gamma = \frac{G_F^2}{30\pi^3} E_0^5}$$

Note the E_0^5 dependence of the decay width – Sargent's law

The E_0^5 dependence can be derived from dimensional arguments:
E_0 is the only "observable" in the process $\rightarrow$ $[\Gamma] = E$ and $[G_F^2] = E^{-4} \rightarrow E_0^5$

For the $^{14}O \rightarrow {}^{14}N^* + e^+ + \nu$ decay

$E_0 = 1.81$ MeV

$\tau = 102$ s ($\Gamma = 9.76 \times 10^{-3}$ s^{-1})

$\rightarrow G_F = 1.136 \pm 0.003 \times 10^{-5}$ GeV^{-2}

Electron Polarization in β Decays

According to the V – A theory, electrons emitted in weak decays are left handed, i.e. they are eigenstates of the chirality projector $P_L = ½(1 - \gamma_5)$:

$$e_L(p) = P_L e(p) = \frac{1}{2}(1-\gamma_5)e(p)$$

To calculate the electron polarization, decompose the chirality eigenstate into helicity eigenstates:

$$e_L(p) = \frac{1}{2}\left(1-\frac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)e = \frac{1}{2}\left(1-\frac{p_z}{E+m}\right)e_{+\frac{1}{2}} + \frac{1}{2}\left(1+\frac{p_z}{E+m}\right)e_{-\frac{1}{2}} \xrightarrow[m\to 0]{p\to\infty} \frac{1}{2}\frac{m}{E}e_{+\frac{1}{2}} + e_{-\frac{1}{2}}$$

The "polarization" $\langle h\rangle = \dfrac{\Pi^+ - \Pi^-}{\Pi^+ + \Pi^-}$ measures the alignment of the electron spin w.r.t. its momentum.

Π^+ probability to be in + helicity state $\qquad \Pi^+ = \left|+\frac{1}{2}\left(1-\frac{p}{E+m}\right)\right|^2$

Π^- probability to be in a – helicity state $\qquad \Pi^- = \left|+\frac{1}{2}\left(1+\frac{p}{E+m}\right)\right|^2$

It follows that

$$\langle h\rangle = \frac{\Pi^+ - \Pi^-}{\Pi^+ + \Pi^-} = \frac{(E+m-p)^2-(E+m+p)^2}{(E+m-p)^2+(E+m+p)^2} = -\frac{p}{E} = -\beta$$

where β is the speed of the electron.

Electron Polarization Measurement

Scatter electrons from β decays on an electron target

This is a QED process, which conserves parity and therefore is not sensitive to the longitudinal electron polarization

This observable $\sigma^{\rightarrow} - \sigma^{\leftarrow}$ violates parity

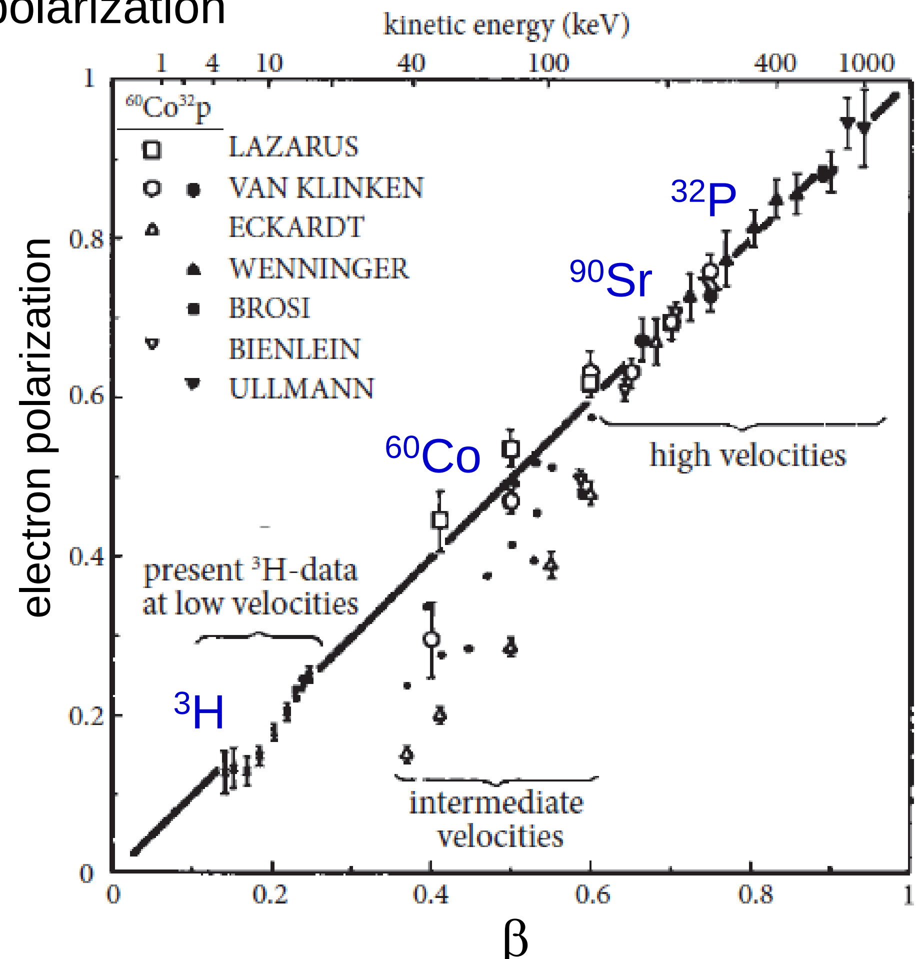
Two solutions:

a) rotate the electron spin from longitudinal to transverse with an electric field
(change the direction of electron without modifying the spin direction)
measure left - right scattering asymmetry

b) scatter longitudinally polarized electrons on a polarized electron target
(polarized i.e. with a magnetic field)
measure

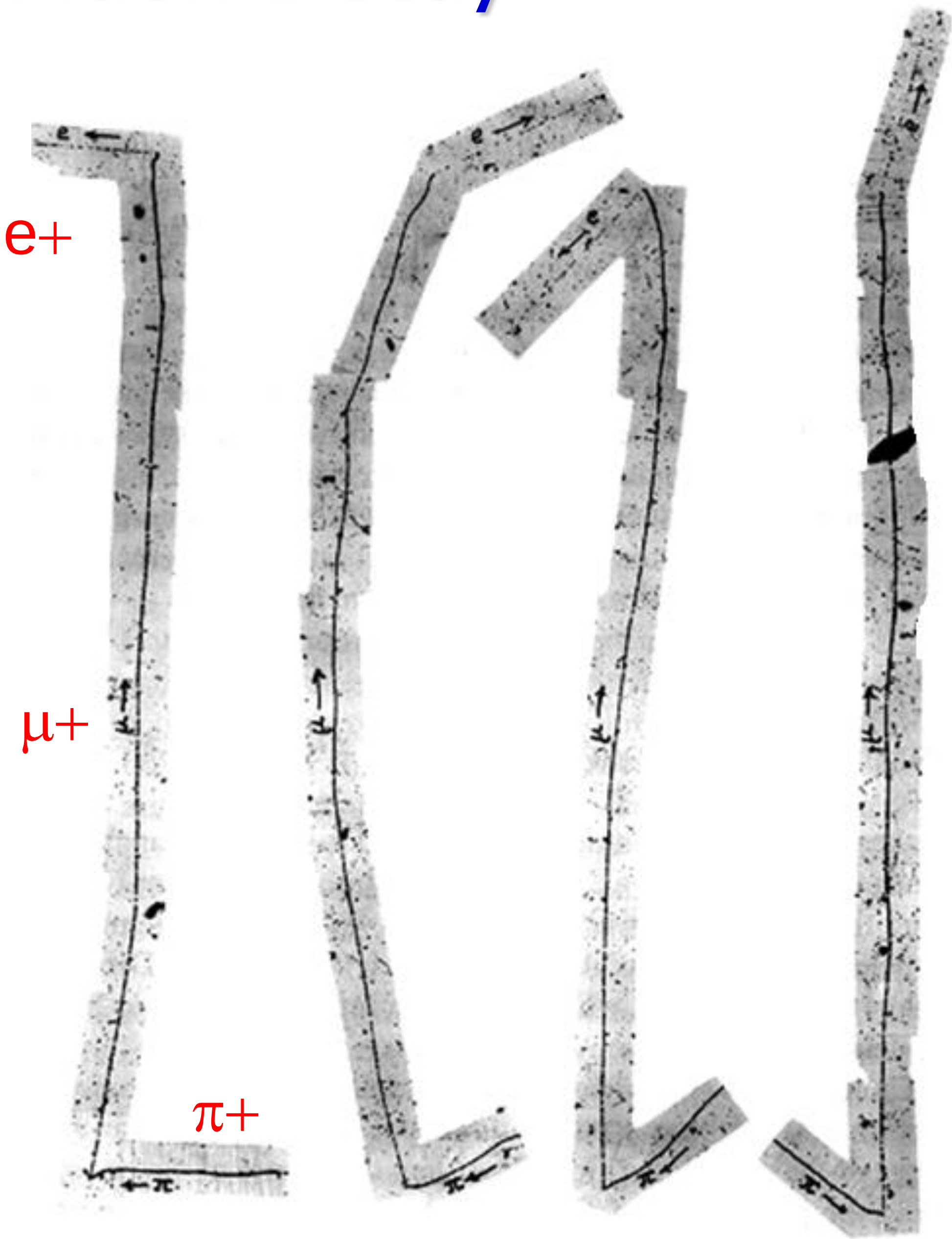
$$\sigma^{\rightarrow\leftarrow} - \sigma^{\leftarrow\leftarrow} / \sigma^{\rightarrow\leftarrow} + \sigma^{\leftarrow\leftarrow} \propto P_e$$

this observable does not violate parity



$$\boxed{P_e = -\beta}$$

Muon Decay



muons discovered in cosmic rays

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

points to note:

dE/dx – Bragg Peak

low dE/dx for fast e^+

constant range for μ (~600μm)
i.e. monochromatic
$\Rightarrow$ 2-body decay

e^+ spectrum not monochromatic
E_e broad range with $0 < E_e < ½\, m_\mu$
$\Rightarrow$ 3-body decay (cfr. β spectrum),
2 neutrinos

Lepton Flavor Conservation

Who ordered that?   I. I. Rabi in 1947 referring to the recently discovered muon
Well, not only this question remains unanswered almost 80 years later,
but we still do not have the slightest clue on the origin of flavor …

Initially thought that the muon could be an excited electron decaying to $\mu \to e + \gamma$

However, experimentally this decay has never been observed, BR < 2.4×10^{-13},
leading to the notion of lepton flavor and lepton flavor conservation. (cfr. baryon cons.)

In the Standard Model ($m_\nu = 0$) **Lepton flavor is conserved absolutely**
not by "principle", but through its structure.
Different leptons (e, μ, τ) are organized in multiplets (families) with the corresponding neutrinos, and to each multiplet a lepton flavor number L_e, L_μ, L_τ is assigned:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$L_e \qquad\qquad L_\mu$$

Transitions across families are strictly forbidden.

$$\mu^- \to \ e^- + \nu_\mu + \bar{\nu}_e$$

L_μ	=	+1		0	+1	0	= +1
L_e	=	0		+1	0	-1	= 0

LFV Searches: Current Situation

The best limits on LFV come from PSI muon experiments

$\mu^+ \rightarrow e^+e^-e^+$

BR < 1×10^{-12}

SINDRUM 1988

$\mu^- + Au \rightarrow e^- + Au$

BR < 7×10^{-13}

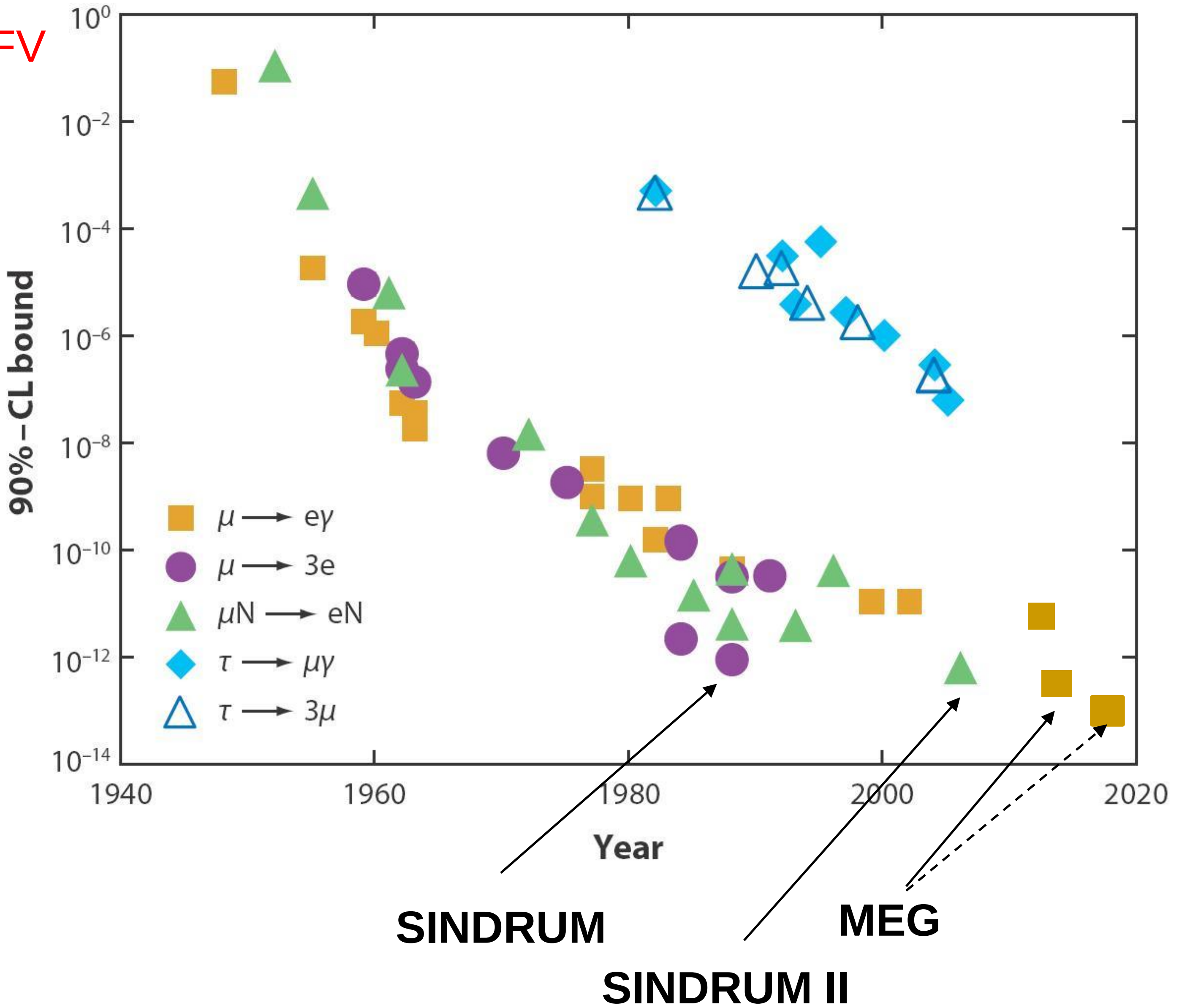
SINDRUM II 2006

$\mu^+ \rightarrow e^+ + \gamma$

BR < 2.4×10^{-13}

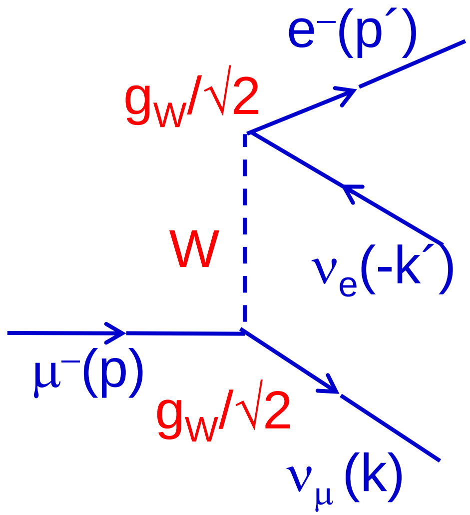
MEG 2016

[90 % C.L.]



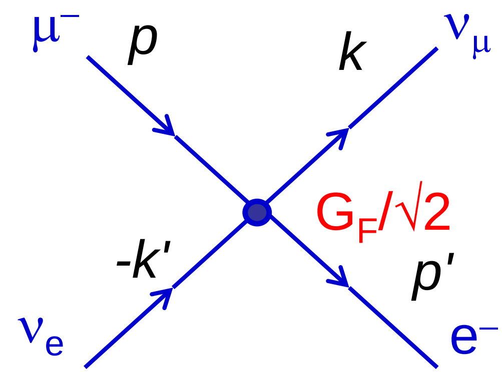
Muon Decay

$\mu^- \to e^- + \nu_\mu + \overline{\nu}_e$



Feynman "prescription":
replace outgoing $\overline{\nu}_e(\omega', \mathbf{k}')$
with incoming $\nu_e\,(-\omega', -\mathbf{k}')$
(i.e. replace *u* spinor with *v* spinor)

$\overline{\nu}_e\;(\omega', \mathbf{k}') \to \nu_e\;(-\omega', -\mathbf{k}')$



effective 4-fermion interaction

invariant amplitude

$$\boxed{M_{fi} = \frac{4G_F}{\sqrt{2}}\left[\bar{u}(k)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p)\right]\left[\bar{u}(p')\gamma_\mu \frac{1}{2}(1-\gamma^5)v(k')\right]}$$

decay rate

$$\boxed{\mathrm{d}\Gamma = \frac{1}{2m_\mu}\overline{\left|M_{fi}\right|^2}\mathrm{d}Q}$$

normalization of wave function
(2 μ⁻ / V; μ⁻ decays at rest)

phase space $\quad \mathrm{d}Q = \frac{\mathrm{d}^3 p'}{(2\pi)^3 2E'}\frac{\mathrm{d}^3 k}{(2\pi)^3 2\omega}\frac{\mathrm{d}^3 k'}{(2\pi)^3 2\omega'}(2\pi)^4\delta^4(p-p'-k-k')$

Invariant Amplitude

$$\left\langle \left|M_{fi}\right|^2 \right\rangle = \frac{1}{2s+1}\sum_{\text{spins}}\left|M_{fi}\right|^2$$

$$= \frac{1}{2}\frac{16G_F^2}{2}\sum_{s,t}\left[\bar{u}^s(k)\gamma^\mu \frac{1}{2}(1-\gamma^5)u^t(p)\right]\left[\bar{u}^t(p)\frac{1}{2}(1+\gamma^5)\gamma^\nu u^s(k)\right]\times$$

"muon" tensor

$$\sum_{s',t'}\left[\bar{u}^{s'}(p')\gamma_\mu \frac{1}{2}(1-\gamma^5)v^{t'}(k')\right]\left[\bar{v}^{t'}(k')\frac{1}{2}(1+\gamma^5)\gamma_\nu u^{s'}(p')\right]$$

"electron" tensor

$$= \frac{1}{2}\frac{G_F^2}{2}L^{\mu\nu}K_{\mu\nu}$$

with $L^{\mu\nu} = \sum_s \underbrace{u_\delta^s(k)\bar{u}_\alpha^s(k)}\left[\gamma^\mu(1-\gamma^5)\right]_{\alpha\beta} \times \sum_t \underbrace{u_\beta^t(p)\bar{u}_\gamma^t(p)}\left[(1+\gamma^5)\gamma^\nu\right]_{\gamma\delta}$

$$\sum_{\text{spins}} u_\delta^s \bar{u}_\alpha^s = (\not{k}+m_\nu)_{\delta\alpha} = \not{k}_{\delta\alpha} \qquad \sum_{\text{spins}} u_\beta^t \bar{u}_\gamma^t = (\not{p}+m_\mu)_{\beta\gamma}$$

Σ initial and final spins completeness relations

$$= \frac{1}{2}\mathrm{Tr}\left\{\not{k}\gamma^\mu(1-\gamma^5)(\not{p}+m_\mu)\gamma^\nu(1-\gamma^5)\right\} = \mathrm{Tr}\left(\not{k}\gamma^\mu\left(\not{p}+m_\mu\right)\gamma^\nu\right) + 4i\varepsilon^{\mu\alpha\nu\beta}k_\alpha\left(\not{p}+m_\mu\right)_\beta$$

and

$$K_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left\{\left(\not{p}'+m_e\right)\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\right\} = \mathrm{Tr}\left(\left(\not{p}'+m_e\right)\gamma_\mu\not{k}'\gamma_\nu\right) + 4i\varepsilon_{\mu\alpha\nu\beta}\left(\not{p}'+m_e\right)^\alpha k'^\beta$$

$$L^{\mu\nu}K_{\mu\nu} = \mathrm{Tr}\left\{\not{k}\gamma^{\mu}(1-\gamma^5)(\not{p}+m_\mu)\gamma^{\nu}(1-\gamma^5)\right\}\mathrm{Tr}\left\{\left(\not{p}'+m_e\right)\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\right\}$$

$$= 256\left(k\cdot p'\right)\left(p\cdot k'\right)$$

$$\left\langle\left|M_{fi}\right|^2\right\rangle = \frac{1}{2}\frac{G_F^2}{2}\mathrm{Tr}\left\{\not{k}\gamma^{\mu}(1-\gamma^5)(\not{p}+m_\mu)\gamma^{\nu}(1-\gamma^5)\right\}\times\mathrm{Tr}\left\{\left(\not{p}'+m_\mu\right)\gamma_\mu(1-\gamma^5)\not{k}'\gamma_\nu(1-\gamma^5)\right\}$$

$$= 64G_F^2(k\cdot p')(k'\cdot p)$$

compare to μ e $\rightarrow$ μ e (neglecting the masses in the extreme relativistic limit)

$$\left\langle\left|M_{fi}\right|^2\right\rangle = \frac{8e^2}{\left(k-k'\right)^4}\left\{(k\cdot p)(k'\cdot p')+(k\cdot p')(k'\cdot p)\right\}$$

which is symmetric in the scattering angle $\cos\theta$.

Finally, the amplitude for $\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e$ In the muon rest frame $p = (m,0,0,0)$ is

$$\boxed{\left\langle\left|M_{fi}\right|^2\right\rangle = 32\ G_F^2\ (m_\mu^2 - 2m_\mu\omega')m_\mu\omega'}$$

Phase Space Factor

$$\mathrm{d}Q = \frac{\mathrm{d}^3 p'}{(2\pi)^3 2E'} \frac{\mathrm{d}^3 k}{(2\pi)^3 2\omega} \frac{\mathrm{d}^3 k'}{(2\pi)^3 2\omega'} (2\pi)^4 \delta^4(p - p' - k - k')$$

Start by integrating over the ν_μ kinematics: transform the d^3*k* (3D) integration into a d^{4}k (4D) integration using the dispersion relation *k* = *p* – *p'* – *k'* (from the δ^4 !),

$$2\pi\delta\left(k^2 - m^2\right)\Theta(\omega)\frac{\mathrm{d}^4 k}{(2\pi)^4} \xrightarrow{\int \mathrm{d}\omega} \frac{1}{2\sqrt{\vec{k}^2 + m^2}} \frac{\mathrm{d}^3 k}{(2\pi)^3}$$

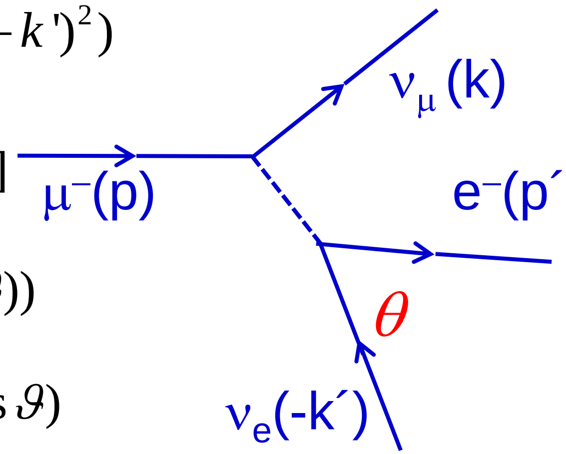
$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

Heaviside Θ function

and we obtain

$$\mathrm{d}Q = \frac{\mathrm{d}^3 p'}{(2\pi)^3 2E'} \frac{\mathrm{d}^3 k'}{(2\pi)^3 2\omega'} 2\pi\, \Theta(E - E' - \omega')\delta((p - p' - k')^2)$$

Then replace $[\mathrm{d}^3 p' \mathrm{d}^3 k']$ with $[4\pi E'^2 \mathrm{d}E' 2\pi\omega'^2 \mathrm{d}\omega' \mathrm{d}(\cos\vartheta)]$



and $\delta((p - p' - k')^2)$ with $\delta(m_\mu^2 - 2m_\mu E' - 2m_\mu\omega' + 2E'\omega'(1 - \cos\vartheta))$

$$\delta(\cdots - 2E'\omega'\cos\vartheta) = \frac{1}{2E'\omega'}\delta(\cdots - \cos\vartheta)$$

m_μ > 200 m_e , neglect m_e

$$\delta(\cdots - 2E'\omega'\cos\vartheta) \Rightarrow \cos\vartheta = \frac{m_\mu^2 - 2m_\mu E' - 2m_\mu\omega' + 2E'\omega'}{2E'\omega'}$$

Transition Rate

Putting everything together

$$\mathrm{d}\Gamma = \frac{G_F^2}{2m_\mu (2\pi)^5} 32(m_\mu^2 - 2m_\mu \omega')m_\mu \omega' \frac{\mathrm{d}^3 p'}{2E'} \frac{\mathrm{d}^3 k'}{2\omega'} \delta(m_\mu^2 - 2m_\mu E' - 2m_\mu \omega' + 2E'\omega'(1-\cos\vartheta))$$

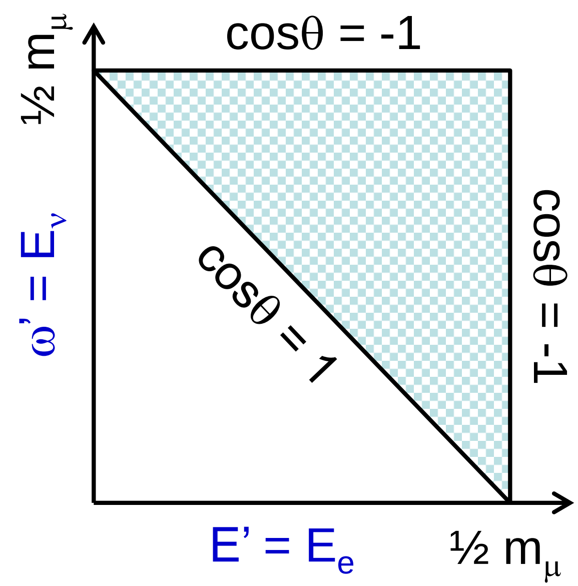
$$= \frac{G_F^2}{2m_\mu \pi^5} (m_\mu^2 - 2m_\mu \omega')m_\mu \omega' 4\pi E'^2 \,\mathrm{d}E' 2\pi\omega'^2 \,\mathrm{d}\omega' \delta(\cdots - \cos\vartheta)\mathrm{d}\left(\cos\vartheta\right)$$

and integrating over cos θ , (i.e. the angle between the electron and the antineutrino)

$$\mathrm{d}\Gamma = \frac{G_F^2}{2\pi^3} m_\mu \omega'(m_\mu - 2\omega')\mathrm{d}E'\mathrm{d}\omega'$$ with $m_\mu - E' - \omega' > 0$

with the following constraints (δ function)

$$\begin{cases} -1 \le \cos\vartheta \le 1 \\ 0 \le E' \le \frac{1}{2} m_\mu \\ \frac{1}{2} m_\mu - E' \le \omega' \le \frac{1}{2} m_\mu \end{cases}$$



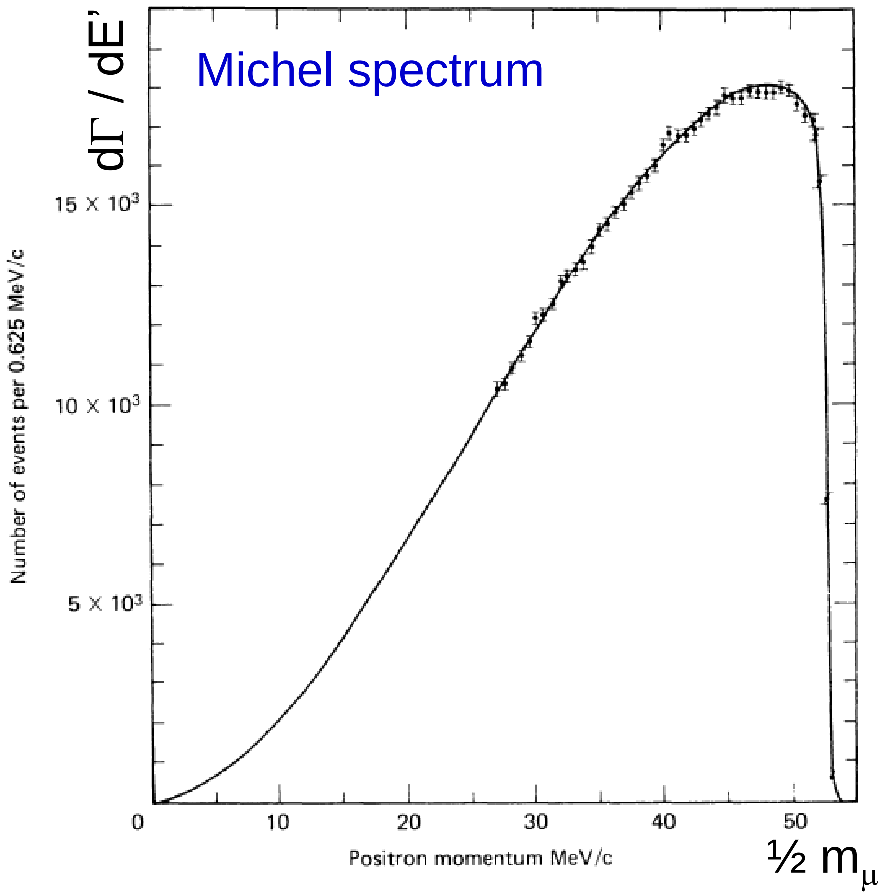
E^{el}_{min} when the neutrinos share all available energy

E^{el}_{max} when the electron recoils w.r.t. the neutrino pair

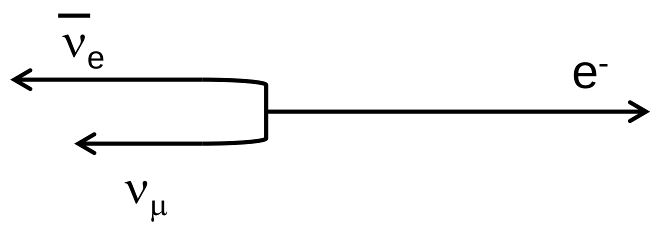
Electron Energy Spectrum

Integrating dΓ over ω' ($\bar{\nu}_e$ energy) gives the electron energy spectrum

$$\boxed{\frac{d\Gamma}{dE'} = \frac{G_F^2}{2\pi^3} m_\mu \int_{m_\mu/2-E'}^{m_\mu/2} d\omega' \omega'(m_\mu - 2\omega') = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)}$$



the electron tends to take the max available energy E' = ½ m_μ ! i.e. the e recoils w.r.t. the ν pair



Muon Lifetime

Integrate dΓ over E´ to obtain the muon decay rate

$$\boxed{\frac{1}{\tau} = \Gamma = \frac{G_F^2}{12\pi^3} m_\mu^2 \int_0^{m_\mu/2} \mathrm{d}E' E'^2 \left(3 - \frac{4E'}{m_\mu}\right) = \frac{G_F^2}{192\pi^3} m_\mu^5 \left[1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right] f\left(\frac{m_e^2}{m_\mu^2}\right)}$$

small correction ~1%:
radiative corrections + m_e

and the muon lifetime is

$$\boxed{\tau_\mu = \frac{192\pi^3 \hbar^7}{G_F^2 m_\mu^5 c^4} = 2.1969811 \pm 0.0000022 \times 10^{-6}\,\mathrm{s}}$$

(i.e. $\Delta\tau$ = 2.2 ps)

which gives the Fermi weak coupling constant
(including m_e and the radiative corrections)

$G_F(\mu) = 1.1663787 \pm 0.0000006 \times 10^{-5}$ GeV^{-2}

Compare to the β decay rate

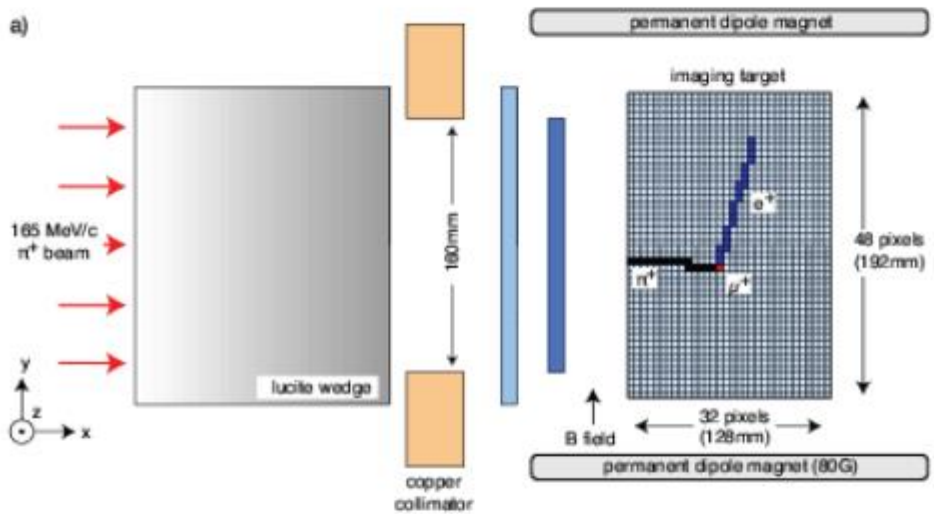
$$\frac{1}{\tau} = \Gamma = \frac{G_F^2}{30\pi^3} E_0^5$$

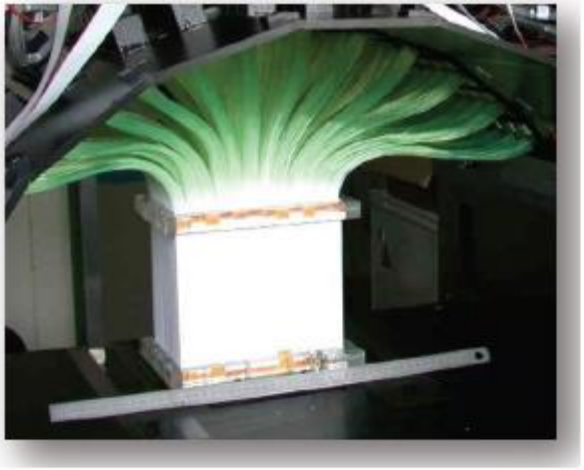
$G_F(\beta) = 1.136 \pm 0.003 \times 10^{-5}$ GeV^{-2}

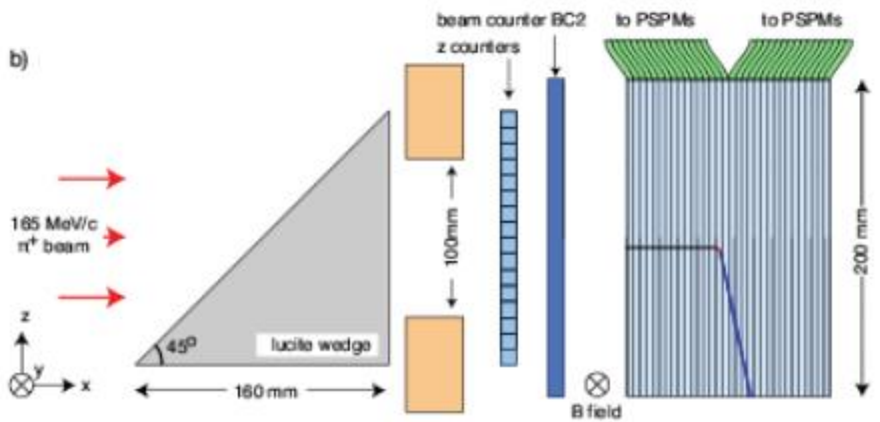
$\Rightarrow G_F(\mu) - G_F(\beta) = 0.030 \pm 0.003 \times 10^{-5}$ GeV^{-2} i.e. 10 σ significance!

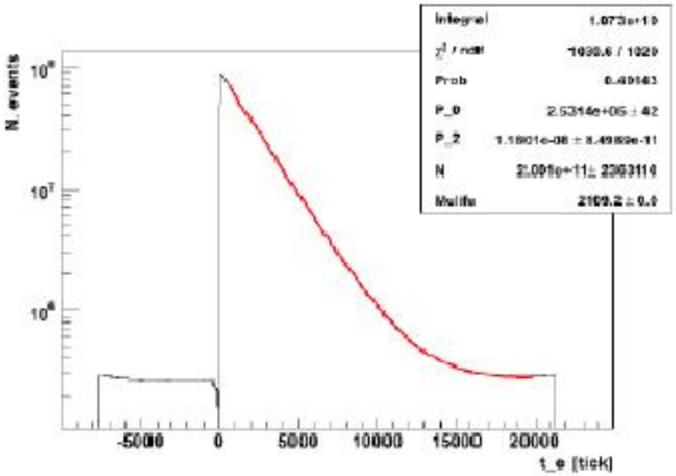
The weak coupling G_F does not seem to be universal. Do we have a problem?

Muon Lifetime Measurements – FAST





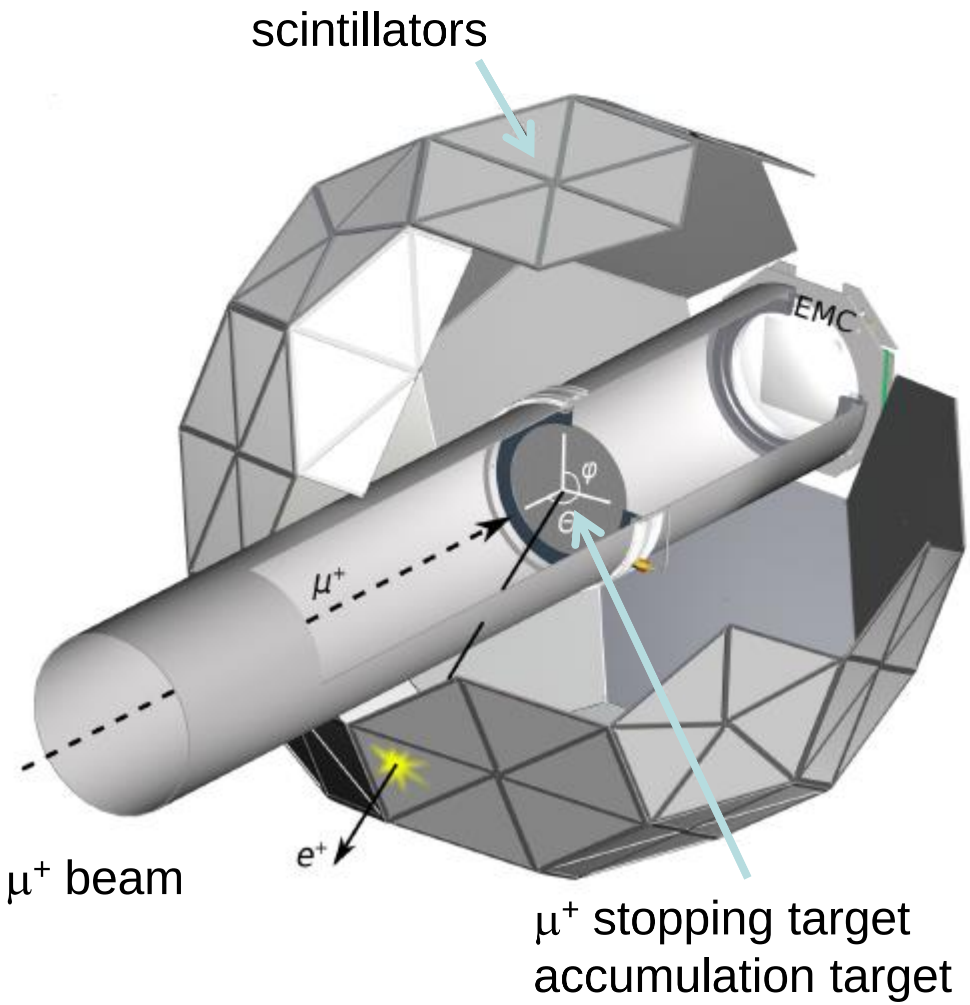


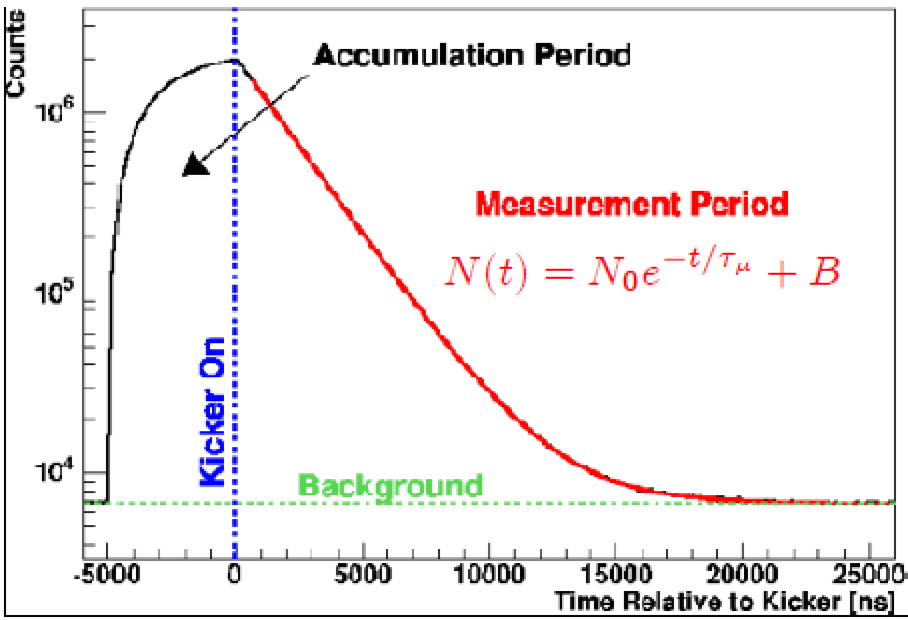


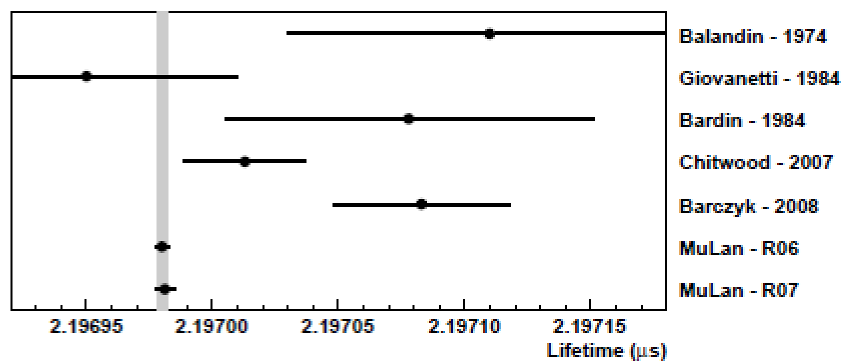
FAST : 2 197 082 ± 35 ps

The muon stops in the active target
measure time difference between stop time and decay time

Muon Lifetime Measurements – MuLAN



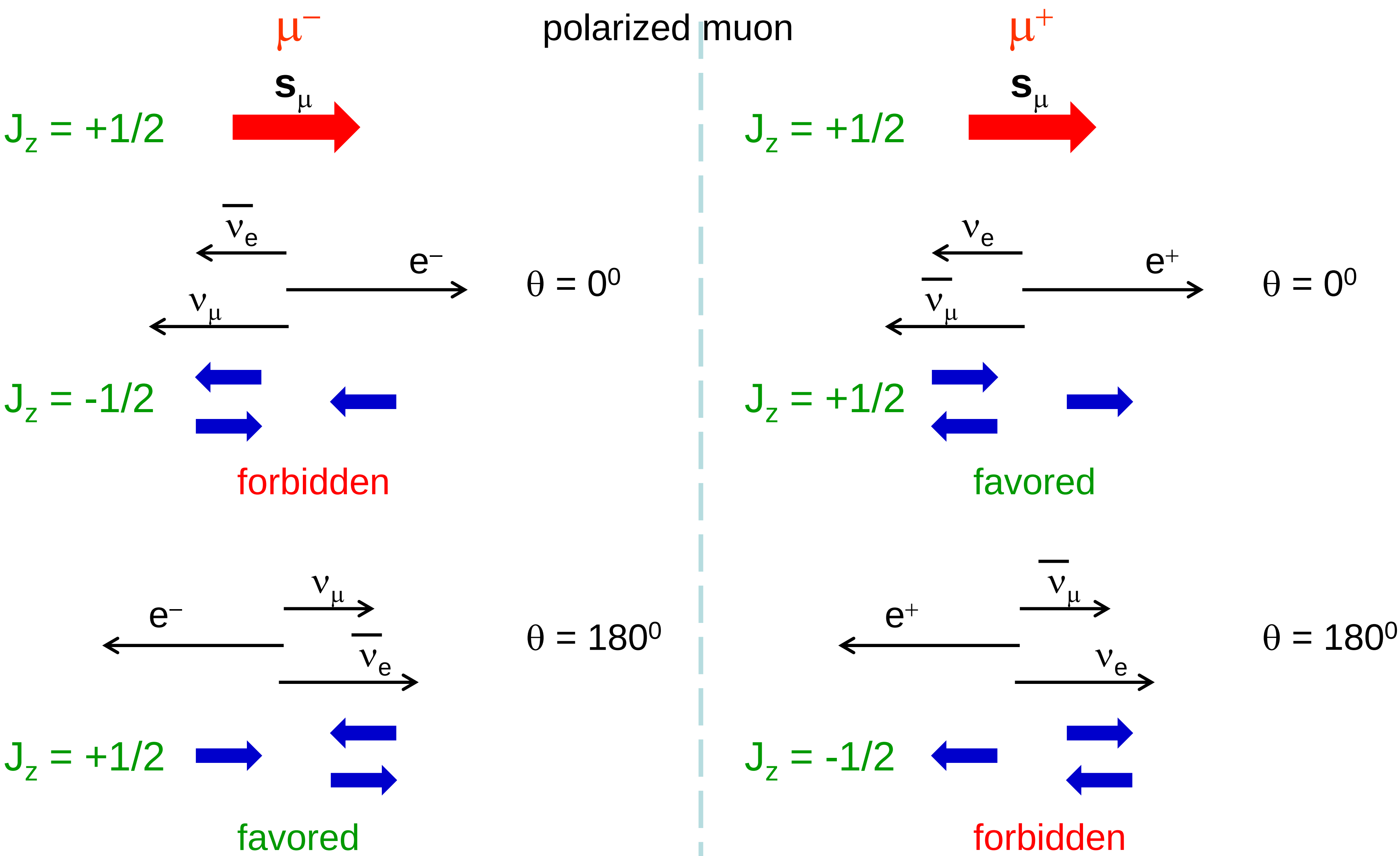




MuLAN : 2 196 980 ± 2 ps

world avarage : 2 196 981.1 ± 2.2 ps $\rightarrow$ $G_F = 1.1663787 \pm 0.0000006 \times 10^{-5}\ \text{GeV}^{-2}$

Helicity Considerations



electrons emitted opposite to μ^- spin

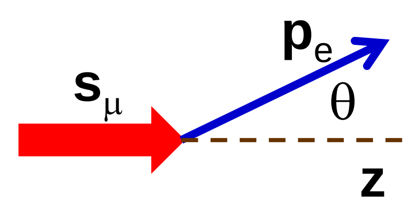
positrons emitted along μ^+ spin

Polarized Muon Decay

We do not average over the initial muon spin.

$$\frac{d\Gamma}{dE'} = \underbrace{\frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)}_{\text{unpolarized spectrum } d\Gamma / dE'} \left[1 + \frac{1 - 4E'/m_\mu}{3 - 4E'/m_\mu} \cos\vartheta\right] \left[\frac{1}{2}\left(1 - \frac{\vec{p}_e \cdot \hat{s}_e}{|\vec{p}_e|}\right)\right] \frac{d(\cos\vartheta)\, d\varphi}{4\pi}$$

θ = angle between e⁻ direction (p') and μ⁻ polarization **z**
ϕ = azimuthal angle
n = unitary vector // p'
$\mathbf{s}_e$ = direction e⁻ spin in e⁻ rest frame



e⁻ angular distribution w.r.t. **z**, direction of μ⁻ polarization

left handed e⁻
for m_e = 0 only helicity -1/2 allowed by V – A
[…] = 1

for E' = $m_\mu/2$ […] = [1 – cos θ] is max for θ = 180⁰
e⁻ preferentially emitted in direction opposite to muon polarization **z**

μ⁺ decay: [1 + …] → [1 – …]; […] = [1 + cos θ] is max for θ = 0⁰
e⁺ preferentially emitted in the direction of muon polarization **z**

Tau Lifetime

muon lifetime $$\frac{1}{\tau_\mu} = \Gamma\left(\mu^- \to e^- \nu_\mu \bar{\nu}_e\right) = \frac{G_F^2}{192\pi^3} m_\mu^5$$ the factor m⁵ comes from phase space (Sargent's law)

Therefore we find the same expression for the transition rate of weak decaying pointlike fermions, for example for the τ lepton (produced e.g. in $e^+e^- \to \tau^+\tau^-$):

$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \mathrm{BR}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \times \Gamma(\tau^- \to \mathrm{all}) = \frac{\mathrm{BR}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\tau_\tau} = \frac{G_F^2}{192\pi^3} m_\tau^5$$

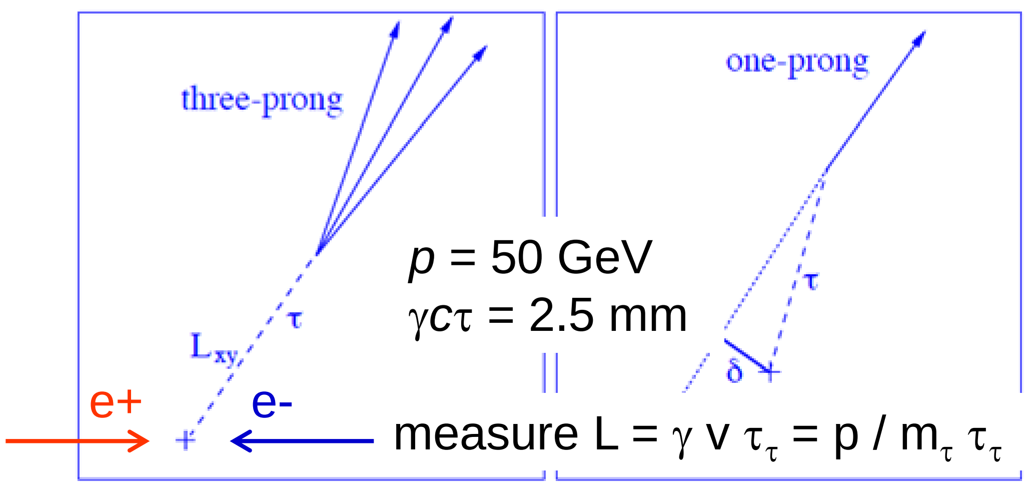
$$\mathrm{BR}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.1782 \pm 0.0004$$

$$G_F = 1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

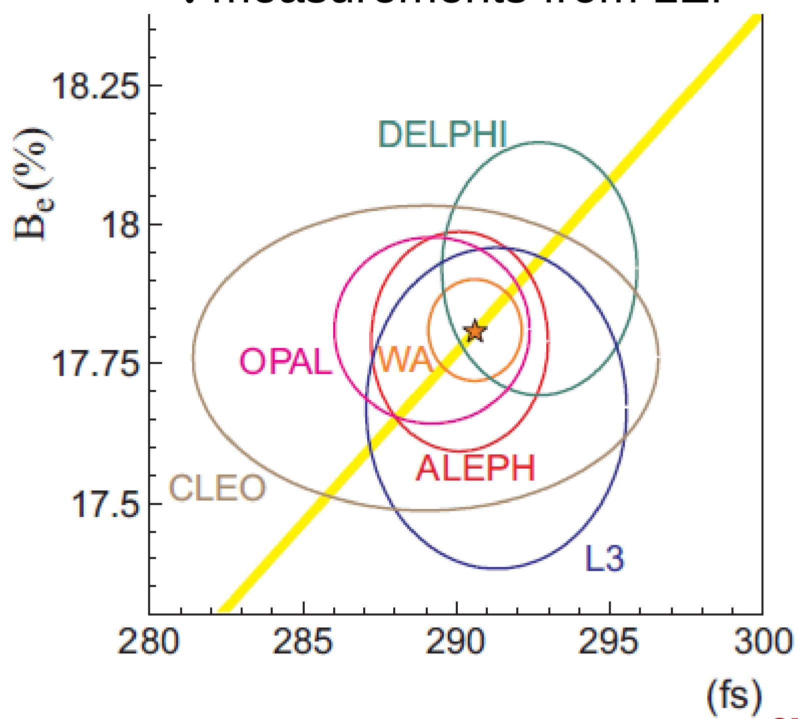
$$m_\tau = (1776.86 \pm 0.12)\ \mathrm{MeV}$$

predicted $$\tau_\tau^{theo} = (290.3 \pm 0.5) \times 10^{-15}\,\mathrm{s}$$

measured $$\tau_\tau^{\exp} = (290.6 \pm 1.0) \times 10^{-15}\,\mathrm{s}$$



τ measurements from LEP



Lepton Universality

Charged current weak interactions is universal and is equal for all fermions (when corrected for the masses of fermions). To test it, consider the following processes:

e - μ universality: compare $\tau^- \to e^- \nu_\tau \bar{\nu}_e$ and $\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu$

$$\Gamma\left(\tau^+ \to e^+ \bar{\nu}_\tau \nu_e\right) \propto \frac{g_\tau^2}{M_W^2}\frac{g_e^2}{M_W^2} m_\tau^5 \qquad \Gamma\left(\tau^+ \to \mu^+ \bar{\nu}_\tau \nu_\mu\right) \propto \frac{g_\tau^2}{M_W^2}\frac{g_\mu^2}{M_W^2} m_\tau^5$$

and take the ratio of the branching ratios (B.R.) (ρ is the phase space)

$$\frac{\Gamma\left(\tau^+ \to \mu^+ \bar{\nu}_\tau \nu_\mu\right)}{\Gamma\left(\tau^+ \to e^+ \bar{\nu}_\tau \nu_e\right)} = \frac{\Gamma_{tot} \cdot BR\left(\tau^+ \to \mu^+ \bar{\nu}_\tau \nu_\mu\right)}{\Gamma_{tot} \cdot BR\left(\tau^+ \to e^+ \bar{\nu}_\tau \nu_e\right)} = \frac{g_\mu^2}{g_e^2}\frac{\rho_\mu}{\rho_e}$$

$$\frac{BR\left(\tau^+ \to \mu^+ \bar{\nu}_\tau \nu_\mu\right)}{BR\left(\tau^+ \to e^+ \bar{\nu}_\tau \nu_e\right)} = \frac{(17.36 \pm 0.05)\%}{(17.84 \pm 0.05)\%} = 0.974 \pm 0.004 \qquad \Rightarrow \qquad \frac{g_\mu}{g_e} = 1.001 \pm 0.002$$

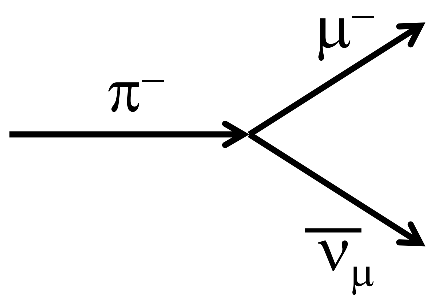
μ - τ universality: compare $\tau^- \to e^- \nu_\tau \bar{\nu}_e$ and $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

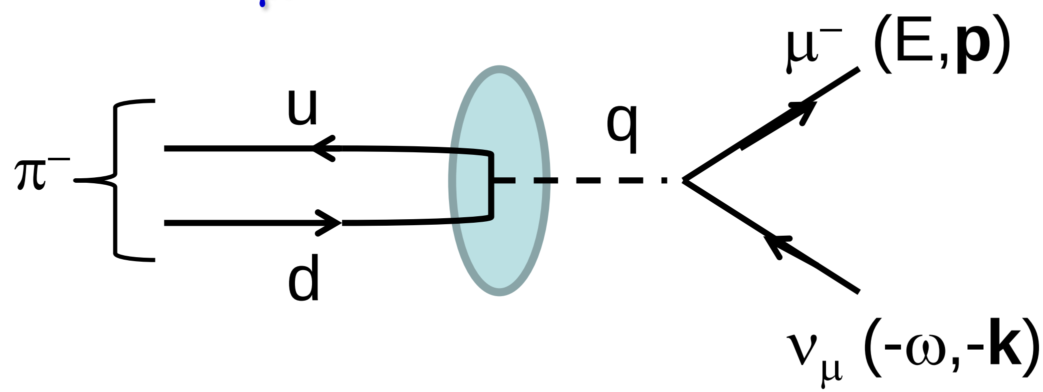
$$\frac{\Gamma\left(\mu^- \to e^- \nu_\mu \bar{\nu}_e\right)}{\Gamma\left(\tau^- \to e^- \nu_\tau \bar{\nu}_e\right)} = \frac{\Gamma_\mu \cdot 1}{\Gamma_\tau \cdot BR\left(\tau^- \to e^- \nu_\tau \bar{\nu}_e\right)} = \frac{\tau_\tau}{\tau_\mu}\frac{1}{BR\left(\tau^- \to e^- \nu_\tau \bar{\nu}_e\right)} = \frac{g_e^2}{g_e^2}\frac{g_\mu^2}{g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau}$$

$$\Rightarrow \frac{g_\mu^2}{g_\tau^2} = \frac{\tau_\tau}{\tau_\mu}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau}\frac{1}{BR\left(\tau^- \to e^- \nu_\tau \bar{\nu}_e\right)} \qquad \Rightarrow \qquad \frac{g_\mu}{g_\tau} = 1.001 \pm 0.003$$

These properties are evident for leptons, but not at all for quarks (quark mixing – CKM matrix).

π^- Decay: $\pi^- \to \mu^- + \bar{\nu}_\mu$





$\bar{\nu}_\mu$ $(\omega,\mathbf{k}) \to \nu_\mu$ $(-\omega,-\mathbf{k})$

Compare π decays to e and μ

$$\boxed{\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = (1.228 \pm 0.022) \times 10^{-4}}$$

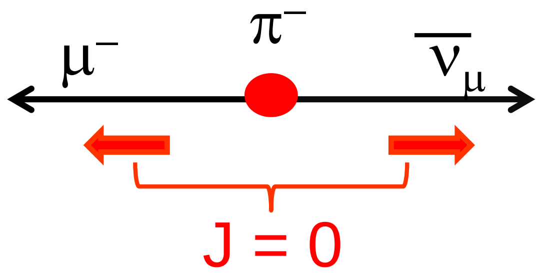
The pion decays by far more likely to a muon rather than to an electron,
while phase space (p_e = 70 MeV, p_μ = 30 MeV) would favor the decay to electrons, the lighter particles.

From phase space $\Gamma \propto p_l = \dfrac{m_\pi^2 - m_l^2}{m_\pi}$ and $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \dfrac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} = 2.34$

There must be some dynamics involved (not only kinematics) to explain this!

Helicity Analysis

According to V – A theory the muons from pion decays are lefthanded and the anti-neutrinos are righthanded.



Angular momentum conservation J = 0 requires the electron (muon) to have positive (*wrong*) helicity, which is forbidden in the limit $m_\mu = 0$.

For $m_\mu \neq 0$, the chirality eigenstates contain also a small component with the *wrong* helicity:

$$\psi_L = \frac{1}{2}\left(1 - \frac{\vec{p}\cdot\vec{\sigma}}{E+m}\right)\psi = \frac{1}{2}\left(1 - \frac{p_z}{E+m}\right)\varphi_{+1/2} + \frac{1}{2}\left(1 + \frac{p_z}{E+m}\right)\varphi_{-1/2}$$

$$\psi_L = \frac{1}{2}\frac{m}{E}\varphi_{+1/2} + \varphi_{-1/2}$$

It is this component that enters into the pion decay.

Probability for a given chirality eigenstate to have the *wrong* helicity: $\frac{1}{4}\frac{m^2}{E^2}$

and $\boxed{\frac{\Gamma(\pi^- \to e^-\overline{\nu}_e)}{\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.28\times 10^{-4}}$ (cfr. experiment !)

So we understand the pion decay!

Invariant Amplitude

$$\boxed{M_{fi} = \frac{G_F}{\sqrt{2}} \langle 0 | \bar{d} \gamma^\mu (1-\gamma^5) u | \pi^+ \rangle \bar{u}(p) \gamma_\mu (1-\gamma^5) v(k)}$$

The weak quark current ≠ "usual form" because quarks are bounded in the π meson, and we don't know how to calculate exactly the quark vertex.

Lorentz covariance → must be a four-vector

π spinless ⇒ only q^μ (= $p^\mu + k^\mu$), no spin (with $q^2 = m_\pi^2$)

most general form for π current $$\boxed{(\cdots\cdots)^\mu \to -q^\mu f_\pi(q^2) = -q^\mu f_\pi(m_\pi^2) = -q^\mu f_\pi}$$

f_π: π disintegration constant (can depend only on q^2, the only scalar available)
f_π = 130 MeV ~ m_π

The matrix element of a vector current between a pion state (pseudo-scalar) and the vacuum is a pseudo four-vector and likewise the matrix element of an axial current between a pion state and the vacuum is a four-vector.
Since q^μ is a true four-vector, and there is no pseudo-vector at our disposal like spin, the two separate contributions are

$$\langle 0 | \bar{d} \gamma^\mu u | \pi^+ \rangle = 0 \quad \text{and} \quad \langle 0 | \bar{d} \gamma^\mu \gamma^5 u | \pi^+ \rangle = f_\pi q^\mu$$

Therefore all contributions come from the axial current.

The invariant amplitude is then given by

$$M_{fi} = \frac{G_F}{\sqrt{2}}(p^\mu + k^\mu) f_\pi \left[\bar{u}(p)\gamma_\mu(1-\gamma^5)v(k)\right]$$

$$= \frac{G_F}{\sqrt{2}} f_\pi \bar{u}(p)(\not{p} + \not{k})(1-\gamma^5)v(k)$$

use Dirac eq. $\quad k^\mu\gamma_\mu v(k) = 0 \quad$ and $\quad \bar{u}(p)(p^\mu\gamma_\mu - m_\mu) = 0$

$$= \frac{G_F}{\sqrt{2}} f_\pi m_\mu \left[\bar{u}(p)(1-\gamma^5)v(k)\right]$$

The initial state has spin 0, sum only over final spin states

$$\overline{\left|M_{fi}\right|^2} = \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \sum_{s,t}\left[\bar{u}^s(p)\gamma_\mu(1-\gamma^5)v^t(k)\right]\times\left[\bar{v}^t(k)\gamma_\mu(1+\gamma^5)u^s(p)\right]$$

spin sums $\quad \sum_{\text{spins}} u_\delta^s \bar{u}_\alpha^s = (\not{p} + m_\mu)_{\delta\alpha} \qquad \sum_{\text{spins}} u_\beta^t \bar{u}_\gamma^t = (\not{k} + m_\nu)_{\beta\gamma} = \not{k}_{\beta\gamma}$

$$= \frac{G_F^2}{2} f_\pi^2 m_\mu^2 \mathrm{Tr}\left\{(\not{p} + m_\mu)(1-\gamma^5)\not{k}(1+\gamma^5)\right\}$$

$$= \frac{G_F^2}{2} f_\pi^2 m_\mu^2 4(p\cdot k)\ 2$$

p = – **k** $\Rightarrow$ p·k = $\omega(E + \omega)$

$$\overline{\left|M_{fi}\right|^2} = 4G_F^2 f_\pi^2 m_\mu^2 \omega\ (E+\omega)$$

with the phase space factor $\quad dQ = \frac{d^3p}{(2\pi)^3 2E}\frac{d^3k}{(2\pi)^3 2\omega}(2\pi)^4\delta^4(q-p-k)$

π Transition Rate

$$\boxed{d\Gamma = \frac{1}{2m_\pi}\overline{\left|M_{fi}\right|^2}dQ}$$

Putting everything together we obtain

$$d\Gamma = \frac{1}{2m_\pi}4G_F^2 f_\pi^2 m_\mu^2 \omega\,(E+\omega)\frac{d^3p}{(2\pi)^3 2E}\frac{d^3k}{(2\pi)^3 2\omega}(2\pi)^4\delta^4(q-p-k)$$

The π transition rate is given by

$$\Gamma = \frac{4G_F^2 f_\pi^2 m_\mu^2}{2m_\pi}\int\frac{d^3p}{(2\pi)^3 2E}\int\frac{d^3k}{(2\pi)^3 2\omega}\;\omega(E+\omega)(2\pi)^4\underbrace{\delta^4(q-p-k)}_{\delta(m_\pi - E-\omega)\delta^3(\vec{p}+\vec{k})}$$

After integration over d³*p*

$$\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2}{2m_\pi(2\pi)^2}\int\frac{d^3k}{E}(E+\omega)\delta(m_\pi - E-\omega)$$

$$d^3k = 4\pi\omega^2 d\omega$$

$$= \frac{G_F^2 f_\pi^2 m_\mu^2}{2m_\pi(2\pi)^2}4\pi\int\omega^2 d\omega\;(1+\omega/E)\;\delta(m_\pi - E-\omega)$$

and integration over dω we arrive finally at

$$\boxed{\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2}{2\pi m_\pi}\left[\frac{m_\pi^2 - m_\mu^2}{2m_\pi}\right]^2 = \frac{G_F^2 f_\pi^2 m_\mu^2}{8\pi}m_\pi\left[1-\left(\frac{m_\mu}{m_\pi}\right)^2\right]^2}$$

π Lifetime and Branching Ratios

Lifetime

f_π is the only unknown, let's guess $f_\pi = m_\pi$

$\rightarrow \Gamma = 4.41 \times 10^7\ \mathrm{s}^{-1}$ $\rightarrow \tau = 22.6$ ns assuming $BR(\pi \rightarrow \mu\nu) \sim 100\%$

$\tau = 26.03 \pm 0.23$ ns (exp.)

but $f_\pi = m_\pi$ was a guess!

Branching ratios

$$\frac{\Gamma(\pi^- \rightarrow e^-\bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^-\bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.28 \times 10^{-4} \rightarrow 1.24 \times 10^{-4}$$ after radiative corrections

$$\frac{\Gamma(\pi^- \rightarrow e^-\bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^-\bar{\nu}_\mu)} = (1.228 \pm 0.022) \times 10^{-4}$$ experimental

Compare this result with previous calculations based on helicity arguments only, i.e. 1.24×10^{-4} vs. 1.28×10^{-4})!

Note that in a scalar theory $$\frac{\Gamma(\pi^- \rightarrow e^-\bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^-\bar{\nu}_\mu)} = 5.5$$

This is yet another proof of parity violation in weak decays.

The V – A theory explains well the charged pion preference to decay to muons rather than electrons despite the unfavorable phase space (p_e = 70 MeV, p_μ = 30 MeV)

For Next Week

Study the material and prepare / ask questions
Study ch. 12 (sec. 3, 5,6) in Halzen & Martin
and / or ch. 11 (sec. 6) and ch. 12 (sec. 1) in Thomson

Do the homeworks

Next week we will study the quark mixing and the CKM matrix
- have a first look at the lecture notes, you can already have questions
- read ch. 12 (sec. 11 to 14) in Halzen & Martin
- and / or ch. 12 (sec. 2 to 5) in Thomson