Advanced Particle Physics 2

Strong Interactions and Weak Interactions

L9 –Quark Mixing and CP Violation

(http://dpnc.unige.ch/~bravar/PPA2/L9)

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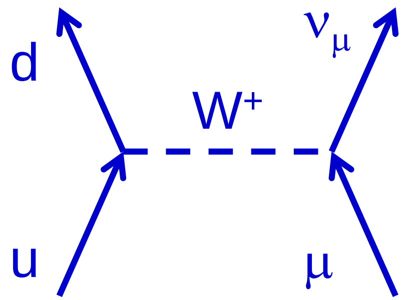
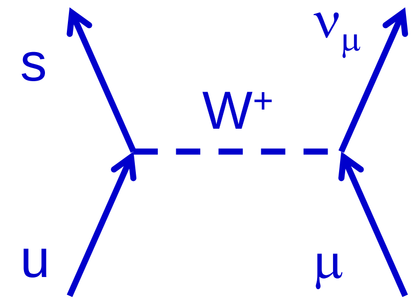
Weak Decays of Strange Particles

π^+ decay
"d" decay
$\Delta S = 0$



K^+ decay
"s" decay
$\Delta S = 1$



experimentally $$\frac{\Gamma\left(K^+ \to \mu^+ \nu_\mu\right)}{\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right)} = \frac{\tau_\pi BR\left(K^+ \to \mu^+ \nu_\mu\right)}{\tau_K BR\left(\pi^+ \to \mu^+ \nu_\mu\right)} = \frac{2.60\times 10^{-8}s \;\cdot 63.6\%}{1.24\times 10^{-8}s \;\cdot 100\%} = 1.33$$

from phase space expect K^+ to decay much faster

$$\frac{\Gamma\left(K^+ \to \mu^+ \nu_\mu\right)}{\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right)} \approx \frac{f_K}{f_\pi} \cdot \frac{\left(\Delta m_K\right)^5}{\left(\Delta m_\pi\right)^5} = \frac{f_K}{f_\pi} \cdot \frac{m_K\left[1-\left(m_\mu / m_K\right)^2\right]^2}{m_\pi\left[1-\left(m_\mu / m_\pi\right)^2\right]^2} = \frac{f_K}{f_\pi} \cdot 8.06 \sim 10$$

$K^+ \to \mu^+ \nu_\mu$ decay suppressed w.r.t. $\pi^+ \to \mu^+ \nu_\mu$ decay ~10 times
same also for $\Lambda \to p e^- \bar{\nu}_e$ compared to $n \to p e^- \bar{\nu}_e$

$\Delta S = 1$ decays suppressed w.r.t. $\Delta S = 0$ decays ~10 times
(i.e. s quark decay suppressed w.r.t. d quark decay)

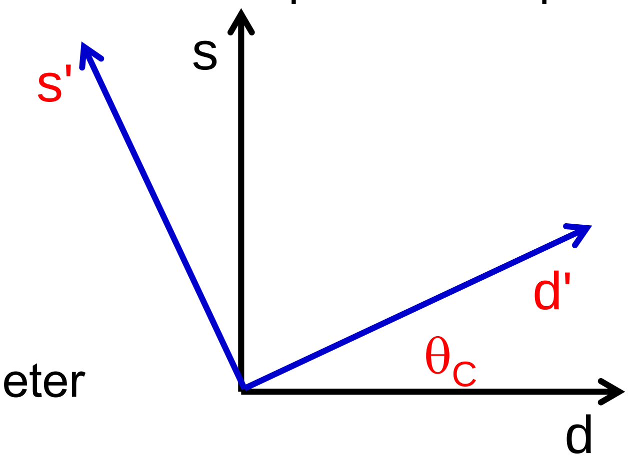
Weak decay universality ??? Do we need 3 different couplings $G_F(\mu)$, $G_F(\beta)$, $G_F(s)$?

Cabibbo Hypothesis (1963)

The "weak" eigenstates are different from mass and flavor eigenstates, and the Weak Interaction couples a "mix" (linear) combination of d and s quarks to u quarks

$$d' = \cos\vartheta_C \, d + \sin\vartheta_C \, s$$

$$s' = -\sin\vartheta_C \, d + \cos\vartheta_C \, s$$



rotation between quarks of charge -1/3 !

where θ_C is the Cabibbo mixing angle ~13.1⁰

To "save" universality, Cabibbo introduced a new parameter that needs to be determined experimentally

The transition amplitude

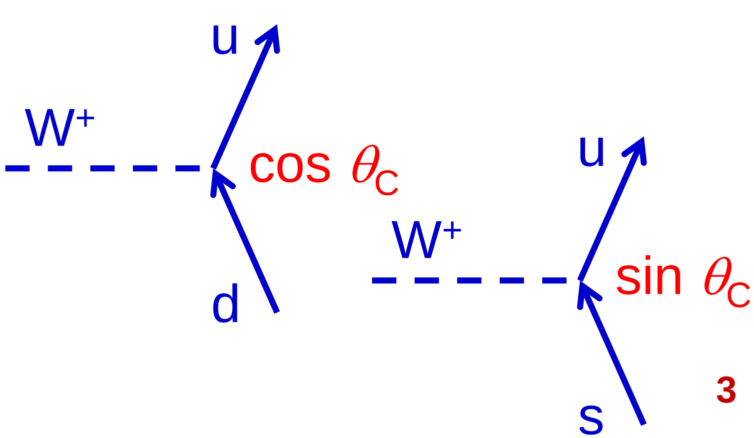
$$M = G_F \left(\bar{e}_L \gamma_\mu \nu_L\right)\left(\bar{d}_L \gamma^\mu u_L\right)$$

is then rewritten as

$$M = G_F \left(\bar{e}_L \gamma_\mu \nu_L\right)\left(\bar{d}'_L \gamma^\mu u_L\right)$$

ΔS = 0 $$= G_F \cos\vartheta_C \left(\bar{e}_L \gamma_\mu \nu_L\right)\left(\bar{d}_L \gamma^\mu u_L\right)$$

ΔS = 1 $$+ G_F \sin\vartheta_C \left(\bar{e}_L \gamma_\mu \nu_L\right)\left(\bar{s}_L \gamma^\mu u_L\right)$$



The decay rate can be rewritten accordingly

$$\frac{\Gamma\left(K^+ \to \mu^+ \nu_\mu\right)}{\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right)} \sim \frac{\sin^2 \vartheta_C}{\cos^2 \vartheta_C} \sim \frac{1}{20}$$

or

$$\frac{\Gamma\left(K^+ \to \pi^0 e^+ \nu\right)}{\Gamma\left(\pi^+ \to \pi^0 e^+ \nu\right)} \sim \frac{\sin^2 \vartheta_C}{\cos^2 \vartheta_C}$$

$$\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right) \sim \cos^2 \vartheta_C$$

Cabibbo "favored" decay

$$\Gamma\left(K^+ \to \mu^+ \nu_\mu\right) \sim \sin^2 \vartheta_C$$

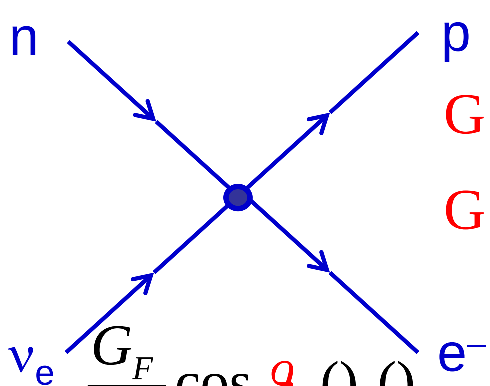
Cabibbo "suppressed" decay

Extraction of θ_C is not strightforward because one needs to take into account also long range strong interaction effects (quarks are not free).

$$\boxed{\vartheta_C = 13.1^0 \qquad \cos\vartheta_C = 0.974 \qquad \sin\vartheta_C = 0.221}$$

β decay

unchanged except for $G_F \to G_F \cos\theta_C$



$$\frac{G_F}{\sqrt{2}} \cos\vartheta_C \, ()_l \, ()_{had}$$

$$G_F(\mu) = 1.1663787 \pm 0.0000006 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$G_F(\beta) = 1.136 \pm 0.003 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$G_\beta = G_F \cos\vartheta_C \qquad \cos\vartheta_C = \frac{G_\beta}{G_F} \approx 0.974$$

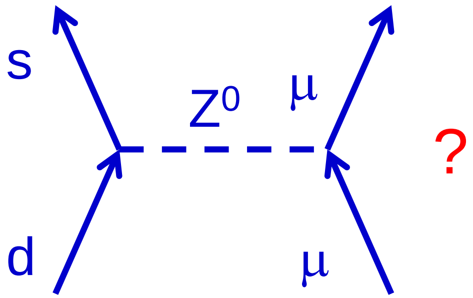
Recovered universality of Weak Interactions (decays)!

but had to introduce a new parameter θ_C (nothing comes for free).

What about s' ?

According to Cabibbo theory the Weak Interaction couples d' to u.
What about the state s'? It couples to what? (answer: c quark, but in 1963 not clear at all)

Let's consider (again)



$$K^0_{"L"} \to \mu^+ \mu^- \qquad K^0(d\bar{s}) \quad J^{PC} = 0^{-+}$$

and also some other K decays:

$$\frac{\Gamma\left(K^0_L \to \mu^+\mu^-\right)}{\Gamma\left(K^0_L \to all\right)} \sim 10^{-8} \qquad \frac{\Gamma\left(K^+ \to \pi^+\nu_e\bar{\nu}_e\right)}{\Gamma\left(K^+ \to all\right)} \sim 10^{-10}$$

"Neutral Current" $\Delta S = 1$ (heavily suppressed!)

$$\frac{\Gamma\left(K^+ \to \mu^+\nu_\mu\right)}{\Gamma\left(K^+ \to all\right)} \sim 0.64 \qquad \frac{\Gamma\left(K^+ \to \pi^0 e^+\nu_e\right)}{\Gamma\left(K^+ \to all\right)} \sim 0.05$$

Charged Current $\Delta S = 1$ (not suppressed!)

K^+ decays $K^+ \to \pi^+\nu_e\bar{\nu}_e$ at quark level $\bar{s} \to \bar{d} + Z^0 \quad (Z^0 \to \nu_e\bar{\nu}_e)$ not allowed !

$K^+ \to \pi^0 e^+\nu_e$ $\qquad \bar{s} \to \bar{u} + W^+ \quad (W^+ \to e^+\nu_e)$ allowed

Let's study the NC d' quark current (for simplicity assume $g_V = -g_A = 1$)

$$\boxed{\begin{aligned}\bar{d}'_L\gamma^\mu d'_L &= \bar{d}_L\gamma^\mu d_L \cos^2\vartheta_C + \bar{s}_L\gamma^\mu s_L \sin^2\vartheta_C \\ &+\left[\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L\right]\cos\vartheta_C \sin\vartheta_C\end{aligned}}$$

$\Delta S = 0$

$|\Delta S| = 1$ experimentally heavily suppressed

GIM Mechanism (1970)

Couple s' to a new quark (charm) to cancel the NC |ΔS = 1| component (Glashow, Iliopulos, and Maiani, 1970, before discovery of charm!)
"Full" Cabibbo theory

$$\boxed{\begin{pmatrix} d' \\ s' \end{pmatrix} = V \begin{pmatrix} d \\ s \end{pmatrix} \qquad V = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C \\ -\sin\vartheta_C & \cos\vartheta_C \end{pmatrix} \qquad d'_i = \sum_j V_{ij} d_j}$$

V – quark mixing matrix (unitary $VV^\dagger = V^\dagger V = 1$ but not necessarily hermitian)
In the case of 4 quarks, we have only 1 real parameter θ_C and $V = V^\dagger$.

The charge rising weak current becomes

$$J_\mu^{CC} = \bar{u}_L \gamma_\mu V d_L = \left(\bar{u}_L, \bar{c}_L\right) \gamma_\mu \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C \\ -\sin\vartheta_C & \cos\vartheta_C \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}$$

and the full NC with 4 quarks is

$$J_\mu^{NC} = \bar{u}_L \gamma_\mu u_L + \bar{d}'_L \gamma_\mu d'_L + \bar{c}_L \gamma_\mu c_L + \bar{s}'_L \gamma_\mu s'_L$$

$$= \bar{u}_L \gamma_\mu u_L + \bar{c}_L \gamma_\mu c_L + \left(\bar{d}_L \gamma_\mu d_L + \bar{s}_L \gamma_\mu s_L\right)\cos^2\vartheta_C + \left(\bar{d}_L \gamma_\mu d_L + \bar{s}_L \gamma_\mu s_L\right)\sin^2\vartheta_C$$

$$\left.\begin{aligned} &+\left(\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L\right)\sin\vartheta_C \cos\vartheta_C \\ &-\left(\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L\right)\sin\vartheta_C \cos\vartheta_C \end{aligned}\right\} = 0!$$

(perfect cancellation)

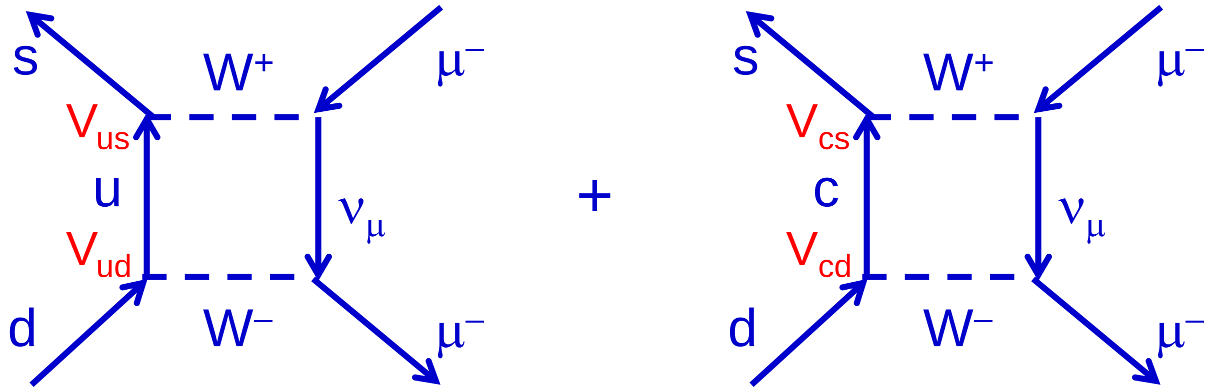
There are no flavor changing neutral currents (FCNC)

flavor changes allowed only within "families" via charged currents

$$\boxed{\begin{pmatrix} u \\ d' \end{pmatrix}\updownarrow \;\not\leftrightarrow\; \begin{pmatrix} c \\ s' \end{pmatrix}\updownarrow}$$

However $K^0_{"L"} \to \mu^+\mu^- = 6.8\times 10^{-9} \neq 0$ (tough very small!)

Decay proceeds via higher order diagrams



$M \sim +g_W^4 \sin\vartheta_C \cos\vartheta_C$ $\qquad$ $M \sim -g_W^4 \sin\vartheta_C \cos\vartheta_C$

$$\boxed{M = M_u + M_c \neq 0}$$

Not perfect cancellation due to different masses of u and c quarks $\to$ $m_c \sim 1 - 3$ GeV

Similar explanation for $B^0 \to \mu^+\mu^-,\;\; B_s^0 \to \mu^+\mu^-$

No FCNC with $\Delta B = 1$, however higher order diagrams, involving u, c, and t quarks, do not cancel exactly

measured and in agreement with SM predictions!

$$BR\left(B^0 \to \mu^+\mu^-\right) = 3.9\times 10^{-10}$$
$$BR\left(B_s^0 \to \mu^+\mu^-\right) = 2.9\times 10^{-9}$$

Charm Meson Decays

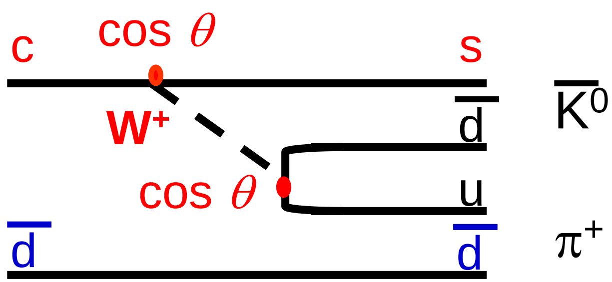
Charm discovered in 1974 $J/\psi(c\bar{c})$

Charm mesons in ~1975 $D^0(c\bar{u})$ $\bar{D}^0(\bar{c}u)$ $D^+(c\bar{d})$ $D^-(\bar{c}d)$ $D_s^+(c\bar{s})$ $D_s^-(\bar{c}s)$

m = 1.864 1.869 1.968 GeV

Cabibbo favored decay

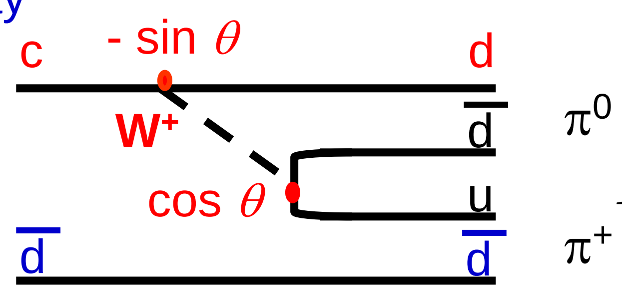
$$D^+ \to \pi^+ \bar{K}^0$$



$$M(c \to s u \bar{d}) \sim \cos^2 \vartheta_C$$

Cabibbo suppressed decay

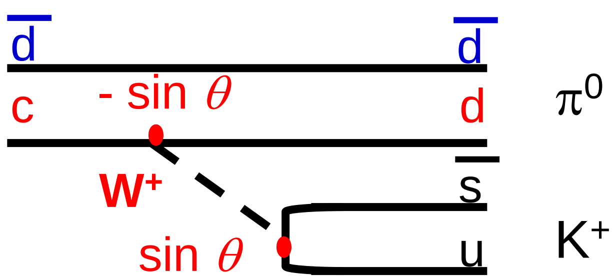
$$D^+ \to \pi^+ \pi^0$$



$$M(c \to d u \bar{d}) \sim -\sin\vartheta_C \cos\vartheta_C$$

Cabibbo doubly suppressed decay

$$D^+ \to K^+ \pi^0$$



$$M(c \to d u \bar{s}) \sim -\sin^2 \vartheta_C$$

Verify the B.R. in the PDG!

Kobayashi – Maskawa Mixing Matrix (1973)

Generalization to N families of the Cabibbo mixing by Kobayashi and Maskawa in 1973.
The quark mass eigenstates are different from the "weak" eigenstates.

Extension to 3 families (6 quarks)

b quark discovered in 1977
t quark discovered in 1994

2008

flavor mixing

$$\boxed{d_i' = \sum_j V_{ij} d_j}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates CKM matrix mass (flavor) eigenstates

By convention CKM matrix defined as acting on quarks of charge -1/3

V – 3 × 3 complex matrix $\rightarrow$ 18 real parameters

V – unitary $V^\dagger V = VV^\dagger = 1$ $\rightarrow$ 9 real parameters

6 arbitrary phases (5 can be absorbed in quarks' fields)

$\Rightarrow$ 4 free real parameters: 3 angles + 1 phase δ

have to be determined from experiment $\rightarrow$ 4 additional SM parameters

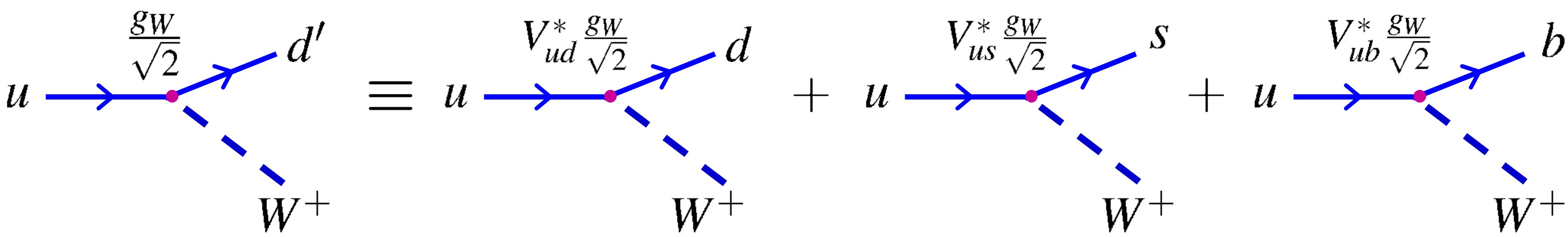
the phase cannot be reabsorbed in the definition of V $\Rightarrow$ V is not real

V is not hermitian $\rightarrow V \neq V^\dagger \Rightarrow M \neq M^\dagger \Rightarrow M_{fi} \neq M_{if} \Rightarrow$ violates T (and CP) (for $\delta \neq n\pi$)

Unitarity of V, i.e. $VV^\dagger = 1$, guarantees that there are no FCNC.

The CKM matrix generalizes the 4 quarks case with 1 single parameter, the Cabibbo angle, to 6 quarks.

The weak eigenstate d' is produced in e.g. weak flavour transitions of an up quark:



Depending on the order of the interaction, $u \rightarrow d$ or $d \rightarrow u$, the CKM matrix enters as either V_{ud}^* or V_{ud} .
Hence, when a quark of charge -1/3 enters as the adjoint spinor, the complex conjugate of the CKM matrix is used.

Within the SM the weak charged current interaction mediated by the $W^{\pm}$ exchange:

- provides the only way to change flavor !
- only way to change from one generation of quarks or leptons to another !

The CKM matrix is almost diagonal (off diagonal elements small):

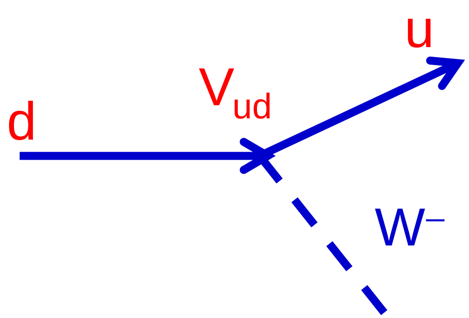
- weak interaction largest between quarks of the same generation
- coupling between first and third generation quarks is very small
- very different from the PNSM neutrino mixing matrix

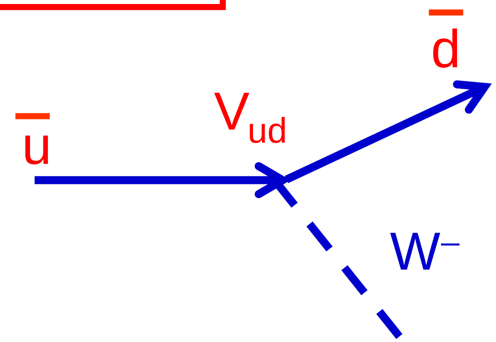
Feynman Rules

charge rising weak current

$$u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad d = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

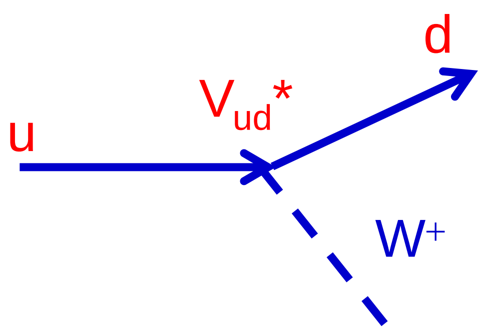
$$\boxed{J_\mu^{CC}(q_i) = \bar{u}_i \frac{1}{2}\gamma_\mu(1-\gamma_5)V_{ij}d_j}$$

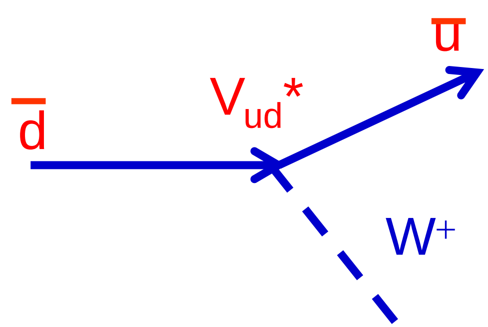




charge lowering weak current $(\bar{d}' = d'^\dagger\gamma^0 \to \left(V_{ij}d_j\right)^\dagger \gamma^0 = V_{ji}^* d^\dagger\gamma^0 = V_{ji}^*\bar{d})$

$$\boxed{\left(J_\mu^{CC}\right)^\dagger(q_i) = \bar{d}_j V_{ji}^* \frac{1}{2}\gamma_\mu(1-\gamma_5)u_i}$$





CKM Matrix Parametrization

$$|V_{CKM}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.220 & 0.995 & 0.041 \\ 0.008 & 0.040 & 0.999 \end{pmatrix}$$

$$\boxed{\begin{aligned} \vartheta_{12} &= 13.1^0 \approx \vartheta_C \\ \vartheta_{23} &= 2.4^0 \\ \vartheta_{13} &= 0.2^0 \\ \delta_{CP} &= 68^0 \end{aligned}}$$

Almost diagonal: transitions within same generation are favored
transitions between generations are suppressed

Standard parametrization, using Euler angles (exact to all orders)

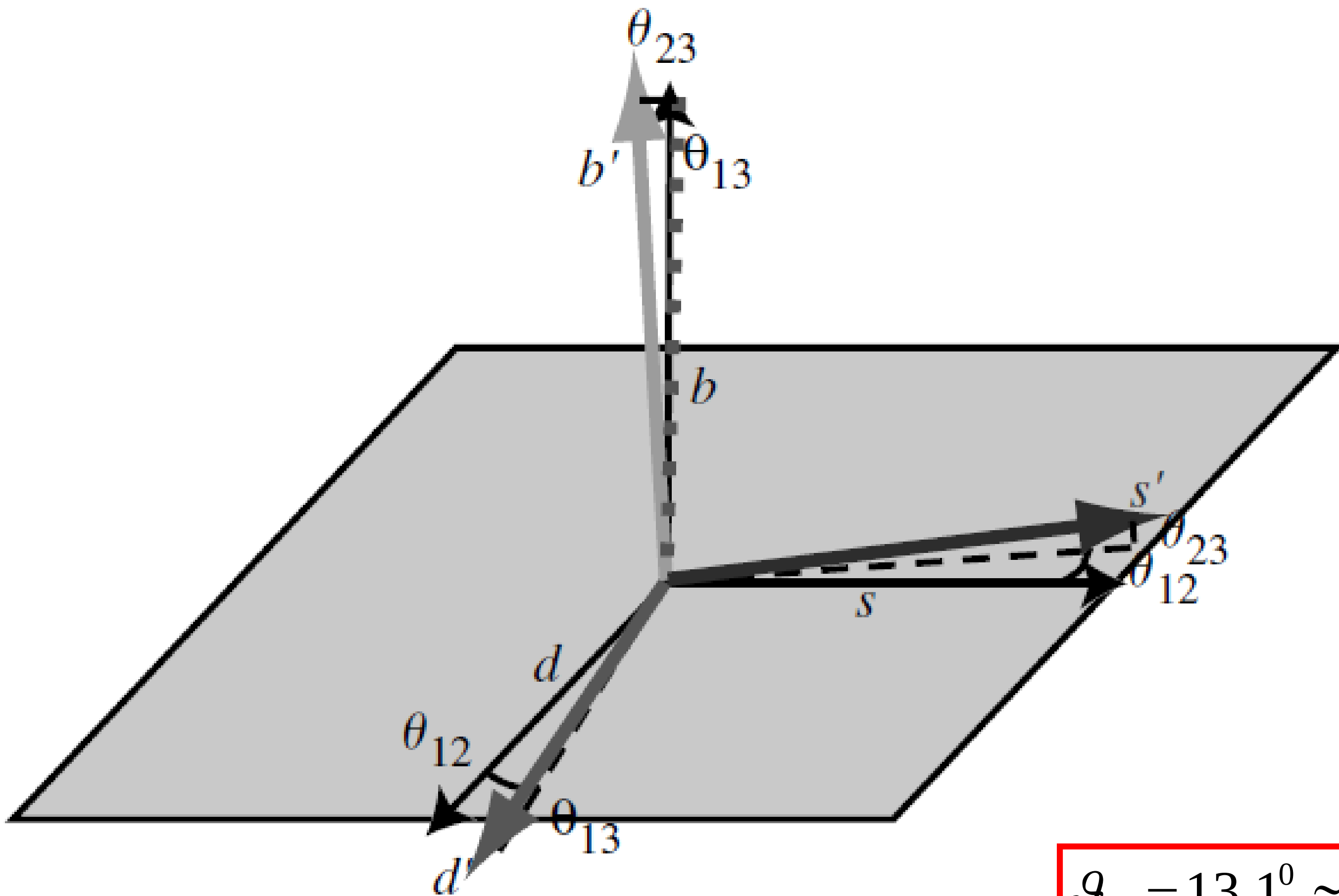
$s_{ij} = \sin \theta_{ij}$ $c_{ij} = \cos \theta_{ij}$

$$V_{CKM} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{"c \to b"} \cdot \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}}_{"u \to b"} \cdot \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{"u \to d"}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

CP effects are small even tough $\delta_{CP} \sim 68^0$ ($\sin \theta_{13} \sim 0.003$)

CKM "Euler" Angles



rotate around "b" axis by θ_{12} $\rightarrow$ d* & s* axes

rotate around "s*" axis by θ_{13} $\rightarrow$ d' & b* axes

rotate around d' axis by θ_{23} $\rightarrow$ s' & b' axes

$$\vartheta_{12} = 13.1^0 \approx \vartheta_C$$

$$\vartheta_{23} = 2.4^0$$

$$\vartheta_{13} = 0.2^0$$

$$\delta_{CP} = 68^0$$

Unitarity of the CKM Matrix

Unitarity requires that the diagonal elements of $VV^\dagger$ and $V^\dagger V$ are equal to one, and that the off-diagonal elements are equal to zero:

$$\sum_i V_{ij} V_{ik}^* = \delta_{jk} \qquad \sum_j V_{ij}^* V_{kj} = \delta_{ik}$$

Can build 6 conditions by multiplying V with $V^\dagger$ and $V^\dagger$ with V for the diagonal:

$VV^\dagger = 1 \Rightarrow$ $\quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \approx 0.9985 \pm 0.0005$ (1st row)

$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \approx 1.025 \pm 0.022$ (2nd row)

$V^\dagger V = 1 \Rightarrow$ $\quad |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \approx 0.9970 \pm 0.0018$ (1st column)

$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \approx 1.026 \pm 0.0022$ (2nd column)

The CKM elements involving the top quark are much less well measured.
Any deviation from 1 would indicate new physics (new particles, more families, FCNC, …)

Similarly we can build six constraints for the off-diagonal elements (must be zero), i.e. in the b sector

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Unitarity Triangle

Consider the off-diagonal elements of $VV^\dagger$ in the b sector. The condition

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td}\, V_{tb}^* = 0$$

can be visualized as the equation of a triangle $ABC: \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

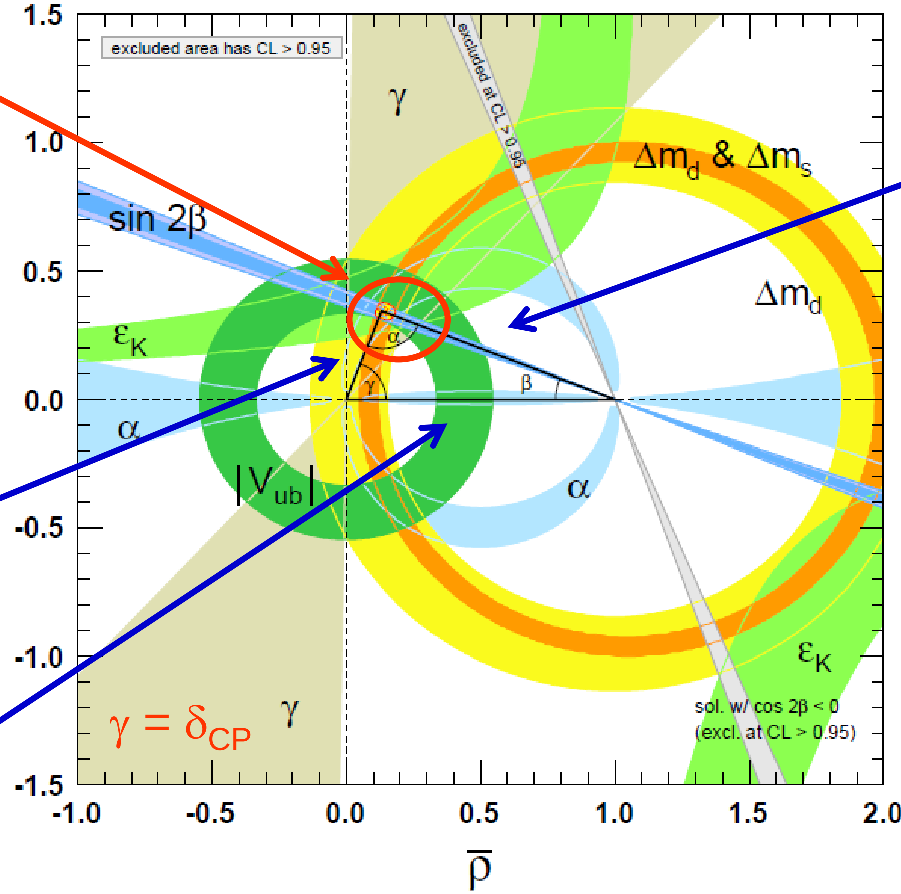
divide each side by the best measured one $V_{cd}V_{cb}^*$ to obtain the unitarity triangle

is the triangle closed ?

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}\, V_{ub}^*}{V_{cd}\, V_{cb}^*}$$

$$\left|\frac{V_{ud}\, V_{ub}^*}{V_{cd}\, V_{cb}^*}\right|$$

$$\left|\frac{V_{cd}\, V_{cb}^*}{V_{cd}\, V_{cb}^*}\right|$$



$$\left|\frac{V_{td}\, V_{tb}^*}{V_{cd}\, V_{cb}^*}\right|$$

$$\alpha = \arg\left(-\frac{V_{td}\, V_{tb}^*}{V_{ud}\, V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}\, V_{cb}^*}{V_{td}\, V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}\, V_{ub}^*}{V_{cd}\, V_{cb}^*}\right)$$

$$\alpha + \beta + \gamma = \left(179^{+7}_{-6}\right)^0$$

Measurement of CKM Matrix Elements

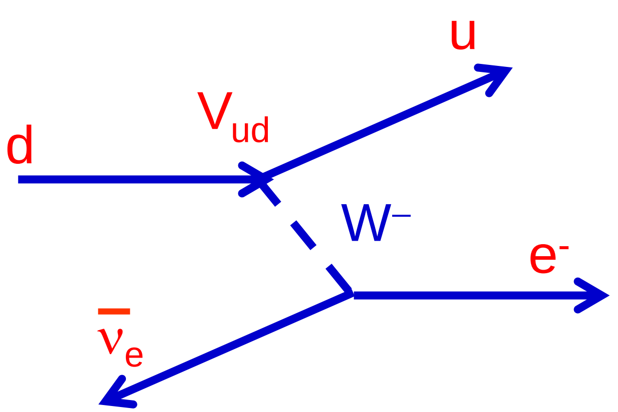
The measurement of the CKM matrix elements is an intense area of research: deviations from unitarity $\Rightarrow$ BSM physics

The experimental determination of the CKM matrix elements comes from measurements of semi-leptonic decays of mesons and ν – DIS.

It is easy to produce mesons and observe their decays, however theoretical uncertainties associated with the decays of bound states (long range strong force) often limits the precision.

$|V_{ud}|$ from nuclear beta decays

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



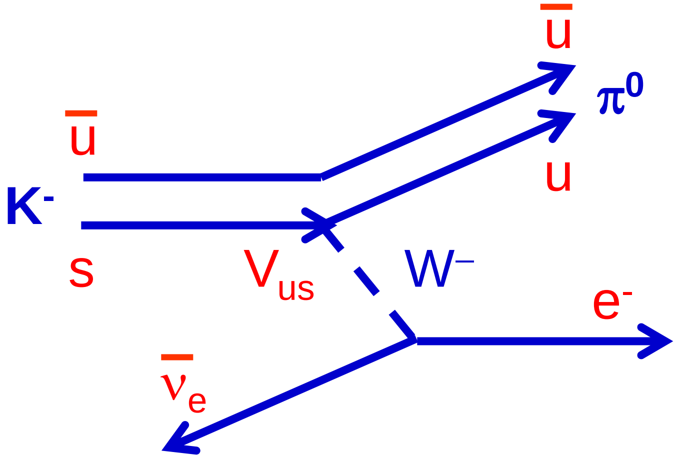
super-allowed $0^+ \rightarrow 0^+$ beta decays (pure vector transitions) are relatively free from theoretical uncertainties

$$\boxed{\Gamma \propto |V_{ud}|^2}$$

$$|V_{ud}| = 0.97417 \pm 0.00021 \quad (\approx \cos\theta_C)$$

$|V_{us}|$ from semi-leptonic Kaon decays

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



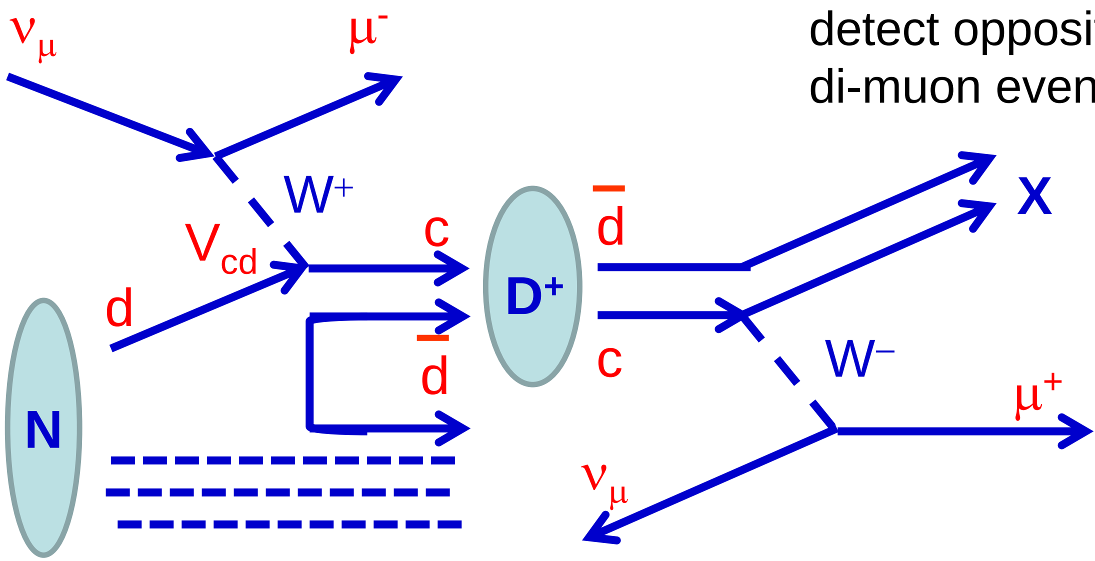
$$\boxed{\Gamma \propto |V_{us}|^2}$$

$$|V_{us}| = 0.2248 \pm 0.0006 \quad (\approx \sin\theta_C)$$

$|V_{cd}|$ from neutrino scattering (charm production off d quarks)

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

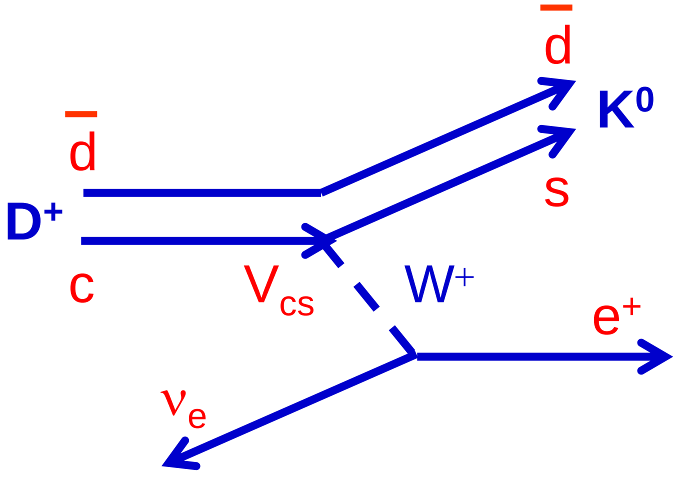
detect oppositely charged
di-muon events ($\mu^- \mu^+$ pair)



$$\boxed{\Gamma \propto |V_{cd}|^2 \, BR(D^+ \to X \mu^+ \nu_\mu)}$$

$$|V_{cd}| = 0.220 \pm 0.005$$

$|V_{cs}|$ from semi-leptonic charmed meson decays



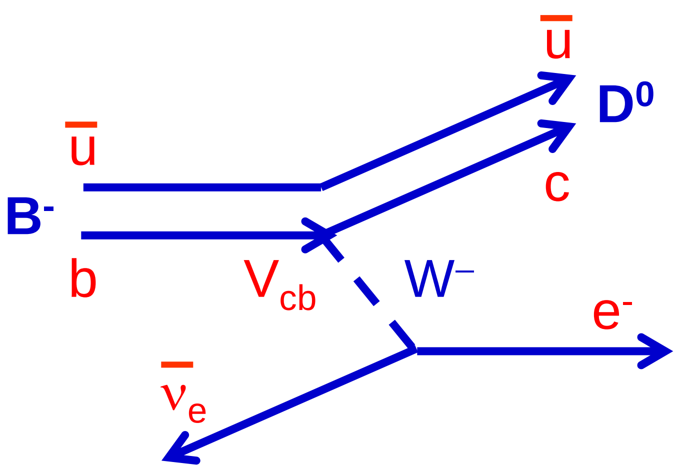
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\boxed{\Gamma \propto |V_{cs}|^2}$$

$$|V_{cs}| = 0.995 \pm 0.016$$

(precision limited from theoretical uncertainties)

$|V_{cb}|$ from semi-leptonic B meson decays



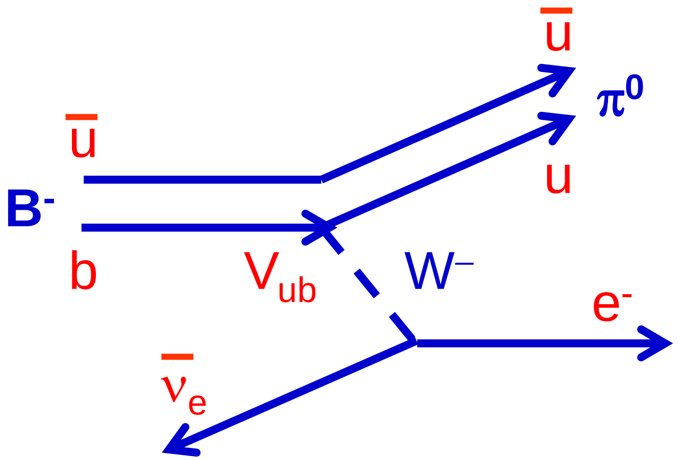
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\boxed{\Gamma \propto |V_{cb}|^2}$$

$$|V_{cb}| = 0.0405 \pm 0.0015$$

(can use Heavy Quark Effective Theories – HQET to model the heavy meson wave function)

$|V_{ub}|$ from semi-leptonic B meson decays

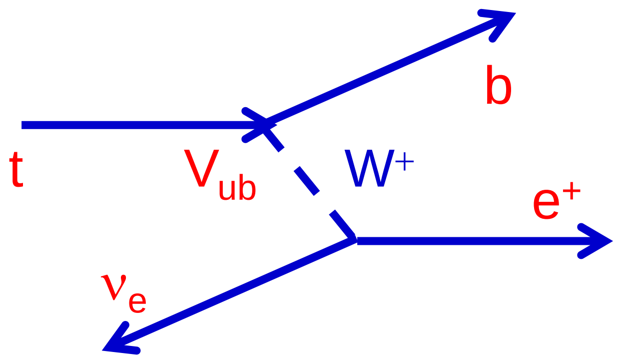


$$\boxed{\Gamma \propto |V_{ub}|^2}$$

$$|V_{ub}| = 0.00409 \pm 0.00039$$

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$|V_{tb}|$ from top quark decays (single top quark production)



$$\boxed{\Gamma \propto |V_{tb}|^2}$$

$$|V_{tb}| = 0.999 \pm 0.003$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \end{pmatrix}$$

$|V_{td}|$ and $|V_{ts}|$ from B – $\bar{B}$ oscillations and rare K and B decays involving loops

$$|V_{td}| = 0.0082 \pm 0.0006 \qquad |V_{ts}| = 0.0400 \pm 0.0027$$

The $K_0 - \overline{K}_0$ System

Let's come back to the K^0 mesons (they belong to the meson octet with $J^{PC} = 0^{-+}$, see L2). The neutral kaons are distinct eigenstates of the strong interaction with definite strangeness:

$$K^0 = K^0(d\bar{s}) \quad S = +1 \qquad\qquad K^+ = K^+(u\bar{s})$$

forms isospin doublet with

$$\bar{K}^0 = \bar{K}^0(s\bar{d}) \quad S = -1 \qquad\qquad K^- = K^-(s\bar{u})$$

One is the antiparticle of the other (not its own!).

Neutral kaons are produced in strong interactions, e.g.

$$\pi^-(\bar{u}d) + p(uud) \to K^0(d\bar{s}) + \Lambda(uds)$$

$$\pi^+(u\bar{d}) + p(uud) \to \bar{K}^0(\bar{d}s) + K^+(u\bar{s}) + p(uud)$$

(different thresholds)

They decay via weak interaction (no conservation of P, C, nor strangeness), e.g.

$$K^0 \to \pi^+\pi^- \qquad\qquad \bar{K}^0 \to \pi^+\pi^-$$

In reality, more subtle than this: these states with definite flavor differ from the states with definite lifetime and mass, i.e. the stationary states.

The Physics of flavored electrically neutral meson-antimeson pairs is an example of quantum two-state systems. There are four such meson doublets:

$$K^0(d\bar{s}) - \bar{K}^0(\bar{d}s);\ \ B^0(d\bar{b}) - \bar{B}^0(\bar{d}b);\ \ B_s^0(s\bar{b}) - \bar{B}^0(\bar{s}b);\ \ D^0(c\bar{u}) - \bar{D}^0(\bar{c}u)$$

$K^0 - \overline{K}^0$ Oscillations

Gell-Mann and Pais (1955): since they can decay to the same final state

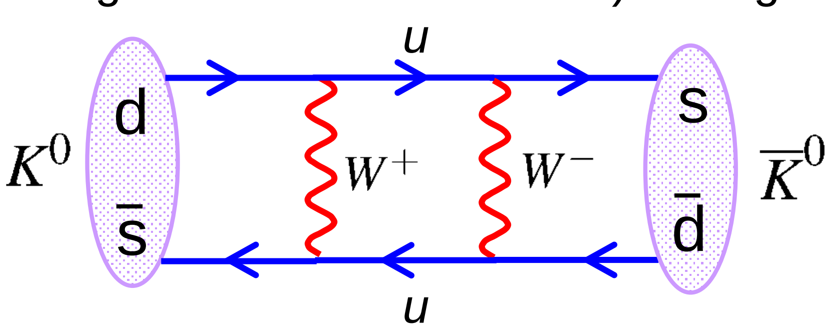
$$K^0 \to \pi^+\pi^- \qquad\qquad \overline{K}^0 \to \pi^+\pi^-$$

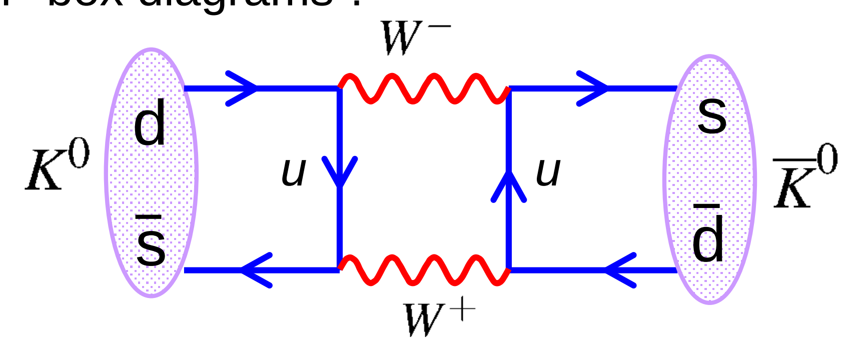
they can mix through the process

$$\boxed{K^0 \to \pi^+\pi^- \to \overline{K}^0 \to \pi^+\pi^- \to K^0}$$

They can mix via a virtual 2 pion state, i.e. $K^0 - \overline{K}^0$ oscillate (mix).
This is a $|\Delta S| = 2$ process, i.e. a higher order weak process (2^{nd} order) or new force.

At the quark level, the K^0 and $\overline{K}^0$ mix through the weak interaction (strangeness is not conserved) via higher order "box diagrams":





Let $+A$ be the amplitude for $\quad K^0 \to \pi^+\pi^-$

then $-A$ (CPT!) is the amplitude for $\quad \overline{K}^0 \to \pi^+\pi^-$

The state $K_1^0 = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) \to 2\pi$ can decay to two pions with amplitude √2 A

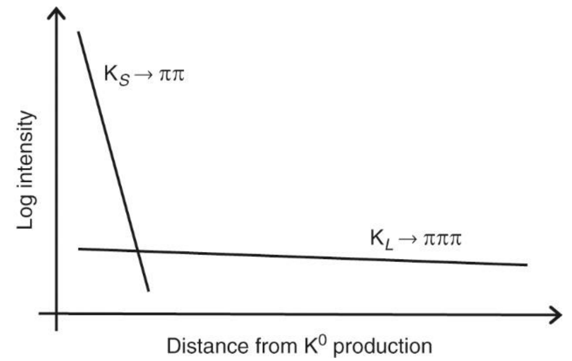
while $K_2^0 = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \cancel{\to 2\pi}$ cannot decay to two pions (amplitude 0)

We can always express the K^0 and $\bar{K}^0$ in terms of K_1^0 and K_2^0

$$\boxed{\begin{aligned} K^0 &= \frac{1}{\sqrt{2}}\left(\left|K_1^0\right\rangle + \left|\bar{K}_2^0\right\rangle\right) \\ \bar{K}^0 &= \frac{1}{\sqrt{2}}\left(\left|K_1^0\right\rangle - \left|\bar{K}_2^0\right\rangle\right) \end{aligned}}$$

Indeed the two states $\boxed{\begin{aligned} K_S^0 &\sim K_1^0 \to 2\pi \\ K_L^0 &\sim K_2^0 \to 3\pi \end{aligned}}$ have been observed experimentally (1961).

Note that K_1^0 and K_2^0 are their own antiparticles.



Which are the particles:

the K^0 and $\bar{K}^0$ produced in strong interactions?

the K_1^0 and K_2^0 that decay?

because of a very different phase space (220 MeV vs 80 MeV) $\tau_{K_1^0} << \tau_{K_2^0}$ also $m_{K_1^0} \neq m_{K_2^0}$

(K_1^0 and K_2^0 have definite lifetime, K^0 and K^0 don't, both are two orthogonal states, …)

K_S^0 Regeneration

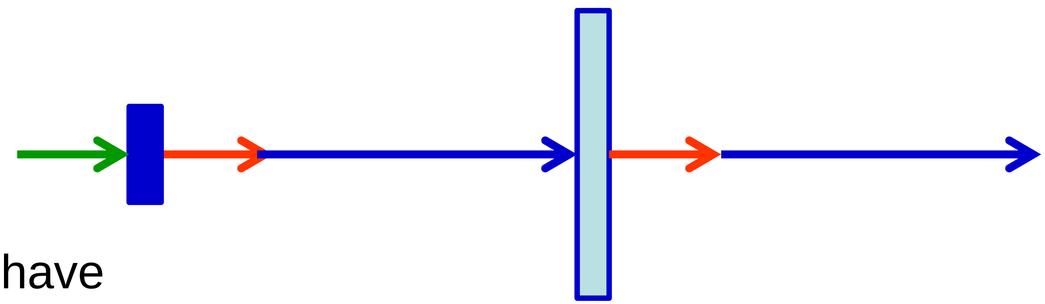
At the production point of a K^0, there is an equal admixture of K_1^0 and K_2^0, the K_1^0 component dies off (decay) quickly and we are left with the K_2^0 only.

However $$\left|K_2^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)$$

and we have a 50% probability to find a $\bar{K}^0$ in the K_2^0 beam after few τ_1.

This can be observed by inserting a target into a K_2^0 beam (Pais and Piccioni 1956)
The K^0 and $\bar{K}^0$ components interact via the strong interactions

$$K^0 p \to K^+ n \qquad K^0 n \to \times$$
$$\bar{K}^0 p \to \pi^+ \Lambda \qquad \bar{K}^0 n \to \pi^- \Lambda$$



The $\bar{K}^0$ components is more absorbed than the K^0 component. At the exit we have

$$\left(f\left|K^0\right\rangle + \bar{f}\left|\bar{K}^0\right\rangle\right) = \frac{f-\bar{f}}{2}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) + \frac{f+\bar{f}}{2}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \qquad \bar{f} < f < 1$$

Since $f \neq \bar{f}$ (different absorption), the K_1^0 state has been regenerated.

Allows also to determine the sign of Δm: $m_L > m_S$.

Matter – Antimatter Oscillations

Strong interaction produces K^0 and $\overline{K^0}$, which are not eigenstates of CP, while P, C, strangeness are globally conserved

How comes that we observe K_S^0 and K_L^0 and not K^0 or $\overline{K^0}$?

What propagates in time is a superposition of K_1^0 and K_2^0 .

Time evolution of a K^0

$$\left|K^0(t=0)\right\rangle = \sqrt{\tfrac{1}{2}}\left(\left|K_1^0\right\rangle + \left|K_2^0\right\rangle\right)$$

controlled by time evolution of K_1^0 and K_2^0

$$\left|K_1^0(t)\right\rangle = \left|K_1^0(t=0)\right\rangle e^{-im_1 t} e^{-\Gamma_1 t/2}$$

$$\left|K_2^0(t)\right\rangle = \left|K_2^0(t=0)\right\rangle e^{-im_2 t} e^{-\Gamma_2 t/2}$$

which give the time evolution of a K^0

$$\left|K^0(t)\right\rangle = \sqrt{\frac{1}{2}}\left(\left|K_1^0\right\rangle e^{-im_1 t} e^{-\Gamma_1 t/2} + \left|K_2^0\right\rangle e^{-im_2 t} e^{-\Gamma_2 t/2}\right)$$

Replace K_1^0 and K_2^0 with K^0 and $\overline{K^0}$ states

$$\left|K^0(t)\right\rangle = \frac{1}{2}\left(\left|K^0 - \bar{K}^0\right\rangle e^{-im_1 t} e^{-\Gamma_1 t/2} + \left|K^0 + \bar{K}^0\right\rangle e^{-im_2 t} e^{-\Gamma_2 t/2}\right)$$

and after rearranging the terms we obtain

$$\boxed{\left|K^0(t)\right\rangle = \frac{1}{2}\left[\left|K^0\right\rangle\left(e^{-im_1 t} e^{-\Gamma_1 t/2} + e^{-im_2 t} e^{-\Gamma_2 t/2}\right) + \left|\bar{K}^0\right\rangle\left(e^{-im_1 t} e^{-\Gamma_1 t/2} - e^{-im_2 t} e^{-\Gamma_2 t/2}\right)\right]}$$

This shows that the K^0 oscillates back and forth between a $\overline{K^0}$ and a K^0.

The probability to find a K^0 at a time *t* in a beam that started as a K^0 is given by

$$\boxed{P\left(K^0(t)\right)=\left|\left\langle K^0 \middle| K^0(t)\right\rangle\right|^2=\frac{1}{4}\left[e^{-\Gamma_1 t}+e^{-\Gamma_2 t}+2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m\cdot t)\right]}$$

Similarly the probability to find a $\overline{K^0}$ in a beam that started as a K^0 is given by

$$\boxed{P\left(\bar{K}^0(t)\right)=\left|\left\langle \bar{K}^0 \middle| K^0(t)\right\rangle\right|^2=\frac{1}{4}\left[e^{-\Gamma_1 t}+e^{-\Gamma_2 t}-2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m\cdot t)\right]}$$

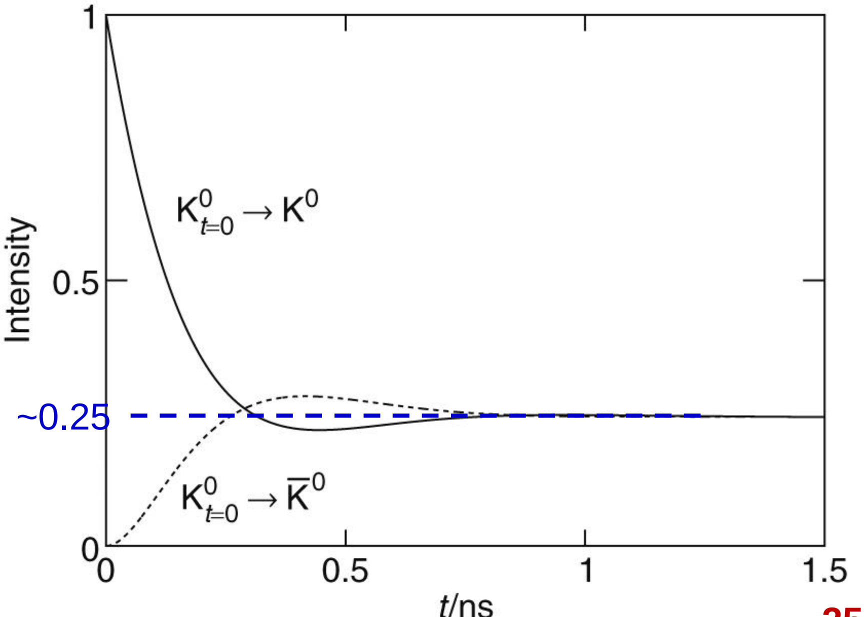
If it were not for the damping factors (decay) the K^0 would oscillate forth and back between a K^0 and a $\overline{K^0}$ with a frequency controlled by Δm ($\Delta m = m_1 - m_2$):

$$T_{osc}=\frac{2\pi\hbar}{\Delta m}\approx 1.2\times 10^{-9}\,\mathrm{s}\Rightarrow \Delta m = 3.484\,\mu\mathrm{eV}$$

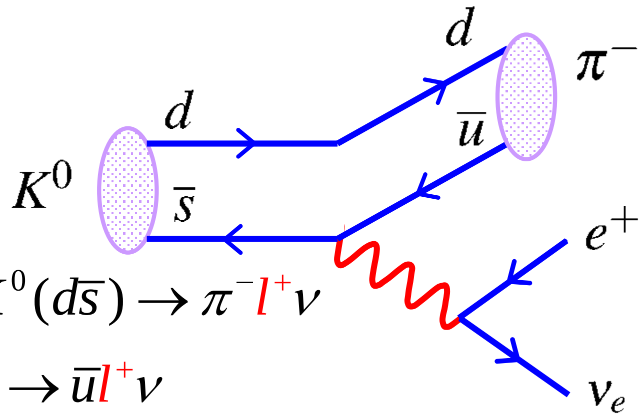
The time *t* can be transformed into a distance travelled by the K^0 knowing its momentum. Since

$$\Gamma_1 = 1/\tau_1 \;\; >> \;\; \Gamma_2 = 1/\tau_2 \quad (\tau_2 >> \tau_1)$$

the oscillation time is relatively long compared to the K_1^0 lifetime and short compared to the K_2^0 lifetime. Consequently do not observe very pronounced oscillations,
after one τ_1 the oscillation pattern dies off.

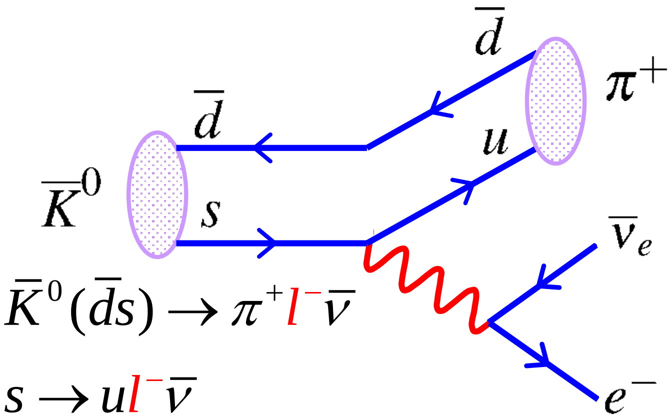


How to distinguish a K^0 from a $\overline{K^0}$ (not K_1^0 from K_2^0)?
Consider the K^0 and K^0 semileptonic decays



$K^0(d\bar{s}) \to \pi^- l^+ \nu$

$\bar{s} \to \bar{u} l^+ \nu$



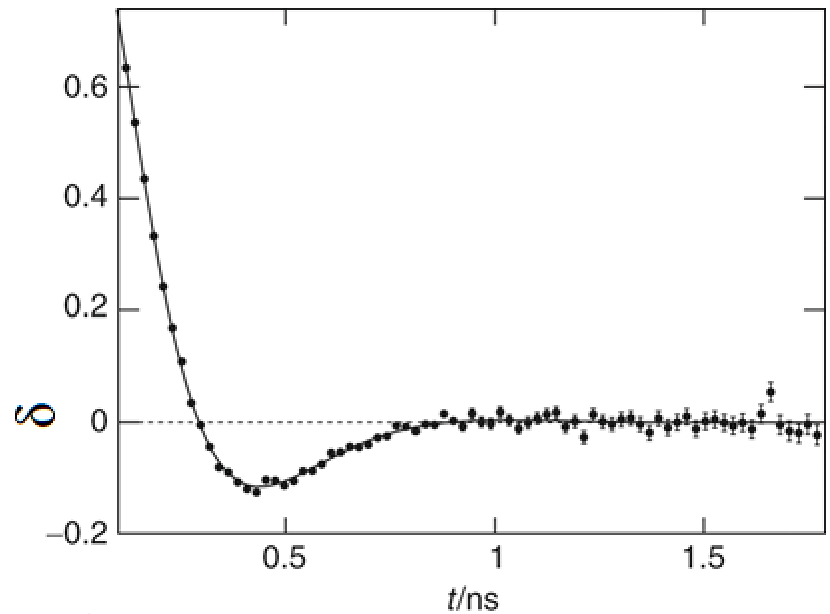
$\bar{K}^0(\bar{d}s) \to \pi^+ l^- \bar{\nu}$

$s \to u l^- \bar{\nu}$

Measure the lepton charge asymmetry

$$\delta(t) = \frac{N^+(t) - N^-(t)}{N^+(t) + N^-(t)} = P\left(K^0(t)\right) - P\left(\bar{K}^0(t)\right)$$

$$\propto e^{-(\Gamma_1+\Gamma_2)t/2} \cos(\Delta m \cdot t) \approx e^{-\Gamma_2 t/2} \cos(\Delta m \cdot t)$$



K_1^0 died off.

In reality, the K_2 state is slightly *contaminated* by K_1^0,
i.e. the content of K^0 and $\overline{K^0}$ are not equal, and $\delta \neq 0$ (even if small deviation)
$\Rightarrow$ CP violation

P, C, and CP of the K^0 System

The strong K^0 eigenstates have

$$P|K^0\rangle = -|K^0\rangle \qquad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$C|K^0\rangle = +|\bar{K}^0\rangle \qquad C|\bar{K}^0\rangle = +|K^0\rangle$$

Not eigenstates of C (not their own antiparticle) and consequently

$$CP|K^0\rangle = -|\bar{K}^0\rangle \qquad CP|\bar{K}^0\rangle = -|K^0\rangle$$

i.e. neither K^0 nor $\bar{K}^0$ are eigenstates of CP.

Let's assume (for now) that CP is conserved in weak decays as in π decays.
The weak force then acts on states with well defined CP properties, i.e. the decaying state must be an eigenstate of CP.

CP eigenstates propagate in time $\rightarrow$ free Hamiltonian, well defined mass, lifetime, etc.

The weak interaction then does not *see* K^0 nor $\bar{K}^0$, but a mix of these two states, which are eigenstates of CP. Consider the simplest mix of K^0 and $\bar{K}^0$:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP|K_1^0\rangle = +1|K_1^0\rangle$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP|K_2^0\rangle = -1|K_2^0\rangle$$

K_1^0 and K_2^0 are eigenstates of CP, i.e. *real particles* as seen by the weak interaction.

K_1^0 and K_2^0 are eigenstates of CP,
i.e. the stationary states of the Hamiltonian describing the $K^0 - \bar{K}^0$ system,
real particles as seen by the weak interaction.
different particles with different lifetimes, mass, and decay modes.

$$K_1^0 \to 2\pi \quad (\pi^+\pi^- \text{ or } \pi^0\pi^0) \qquad CP = +1 \qquad \tau_1 = 0.895\times 10^{-10}\,\text{s} \qquad \bar{m} = 497.614\ \text{MeV}$$

$$K_2^0 \to 3\pi \quad (\pi^+\pi^-\pi^0 \text{ or } \pi^0\pi^0\pi^0) \qquad CP = -1 \qquad \tau_2 = 0.512\times 10^{-7}\,\text{s} \qquad \Delta m = 3.484\ \mu\text{eV}$$

Note $\quad CP\left(\left|\pi^+\pi^-\right\rangle\right) = +1 \quad \text{and} \quad CP\left(\left|\pi^+\pi^-\pi^0\right\rangle\right) = -1$

In 1964 Cronin and Fitch observed that the K_2^0 state sometime decays also to 2π
i.e. 2×10^{-3} times $\Rightarrow$ violation of CP (small but non-zero!)

The physics states are then given by (ε measures the amount of CP violation)

$$\boxed{\begin{aligned} \left|K_1^0\right\rangle &\to \left|K_S^0\right\rangle \equiv \left(\left|K_1^0\right\rangle - \varepsilon\left|K_2^0\right\rangle\right) \Big/ \sqrt{1+|\varepsilon|^2} \\ \left|K_2^0\right\rangle &\to \left|K_L^0\right\rangle \equiv \left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right) \Big/ \sqrt{1+|\varepsilon|^2} \end{aligned}}$$

with

$$|\varepsilon|^2 = \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} \to |\varepsilon| = (2.228 \pm 0.011)\times 10^{-3}, \quad \phi = (43.51 \pm 0.05)^\circ$$

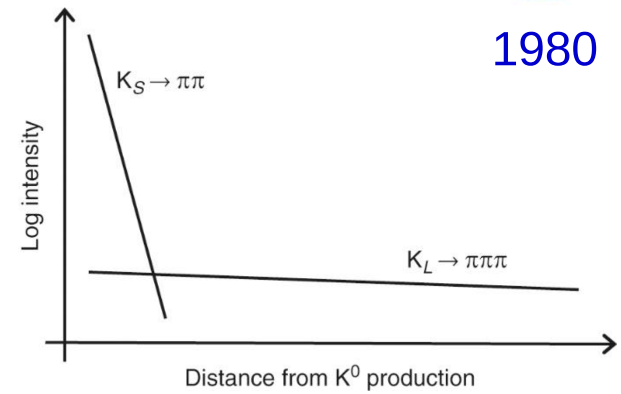
K_S^0: K^0-short, K_L^0: K-long are no longer eigenstates of CP
they contain a a small admixture of the opposite CP state.

The Experiment

1980

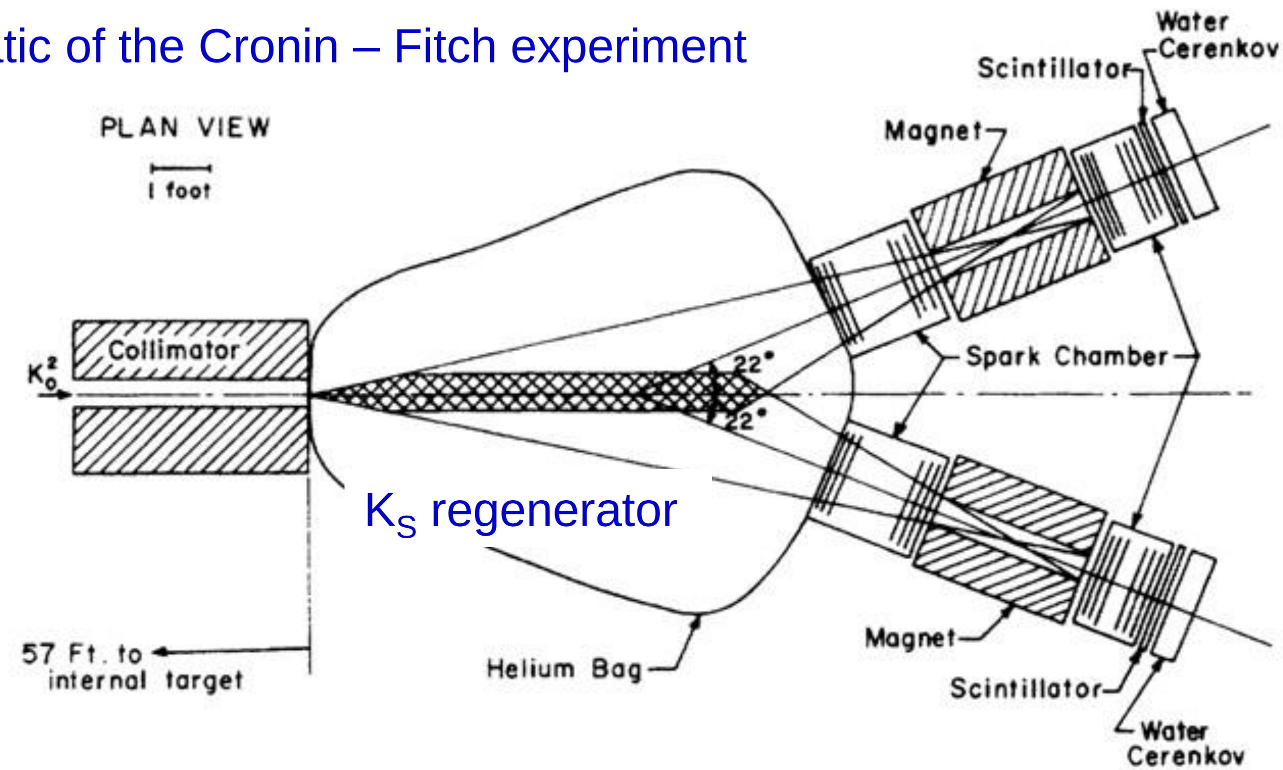
In 1964 Cronin and Fitch observed 23,000 K_2^0 decays,
of which 45 decayed to $\pi^+\pi^-$,
i.e. 2×10^{-3} times $\Rightarrow$ violation of CP

How can we produce a "K_2^0" beam?
Far away from target all K_1^0 have decayed,
the beam contains only K_2^0 (K_L^0)



schematic of the Cronin – Fitch experiment

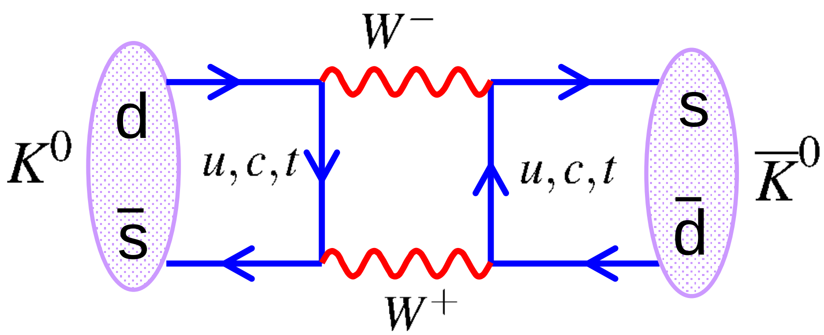
K_L^0 beam

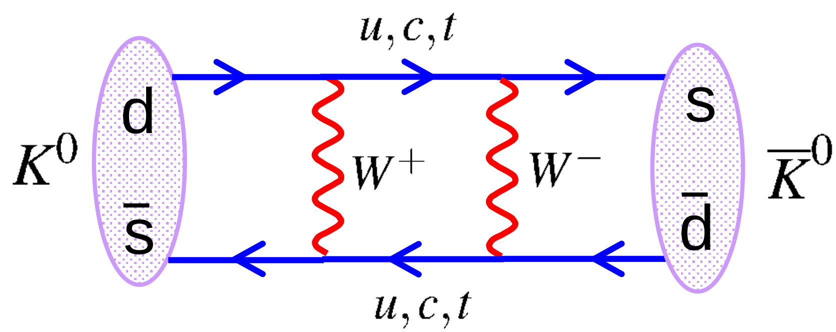


CP Violation and the CKM Matrix

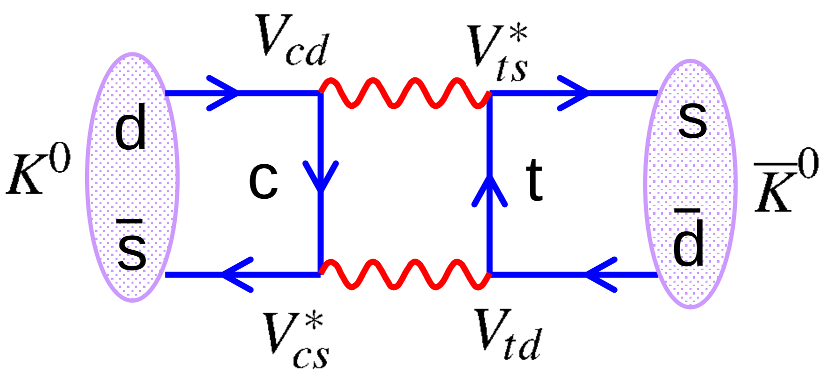
How can we explain $\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \neq \Gamma\left(K^0_{t=0} \to \bar{K}^0\right)$ in terms of the CKM matrix?

The week interaction allows mixing of neutral Kaons through box diagrams (higher order process "G_F^4", $|\Delta S| = 2$)

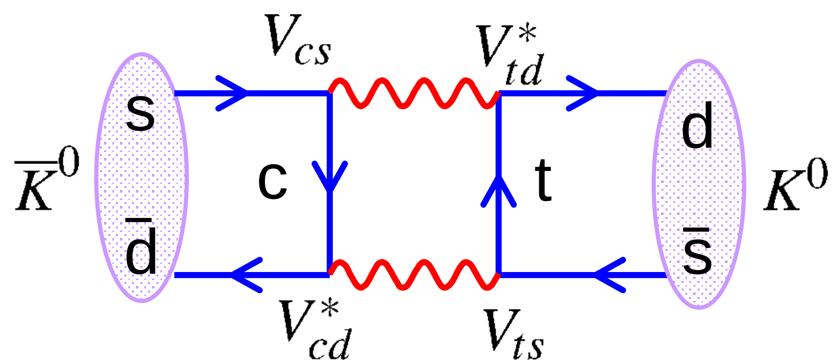




Each *qWq* vertex weighted by the corresponding V_{CKM} element.
We have to sum over all possible quark exchanges in the box, for simplicity let's look at one diagram involving the phase δ_{CP} in the V_{CKM} elements.



$$\boxed{M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*}$$



$$\boxed{M'_{fi} \propto A_{ct}^* V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*}$$

Since V is not hermitian, $\boxed{M_{fi} \neq M_{fi}^*}$ and CP is violated (V contains the phase $e^{i\delta}$)

Time reversal violation leads to the difference in the decay rates

$$\boxed{\Gamma\left(K^0 \to \bar{K}^0\right) - \Gamma\left(\bar{K}^0 \to K^0\right) \propto M_{fi} - M_{fi}^* = 2\Im\left(M_{fi}\right)}$$

The rates can be different only if the CKM matrix has imaginary components, i.e. mixing leads to CP violation with

$$\boxed{|\varepsilon| \propto 2\Im\left(M_{fi}\right)}$$

In the kaon system, adding all terms containing the phase δ_{CP}

$$\boxed{|\varepsilon| \propto A_{ut}\Im\left(V_{ud}V_{us}^*V_{td}V_{ts}^*\right) + A_{ct}\Im\left(V_{cd}V_{cs}^*V_{td}V_{ts}^*\right) + A_{tt}\Im\left(V_{td}V_{ts}^*V_{td}V_{ts}^*\right)}$$

This shows that CP violation is related to the imaginary parts of the CKM matrix.
In the Standard Model, CP violation is associated with the imaginary components of the CKM matrix.

Can distinguish matter from anti-matter.
ET calls home: in a matter dominated region of the Universe, the charge of the most abundant lepton and of the atomic nucleus are the same.

CP Violation

CPT theorem:

under very general assumptions

all physics laws are invariant under simultaneous C, P, and T transformations

in whatever order, provided the local quantum field theory is Lorentz invariant with a hermitian hamiltonian

consequences:

1. particle – antiparticle have same mass, lifetime, decay modes, …
2. magnetic moments same magnitude but opposite sign, opposite charge, …
3. integer spin – Bose-Einstein statistics
4. half integer spin – Fermi statistics

Weak decays violate P and C

$$P\left[\Gamma\left(\pi^+ \to \mu^+ \nu_L\right)\right] = \Gamma\left(\pi^+ \to \mu^+ \nu_R\right) = 0$$

$$C\left[\Gamma\left(\pi^+ \to \mu^+ \nu_L\right)\right] = \Gamma\left(\pi^- \to \mu^- \overline{\nu}_L\right) = 0$$

but CP is conserved, at least in π decays

$$CP\left[\Gamma\left(\pi^+ \to \mu^+ \nu_L\right)\right] = C\left[\Gamma\left(\pi^+ \to \mu^+ \nu_R\right)\right] = \Gamma\left(\pi^- \to \mu^- \overline{\nu}_R\right) = \Gamma\left(\pi^+ \to \mu^+ \nu_L\right)$$

In reality also CP violated at the level of 0.3% in weak interactions (K^0 system, 1964)

CP violated $\rightarrow$ T violated $\rightarrow$ time arrow

1980

CP Violation

CP violation exists, but is a small effect.

Three ways to violate CP:

1) Violation in the wave function (violation in mixing or indirect CP violation, i.e. $|\Delta S| = 2$).
It occurs when the stationary sates of the free Hamiltonian are not CP eigenstates i.e.

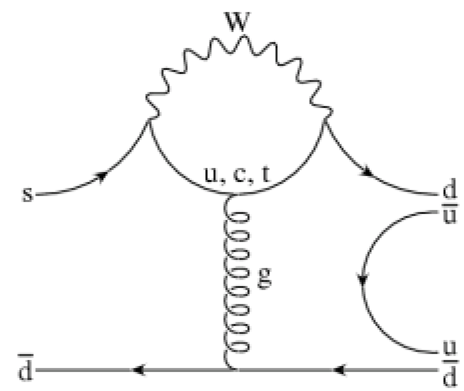
$$\left|K_L^0\right\rangle \equiv \left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right) \Big/ \sqrt{1+|\varepsilon|^2}$$

2) Violation in decays (direct CP violation, i.e. $|\Delta S| = 1$)

$$\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$$

but suppressed relative to CP violating $K^0 - \overline{K}^0$ mixing.
Observed in $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ systems.
Can happen also in the decays of charged particles.
Conserved if

$$H = \underset{i\to f}{M} + \underset{f\to i}{M^\dagger} \quad \Rightarrow \quad \mathrm{M} = M^\dagger$$



"penguin diagram"
direct CP violation

3) Violation in the interference between decays with and without mixing (even if CP is conserved in 1) and 2)).
Observed in the $B^0 - \overline{B}^0$ system as $B^0 \to f$ and $B^0 \to \overline{B}^0 \to f$

CP and Cosmology

Matter over anti-matter abundance in our Universe, Universe is matter dominated
(no indications of anti-matter).

Expect same number of baryons and anti-baryon in the very early universe.

From Big Bang Nucleosynthesis matter / anti-matter asymmetry
(baryon and anti-baryon annihilate in 1 photon)

$$\xi = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every 10^9 anti-baryons there were 10^9 + 1 baryons,
what we see today is the +1 baryon, the rest annihilated producing the photons

How could this happen?

Sakharov conjecture (1967)

To generate this initial asymmetry 3 conditions are required:

1. baryon number violation
   (i.e. proton decay, not observed yet! $\tau_p > 10^{35}$ y)
2. C and CP violation
   (if δ_{CP} in CKM only source, too small to generate the observed asymmetry)
3. departure from thermal equilibrium
   (in thermal equilibrium any reaction is balanced by inverse reaction)

For Next Week

Study the material and prepare / ask questions
Study ch. 12 (sec. 11 to 14) in Halzen & Martin
and / or ch. 14 in Thomson

Do the homeworks

Next week we will study neutrino interactions
have a first look at the lecture notes, you can already have questions
read ch. 12 (sec. 7 to 10) and ch. 13 (sec. 5) in Halzen & Martin
and / or ch. 12 (sec. 2 to 5) in Thomson